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A Finite-difference Modeling of Love Channel Waves in Transversely Isotropic Medium

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ABSTRACT: The present paper deals with numerical modeling of Love channel waves in transversely isotropic elastic medium. First, an explicit finite-difference scheme of second order approximation is formulated with the wave equation of SH particle displacement in transversely isotropic medium. Since it is a heterogeneous formulation, it should enable efficient modeling of complex model structures without additional treatment of the internal boundary matching. With a model of isotropic coal seam embedded in high velocity host rock, seismograms are synthesized and turn out to be essentially identical with published ones of Korn and Stöckl. Next, anisotropic coal seams are investigated. It is found that the horizontal velocity of the seam appears to play a major role of determining the group velocity of Love channel waves. The group velocity increases with the increase of the horizontal velocity or vice versa. However, further study will be needed to exploit fully Love channel waves for the determination of lithology, stratification, fracture in sedimentary rocks, for instance, for hydrocarbon exploration and development.

INTRODUCTION

The existence of the guided modes of seismic waves in low velocity channel has long since been recognized. As early as in 1955, Evision showed that a coal seam can play a role of a guide for seismic energy (Evison, 1955). Subsequently, Evans (1959) identified Love channel waves in his seismograms obtained with physical modeling experiments. But it was Krey in 1963 who laid a firm foundation for channel wave seismics as a new method of seismic survey. He showed theoretically in his classic paper that Love and Rayleigh channel waves can be developed in a low velocity channel sandwitched between higher velocity media (Krey, 1963). Ever since the inception of the basic idea, follow-up research efforts have been rendered to perfect the method. By the end of 1970s, the so-called in-seam seismics was well established as a technique for probing coal seams ahead of a mining face, particularly in the Great Britain (Buchanan et al., 1981) and Germany (Rüter et al., 1979). In recent years, its application has been extended to investigate the continuity of petroleum geological formations (Chon, 1992).

Freystätter (1974) carried out scaled model experiments. Mason did tomographic survey of channel waves in Thoresby colliery (Mason, 1981). As for numerical modeling, Guu (1975), Korn and Stöckl (1981) used finite-difference method, and Su (1976) a hybrid finite-element, finite-difference method. Asten et al. (1984) adopted a finite-element approach. Lagasse and Mason (1975) applied a transfer matrix method to investigate dispersions in multi-layered formations. In Korea, attention has been paid to channel waves only in recent years. Kim et al. (1988, 1990) introduced scaled model technique into Korea. Lee, S.S (1992), one of the present authors, and Lee, S.W. (1993) wrote their M.Sc. theses on Love channel wave problems, using finite-difference method. Through the research efforts of the past decades, seismic channel waves are now well understood.

In this paper, finite-difference modeling is performed to investigate Love channel waves guided in transversely isotropic medium. It is now accepted that transverse isotropy is a standard situation in sedimentary formations (Chi, 1992). And an extensive literature exists in connection with seismic transverse isotropy and its application to petroleum exploration and development. Mostly, however, seismic body waves are in consideration. To our knowledge, the present study may be one of the first efforts solely devoted to channel waves in transversely isotropic media. Coal seams as a transversely isotropic channel are taken here for study.

FINITE-DIFFERENCE FORMULATION OF SH WAVES IN TRANSVERSELY ISOTROPIC MEDIUM

The wave equation of transversely isotropic elastic medium is well known and that for SH particle disp-

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lacement is as follows (Grant and West, 1965a).

$$\rho\frac{\partial^2 v}{\partial t^2}=\mu\frac{\partial^2 v}{\partial x^2}+\nu\frac{\partial^2 v}{\partial z^2} \tag{1}$$

where ρ=density, μ=shear modulus in the horizontal direction (x), ν=shear modulus in the vertical direction (z). SH waves will propagate with a velocity $\beta_{\parallel}=\sqrt{\mu/\rho}$ in the horizontal direction but with $\beta_{\perp}=\sqrt{\nu/\rho}$ in the vertical direction. In an oblique direction, its velocity will lie between these two values.

At a grid point ($m\Delta x$, $n\Delta z$) at the p-th time step, the displacement $v^p_{m,n}$ must satisfy Eq. (1). Second order finite difference approximation of both spatial and temporal differential operators in Eq. (1) yields the following explicit scheme.

$$\begin{aligned} v^{p+1}_{m,n} &= -v^{p\ 1}_{m,n}+2\left\{1-\left(\frac{\beta_{\parallel}\Delta t}{h}\right)^2-\left(\frac{\beta_{\perp}\Delta t}{h}\right)^2\right\}v^p_{mn} \\ &\quad +\left(\frac{\beta_{\parallel}\Delta t}{h}\right)^2\{v^p_{m+1,n}+v^p_{m-1,n}\} \\ &\quad +\left(\frac{\beta_{\perp}\Delta t}{h}\right)^2\{v^p_{m,n+1}+v^p_{m,n-1}\} \end{aligned} \tag{2}$$

where $h=\Delta x=\Delta z$ is the spatial grid interval. And the solution of Eq. (2) is stable if $\beta\Delta t/h$. This difference equation will be reduced exactly to that of Korn and Stöckl if $\beta_{\parallel}=\beta_{\perp}$ (homogeneous medium) (Korn and Stöckl, 1981).

At an internal interface, the boundary conditions of the continuity of stress and displacement are to be matched. At a horizontal interface, $\mu\partial v/\partial x$ (stress) and v (displacement) are continuous. Likewise, $\mu\partial v/\partial z$ and v are continuous at a vertical interface. To make our formulation as general as possible, each grid point can be surrounded by four closed interfaces, i.e., two horizontal and two vertical, which lie half way between the grid point in consideration and its immediate neighboring grid points in both directions (Korn and Stöckl, 1981).

With respect to the grid point ($m\Delta x$, $n\Delta z$), the four neighboring grid points are fictitious while the situation is reserved, i.e., ($m\Delta x$, $n\Delta z$) is fictitious, when the four neighboring points are real as far as the boundary matching is concerned. Now Eq. (2) is written in terms of those four fictitious displacements and the point ($m\Delta x$, $n\Delta z$). The fictitious displacements are to be eliminated with the use of boundary conditions. Somewhat tedious but straightforward calculation results in

$$\begin{aligned} v^{p+1}_{m,n} &= -v^{p\ 1}_{m,n}+2v^p_{m,n} \\ &\quad +2\left(\frac{\beta^{\parallel}_{mn}\Delta t}{h}\right)^2\{M_1(v^p_{m-1,m}-v^p_{mn})+M_2(v^p_{m+1,m}-v^p_{mn})\} \\ &\quad +\left(\frac{\beta^{\perp}_{m,n}\Delta t}{h}\right)^2\{M_3(v^p_{mn-1}-v^p_{mn})+M_4(v^p_{mn+1}-v^p_{mn})\} \end{aligned} \tag{3}$$

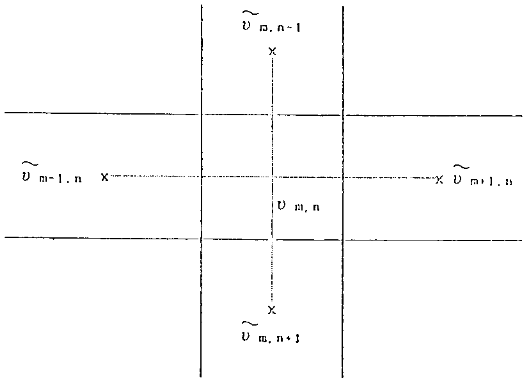


Fig. 1. Arrangement of four fictitious grid points in the neighborhood of a real grid point ($m\Delta x$, $n\Delta z$).

where

$$M_1=\frac{\mu_{m\ 1,n}}{\mu_{m\ 1,n}+\mu_{m,n}}$$

$$M_2=\frac{\mu_{m+1,n}}{\mu_{m+1,n}+\mu_{m,n}}$$

$$M_3=\frac{\nu_{m,n\ 1}}{\nu_{m,n\ 1}+\nu_{m,n}}$$

$$M_4=\frac{\nu_{m,n+1}}{\nu_{m,n+1}+\nu_{m,n}}$$

If $\mu_{m,n}=\nu_{m,n}$, i.e., homogeneous, Eq. (2) becomes indentical with Korn and Stöckl (1981). Eq. (3) is an equation of heterogeneous explicit finite-difference scheme of second order approximation. It should cope even with very complicated model structures without further difficulty. All that is additionally needed is just more computer memory.

Unwanted reflections from artificial boundaries are suppressed in the present study, introducing strongly dissipative medium that surrounds the region of our interest. In this way, of course, the boundary has no physical meaning. But it works and meets our expectation.

MODEL AND PARAMETERS

As shown in Fig. 2, the model chosen for the present study is a hard coal seam of 2 m thickness embedded in otherwise homogeneous elastic host rock. The right end of the seam is truncated by a fault face. A region immediately adjacent to a part of the seam is elastic but there is a dissipative region bey-

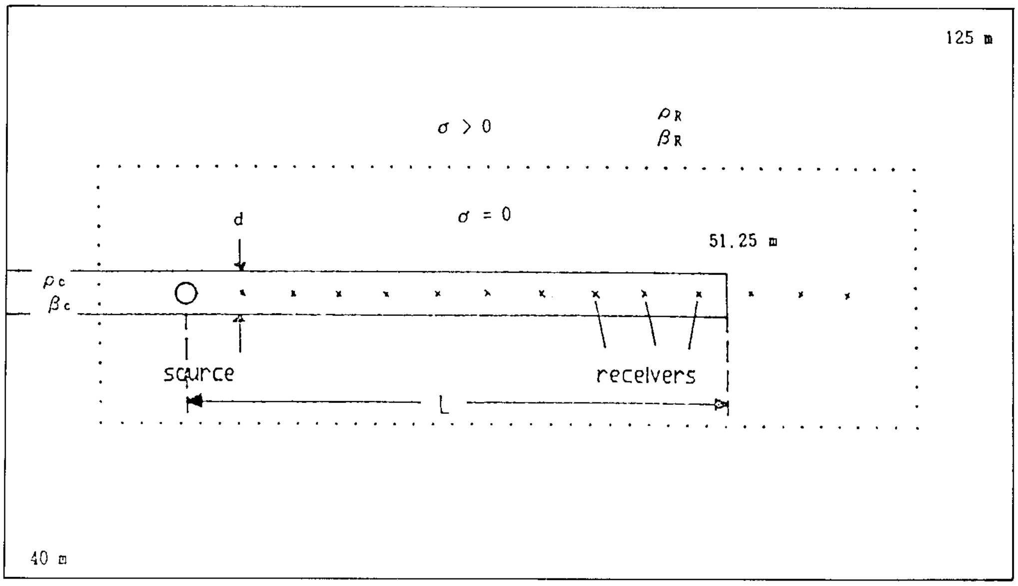


Fig. 2. Geometry of the model. Inside the dotted line lies the elastic host rock and the coal seam.

Table. 1. Physical dimension of the model.

(1) Horizontal size of the model	125 m
(2) Vertical size of the model	40 m
(3) Thickness of the coal seam	2 m
(4) Distance between the source & the fault	51.25 m
(5) Spatial grid interval	0.5 m
(6) Total number of the grid points	250 (horizontal) ×80(vertical)
(7) Time step interval	0.15 msec
(8) Geophone interval	5 m
(9) Source pulse duration	6.25 msec

ond the elastic one. The dissipative region is characterized by a constant σ which is the coefficient of the $\partial v/\partial t$ term artificially added to Eq. (1). In Table 1, the related dimension of the model is summarized.

The seismometers are planted in series at 5 m intervals, right of the source, inside the seam and beyond the truncated face as well. In fact , the model is the same one as Korn & Stöckl. It may be better to do this way to verify our formulation.

The shear wave velocities and densities of the seam and the host rock are 1000 m/sec, 2000 m/sec, and 1.5 g/cm^3, 3.0 g/cm^3 respectively. Later, the velocity anisotropy of the seam is introduced. The high reflection coefficient of 0.6, combined with the lower velocity of the seam, should give rise to channel waves guided in the seam.

The interval of time step is chosen 0.15 msec so that the stability of the numerical solution is ensured with the choice of 0.5 m grid interval. The condition for numerical stability is known as $\beta\Delta t/h \leq 1/\sqrt{2}$ for an explicit second order difference scheme of a wave equations in two space dimensions (Mitchell, 1969).

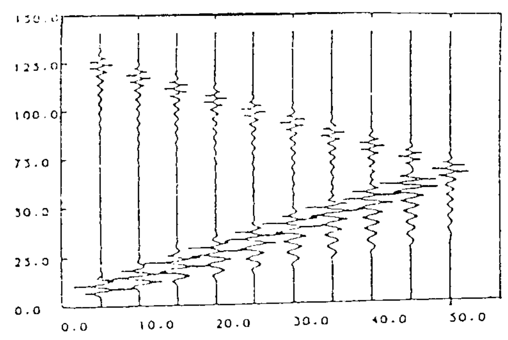
The line source function employed in the present study is

$$S(t)=\sin(2\pi t/T_s)-\frac{1}{2}\sin(4\pi t/T_s) \qquad (4)$$

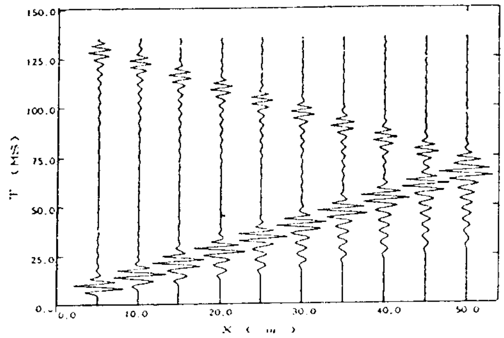
where T_s=pulse duration. The pulse duration as defined here is very nearly the period of the fundamental Fourier components of the source function.

DISCUSSION

In Fig. 3 is shown a comparison of the seismograms obtained with the homogeneous coal seam in the present study to those of Korn & Stöckl. Both are essentially identical even though their seismograms look more spiky. In the context of wave form, our seismograms are more reliable than those of Korn and Stöckl. With the increase of source-geophone off-set distance, degree of the dispersion increases. On the other hand, the amount of energy fairly remains constant and the Airy phase is more or less intact in its shape and amplitude as shown in Fig. 3 (Grant and West, 1965b). The frequency of the Airy phase of the fundamental mode is about 320 Hz. The primary channel waves are followed by reflected channel waves coming from the truncated end of the seam. But their amplitudes have been considerably reduced, suggesting that main loss of energy occurred



(a) Seismograms of Korn and Stöckl (1981).



(b) Seismograms of the present study.

Fig. 3. Comparison of seismograms of the present study with those of M. Korn and Stöckl. Horizontal velocity of coal seam = 1000 m/sec, vertical velocity of coal seam = 1000 m/sec.

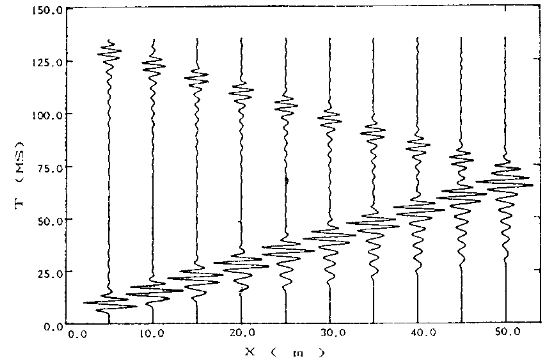


Fig. 4. Seismograms of an isotropic coal seam; horizontal and vertical velocities are all 1000 m/sec.

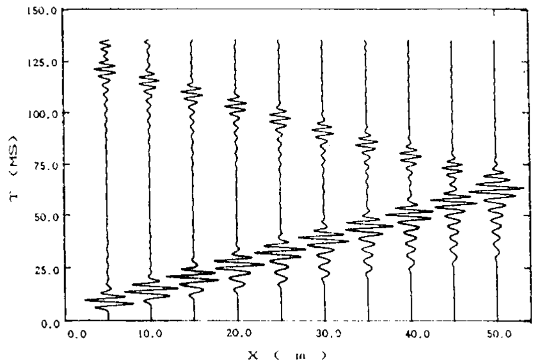


Fig. 5. Seismograms of an anisotropic coal seam; horizontal velocity = 1050 m/sec, vertical velocity = 1000 m/sec.

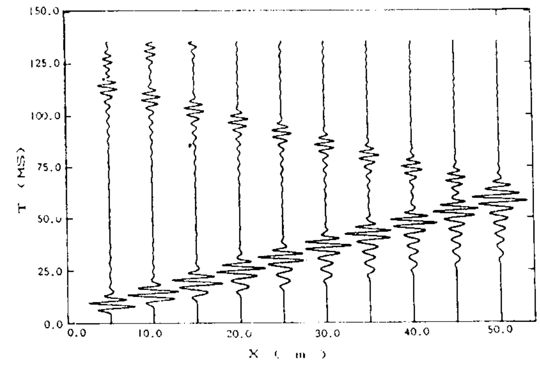


Fig. 6. Seismograms of an anisotropic coal seam; horizontal velocity = 1100 m/sec, vertical velocity = 1000 m/sec.

at that end of the seam. Much energy must have leaked into the high velocity host rock beyond the end face.

Next, seismograms associated with transversely isotropic coal seams are shown in Figs 4, 5, 6, 7, 8, 9, 10. The index of anisotropy, $\alpha = v_\parallel / v_\perp - 1$, is used here. $v_\parallel$ is the velocity in the horizontal direction, $v_\perp$ in the vertical direction. For $v_\perp > v_\parallel$, α is negative, but it becomes positive for $v_\parallel < v_\perp$. It is generally known that $v_\parallel > v_\perp$ for coal seams and many other sedimentary formations. The opposite case looks unreal in the present case. However, this somewhat puzzling situation is also reported in connection with other sedimentary formations. Therefore, both cases are considered in the present study although they are not physically very consistent with coal seams.

For the anisotropy ratio greater than zero, the horizontal velocity of the seam is increased at three steps, i.e., 1050, 1100, 1150 m/sec, while the vertical velocity is held constant at 1000 m/sec. Compared with the isotropic case of Fig. 4, all the three seismograms (Fig. 5, 6, 7) of anisotropic velocity look alike at a first glance. Their shapes and amplitudes remain almost same. But superposition of an anisotropic case with the isotropic one clearly reveals their differences, i.e., their phases differ each other consistently. With the increase of the anisotropy, the group velocity of channel waves increase slightly. Here, the group ve-

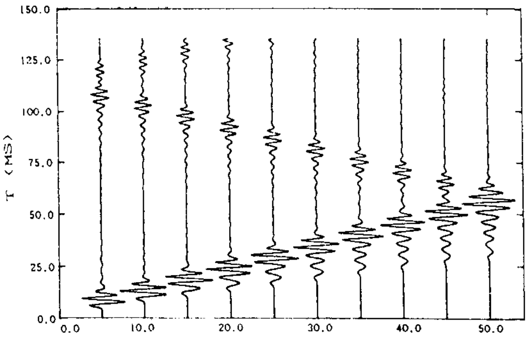


Fig. 7. Seismograms of an anisotropic coal seam; horizontal velocity = 1150 m/sec, vertical velocity = 1000 m/sec.

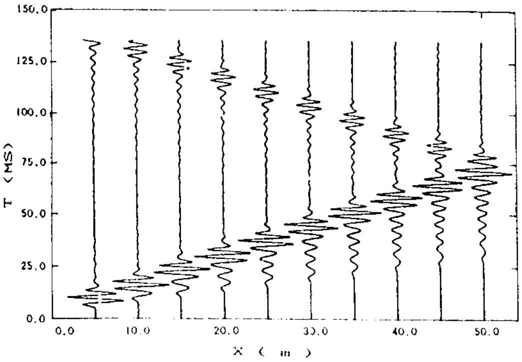


Fig. 8. Seismograms of an anisotropic coal seam; horizontal velocity = 950 m/sec, vertical velocity = 1000 m/sec.

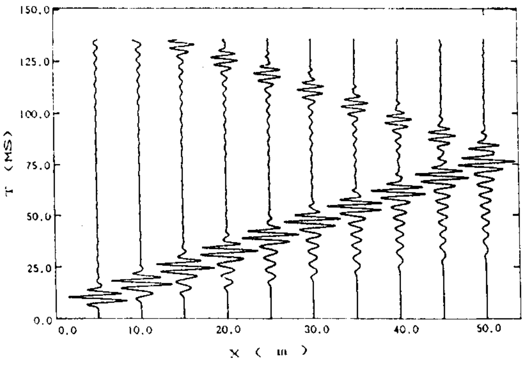


Fig. 9. Seismograms of an anisotropic coal seam; horizontal velocity = 900 m/sec, vertical velocity = 1000 m/sec.

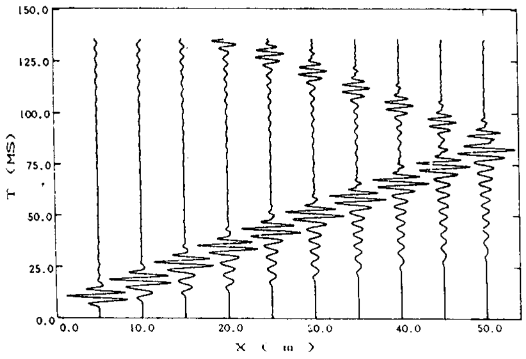


Fig. 10. Seismograms of an anisotropic coal seam; horizontal velocity = 850 m/sec, vertical velocity = 1000 m/sec.

Table 2. Physical parameters of the model.

(1) Shear wave velocity of the seam	1000 m/sec
(2) Shear wave velocity of the host rock	2000 m/sec
(3) Density of the seam	1.5 g/cm^3
(4) Density of the host rock	3.0 g/cm^3

Table 3. Comparison of the group velocity in isotropic medium with those in anisotropic media.

Fig.	Horizontal velocity of coal seam ($V_\parallel$) (m/sec)	Vertical velocity of coal seam ($V_\perp$) (m/sec)	Anisotropy ratio (α)*	Group velocity (m/sec)
Fig. 4	1000	1000	0.000	833
Fig. 5	1050	1000	0.050	888
Fig. 6	1100	1000	0.100	952
Fig. 7	1150	1000	0.150	1006
Fig. 8	950	1000	−0.050	751
Fig. 9	900	1000	−0.100	683
Fig. 10	850	1000	−0.150	627

*The anisotropy ratio $(\alpha)=\frac{V_\parallel}{V_\perp}-1$

locity are measured as that of Airy phases around which a predominent wave packet is formed. Consequently, channel waves are seen to travel faster in an positively anisotropic medium. This fact can also be confirmed better with the reflected channel waves. The third group of arrivals, for instance, those centered at 125 msec of the 10 m offset station, are interpreted as waves reflected at the artificial boundary on the left of the seam. This boundary, relatively near the source, was "transparent" up to the times taken by the second group of arrivals, but it did not work very well after this. Probably, the dissipative medium was not thick enough.

Next, the negative anisotropic cases are considered. The horizontal velocity is decreased at three steps, i.e., 950, 900, 850 m/sec, with the vertical velocity kept constant at 1000 m/sec. Overall pictures look alike (Fig. 8, 9, 10). However, the group velocity tends to decrease with the decrease of the horizontal velocity. This is more clearly seen in reflected waves.

In Table 3 is summarized the anisotropic parameters and the measured group velocities. As analized before, the group velocity of Love channel waves increases (or decreases) with the increase (or decrease) of the horizontal velocity of the seam. It is thus concluded that the horizontal component of the velocity may be a decisive factor to determine the group velocity of channel waves. Apart from this result, however, no more information could be extracted in the present study. To exploit channel waves for the identification, lithology and fracture in sedimentary formations, particularly for hydrocarbon exploration and development, further studies in detail are to be undertaken with more general equation of elasticity.

CONCLUSION

To study Love channel waves in transversely isotropic medium, an explicit heterogeneous finite-difference scheme of second order approximation is newly formulated with the wave equation of SH particle displacement in transversely isotropic medium. The heterogeneous approach should enable to handle complicated model structures automatically almost without paying special attention to boundary matching problem. This can be a great assets especially to deal with conventional SH body wave propagation in transversely isotropic media.

Comparison of the seismograms of the present study with some published ones, i.e., Korn & Stöckl, shows practically identical features. It is our belief that our seismograms are more reliable than they are. Theirs look too spiky.

Next, it is found that the group velocity tends to increase (or decrease) with the increase (or decrease) of the horizontal velocity of the coal seam. It appears therefore evident that the horizontal velocity plays a major role in determining the group velocity of Love channel waves. However, further study in more detail is to be undertaken to exploit channel waves in the determination of lithology, stratification, fracture, etc. in sedimentary formations, for instance, for hydrocarbon exploration and development.

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유한차분식을 이용한 Transverse 異方性 매질내 Love 채널파동 연구

趙東行・李誠洙

요 약: 이 논문은 transverse 異方性 매질내 Love 채널파동 수치해석에 관한 연구를 요약한 것이다. 먼저, 이를 위하여 상기한 매질내 SH파동방정식으로부터 2차 근사 陽유한차분식을 유도하였다. 이 유한차분식은 매 격자점마다 상이한 물성을 정해줄 수 있기 때문에, 복잡한 모델구조를 추가로, 내부 경계조건 처리를 함이 없이, 매우 효율적으로 해석할 수 있을 것이다. 等方性 석탄층에 대한 해석결과, 본 연구에서 작성한 탄성파기록이 기존의 Korn and Stöckl의 것과 본질적으로 동일함을 확인하였다. 다음, 異方性 석탄층에 대한 해석에서는, Love 채널파동의 群속도가 석탄층의 수평방향 속도의 증감에 따라 증감됨을 알 수 있었다. 그러나 현재의 연구단계에선, Love 채널파동을 이용하여 低 속도층의 組成, 層序, 균열등에 관한 정보를 이끌어 낼 수는 없었다. 이러한 정보는 석유탐사 및 개발의 측면에서 중요하기 때문에, 앞으로 이와 관련된 채널파동 연구가 기대된다.