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A New Approach for Forest Management Planning : Fuzzy Multiobjective Linear Programming[1]

Woo, Jong Choon[2]

森林經營計劃을 위한 새로운 接近法 : 퍼지 多目標線型計劃法[1]

禹 鍾 春[2]

ABSTRACT

This paper descbibes a fuzzy multiobjective linear programming, which is a relatively new approach in forestry in solving forest management problems. At first, the fuzzy set theory is explained briefly and the fuzzy linear programming(FLP) and the fuzzy multiobjective linear programming(FMLP) are introduced conceptionally. With the information obtained from the study area in Thailand, a standard linear programming problem is formulated, and optimal solutions(present net worth) are calculated for four groups of timber price by this LP model, respectively. This LP model is reformulated to a fuzzy multiobjective linear programming model to accommodate uncertain timber values and with this FMLP model a compromise solution is attained. Optimal solutions of four objective functions for four timber price groups and the compromise solution are compared and discussed.

Key words : *fuzzy multiobjective linear programming, fuzzy linear programming, fuzzy set theory, MAXMIN approach, forest plantation management.*

要 約

本 論文에서는 人工造林林分經營計劃문제를 해결하기 위하여 林業에서 최근 새롭게 적용되고 있는 퍼지 多目標 線型計劃法을 소개한다. 우선 퍼지 集合理論이 간략하게 설명되고 퍼지 線型計劃法 및 퍼지 다목표 선형계획법이 개념적으로 소개된다. 그 다음 연구대상지역의 자료를 분석하여 標準的 線型計劃모델이 구성되고 이 모델의 컴퓨터 계산에 의해 4개의 木材價格 범위에 대한 最適解(純現在價)가 각각 계산된다. 이 선형계획모델은 불확실한 4개의 목재가격정보를 조정하기 위해서 퍼지 多目標線型計劃모델로 재구성된다. 그리고 이 퍼지 모델의 계산에 의해 折衷最適解가 얻어지게 된다. 4개의 목재가격 범위에 대해 얻어진 4개의 최적해와 하나의 절충최적해가 比較되고 論議된다.

[1] 接受 1994年 1月 19日 Received on Jan. 19, 1994.
[2] 江原大學校 林科大學 森林經營學科 Department of Forest Management, College of Forestry, Kangweon National University, Chuncheon 200-701, Republic of Korea.

INTRODUCTION

Forest planning is made generally with uncer-atinty, because of long planning horizons, variable future timber prices, the incorporation of human subjectivity and so on. Therefore, the optimization models, which can be used to incorporate imprecise information, are preferred to comprehensive planning, particularly in complex planning environments, such as forestry. Several methods have been suggested to deal with imprecision and uncertainty in forest planning. That is, parametric linear programming, probabilistic or stochastic programming and so on. There are few application examples of such a mathematical forest planning in korean forestry.

Woo and Prasomsin(1993) have described the use of linear programming(LP) under uncertainty for forest plantation management in northeast Thailand, with the goal to maximize the expected present net monetary income of timber harvested from the plantation over the planning horizon. Among the various sources of uncertainty affecting the timber management problems, i.e. the future volume yield, the future price, the appropriate discount rate and so on, the future price was considered for LP model under uncertainty. It was investigated that the tree value has been varied from 300 to 700 Baht/m^3(1 US \$=25 Baht) for the merchantable timber. There was few difference in tree value between each age class until age class 10. The tree value was devided to four groups(300-400 : 350 Baht/m^3, 400-500 : 450 Baht/m^3, 500-600 : 550 Bath/m^3, 600-700 : 650 Baht/m^3) subjectively in order to estimate the value of the coefficients for 4 objective functions. Therefore, with one LP model the 4 optimal solutions had to be calculated respectively. There are here weak aspects in LP model. That is, although the tree value was described with interval values, one certain, deterministic value had to be used in LP model. The coefficients of 4 objective functions were imprecise or uncertain, because the tree value was devided randomly. Therefore, the inherently deterministic nature of LP models and the use of precise coefficients are mainly criticized. The coefficients or parameters in conventional LP models are assumed to be known with certainty. However, in many real world forest planning problems it has often to be dealt with insufficient or imperfect information. Therefore, to enhance model utility, it is necessary to be able to incorporate imprecise or uncertain information into the model(Mendoza et al. 1993).

Recently a fuzzy programming, which is applied with fuzzy set theory, is used to solve the problem under uncertainty. Hof et al.(1986) investigated an alternative approach, i.e. MAXMIN approach, to scheduling timber harvests when nondeclining yield is desired. The background for this approach is found in the literature on fuzzy set and fuzzy goal programming. Mendoza and Sprouse(1989) have described two modeling approachs, generation of alternatives, and evaluation and prioritization of these alternatives. The methods for generating alternative solutions adopts a fuzzy approach. Pickens and Hof(1991) documented a practical application of fuzzy goal programming in forestry, which is extremely difficult to solve using standard linear programming codes on mainframe computers. Yang et al.(1991) have described the application of fuzzy goal programming to forest management planning under a fuzzy environment. Bare et al.(1993) have described a fuzzy approach to natural resource management from a regional perspective. Mendoza et al.(1993) have described the use of fuzzy multiple objective linear programming(FMOLP) in forest planning where imprecise objective function coefficients are present.

The purpose of this study is to develop a fuzzy multiobjective linear programming model for the forest plantation management planning of Thailand that can accommodate the imprecise coefficients of objective functions. First, fuzzy set theory is introduced for the better understanding of a fuzzy approach, and a fuzzy linear programming and a fuzzy multiobjective linear programming is described conceptionally. Secondly, LP model is introduced for the forest plantation management planning as Woo et al.(1993) have developed. And then, with this LP model a FMLP model is developed to accommodate uncertain timber value. For this model the source of the imprecision is the timber price, as explained above, there are four groups of timber price. From the optimal solutions of 4 objective functions the interval values are estimated by the method of pay-off. With this information a fuzzy multiobjective linear programming model is operated. At the end the results calculated are analyzed and discussed.

FUZZY SET THEORY

A fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Because of that, the concept of fuzziness has generally been associated with complexity, vagueness, ambiguity, and imprecision. Fuzziness has also been interpreted either in the context of imprecise numbers(i.e. coefficients or parameters), or in the functional relationships, such as in the objective function or constraints. Now, let $X=\{x\}$ denote a collection of objects denoted generically by x. A fuzzy set A in X, then, is a set of ordered pairs

$$A=\{(x,\ \mu_A(x))\},\ x\in X \qquad (1)$$

where $\mu_A(x)$ is called the membership function or grade of membership, also degree of compatibility or degree of truth, of x in A which maps X to the membership space M. The range of the membership function is a subset of the non-negative real numbers in the interval [0, 1] whose supremum is finite. When M contains only two points, 0 and 1, A is nonfuzzy and its membership function becomes identical with a characteristic function of a nonfuzzy set (Zimmermann, 1983).

The optimal solution to a linear programming problem is a solution which satisfies both objective functions and constraints, therefore, a solution to a fuzzy decision problem, i.e. a decision problem in which objectives as well as constraints are represented by fuzzy set, has to belong to all fuzzy sets involved. The space of these solutions, which will be called decision, is again a fuzzy set. This set is ordered by its membership function. The highest degree of membership in this set is reached for the fuzzy set decision.

The logical "and" corresponds to the set-theoretic intersection. The set of solutions in the fuzzy set decision is therefore defined as the intersection of all fuzzy sets representing constraints or objective functions. The intersection of fuzzy sets by its membership function $m_D(x)$ is defined as follows(Zimmermann, 1983) :

$$\mu_D(x)=\min\{\mu_i(x)\} \qquad (2)$$

when $\mu_i(x)$ are the membership functions of all fuzzy sets involved.

The maximizing decision is then defined as follows :

$$\max_x \mu_D(x)=\max_x \min_i\{\mu_i(x)\} \qquad (3)$$

FUZZY LINEAR PROGRAMMING

When the system and fuzzy coefficients are linear, it is called a fuzzy linear programming(FLP) problem. Under the fuzzy circumstances, the conventional linear programming problem can be described as follows :

$$\begin{aligned}&\text{Find } x \text{ such that}\\ &C^t x \lesssim Z_o\\ &Ax \lesssim b_o \qquad (4)\\ &x \geq 0\end{aligned}$$

The symbol $\lesssim$ is a fuzzy inequality which represents essentially smaller than or equal to. Z_o is an aspiration level for the objective function and b_o are tolerance levels for the constraints. Since the objectives and constraints are expressed by inequalities, these can be consolidated and rewritten as follows :

$$\begin{aligned}&\text{Find } x \text{ such that}\\ &Bx \lesssim b \qquad (5)\\ &x \geq 0\end{aligned}$$

The ith inequality about b_i or less is defined by the following membership functions, if the constraints are linear :

$$\mu_i([Bx]_i)=\begin{cases}1 & \text{for } [Bx]_i \leq b_i\\ 1-([Bx]_i-b_i)/d_i & \text{for } b_i \leq [Bx]_i \leq b_i+d_i\\ 0 & \text{for } [Bx]_i \geq b_i+d_i\end{cases} \qquad (6)$$

where $[Bx]_i$ is the ith element of the vector, μ_i the membership function of the ith inequality, and d_i the maximum possible value for the right-hand side of the inequality.

The problem of the maximized decision for equation (3) is finding x such that

$$\max_{x\geq 0} \min_i \{\mu_i([Bx]_i)\} \qquad (7)$$

If it is normalized with $b_i' = b_i/d_i$, $[B'x]_i = [Bx]_i/d_i$ and considered that the constraints are linear, problem (7) comes out as

$$\max_{x \geq 0} \min_i \{b_i' - [B'x]_i\} \quad (8)$$

Problem (8) turns out to be the following standard LP problem :

$$\text{maximize } \lambda$$
$$\text{subject to } \lambda \leq b_i' - [B'x]_i \quad (9)$$
$$x, \lambda \geq 0$$

In this information, the solutions to fuzzy LP problems can be obtained using standard LP.

Fuzzy Multiobjective Linear Programming

If a problem contains crisp constraints or objective functions with crisp aspiration levels, these can be directly added in their original form to the equivalent crisp model (9).

For example, Mendoza et al. (1993) have developed fuzzy multiple objective linear programming (FMOLP) in forest planning where imprecise objective function coefficients are present. That is, consider a multiple objective problem :

$$\text{maximize } Z_i = C_i x$$
$$\text{subject to } Ax \leq b_0 \quad (10)$$
$$x \geq 0$$

If the objective functions are more loosely defined and have imprecise coefficients, the coefficients could be represented by interval values instead of exact values. The membership functions of the objectives can be defined like the equation (6) as follows :

$$\mu_i(x) = \begin{cases} 0 & \text{if } Z_i(x) \leq f_{1i} \\ (Z_i(x) - f_{1i})/(f_{0i} - f_{1i}) & \text{if } f_{1i} \leq Z_i(x) \leq f_{0i} \\ 1 & \text{if } f_{0i} \leq Z_i(x) \end{cases} \quad (11)$$

where f_{0i} is the optimal or most desirable value for objective i, and f_{1i} is the least desirable or tolerant value for objective i.

Now, the problem is to simultaneously satisfy all objective functions represented by their corresponding membership functions. For this problem the multiobjective linear programming model (10) can be formulated like the FLP model (9) as follows :

$$\text{maximize } \lambda$$
$$\text{subject to } \lambda \leq (Z_i(x) - f_{1i})/(f_{0i} - f_{1i}) \quad (12)$$
$$Ax \leq b_0$$
$$x, \lambda \geq 0$$

This formulation (12), so called FMLP model, follows the MAXMIN approach where the objective is to find a solution that yields the maximum membership function value, λ, which satisfies the constraints described in formulation (12). That is, λ is the highest minimum degree of satisfaction considering all objectives and their respective desirable limits denoted by f_{0i} and f_{1i}.

A CASE STUDY

Study area

To demonstrate the role of a fuzzy multiobjective linear programming (FMLP) in the process of plantation planning, the plantations of FIO in northeast Thailand were selected to serve as the basis for a case study. There are now 35 plantations in serveral provinces with area of 47,547.13 rais (7,607.54ha) for non-productive area and 93,547.25 rais (14,967.56 ha) for productive area. The main species are *Eucalyptus camaldulensis*, *Tectona grandis*, *Pterocarcus marcrocarpus*. For the productive area, *E. camaldulensis* had been planted mostly. It occupies 66,164 rais (10,586ha), or 71 percent of the total productive area as a pure plantation (Table 1). Therefore, this study is only concerned with *E. camaldulensis* plantations because they are used mostly to provide the needed amount of timber to various forest industries.

Linear programming model

Woo et al. (1993) have developed one linear programming (LP) model belonging to the "Model I"

Table 1. Area and volume distribution for each age class of Eucalypt (*E. camaldulensis*) plantations in northeast Thailand.

Age class (yrs.)	Area (rai)*	Area (ha) (%)	Volume (m^3) (%)
1 (1- 3)	6,806	1,089 (10.3)	-
2 (4- 6)	10,185	1,630 (15.4)	29,148 (6.5)
3 (7- 9)	22,822	3,651 (34.5)	147,320 (32.9)
4 (10-12)	20,289	3,246 (30.7)	196,997 (44.0)
5 (13-15)	6,062	970 (9.1)	73,941 (16.6)
Total	66,164	10,586 (100)	447,406 (100)

* 1ha = 6.25 rais.

group in order to demonstrate the role of linear programming under uncertainty in the process of forest plantation planning. For this LP model, it is assumed that each space of 5 age classes for Eucalypt plantations is 3 years, there are 5 planning periods(a 15-year planning horizon), rotation age ranges from 5 to 15 years without thinning and after harvesting the regeneration is immediately followed by planting. Any action taken should occur at the beginning of each period. The merchantable age is beginning from 4 years old(the second age class). Therefore, the harvesting by clear cutting is to be made from the second age class. There is a minimum of one period between regeneration harvests, that is, it takes only one period for a new stand to reach merchantability. To use Model I in solving this problem, it is to be formulated an activity for each possible management prescription of regeneration harvests that can occur over the next 5 periods in each age class. There are 13 such management prescriptions, including the option of not harvesting at all over the 5 periods. It was also investigated that the tree value has been varied. However, there was few difference in tree value between each age class until age class 10(Table 2). Another assumptions are the same as Woo et al. (1993) have described.

As required from Forest Industry Organization (FIO) of Thailand the objective function was formulated as one of maximizing the present net worth of timber produced from the forest plantations under uncertainty of future timber price during the planning horizon. With the data of FIO the LP model can be formulated as follows:

$$\text{maximize } Z=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\,x_{ij}$$

subject to

$$\text{(a)}\ \sum_{j=1}^{n} x_{ij}=A_i$$

$$\text{(b)}\ (1-\alpha)\sum_{i=1}^{m}\sum_{j=1}^{n} V_{ijk}x_{ij}-\sum_{i=1}^{m}\sum_{j=1}^{n} V_{ij(k+1)}x_{ij}\leq 0 \qquad (13)$$

$$\text{(c)}\ (1+\beta)\sum_{i=1}^{m}\sum_{j=1}^{n} V_{ijk}x_{ij}-\sum_{i=1}^{m}\sum_{j=1}^{n} V_{ij(k+1)}x_{ij}\leq 0$$

$$\text{(d)}\ V^{t}_{ijH}x_{ij}\geq E^{t}$$

$$\text{(e)}\ \sum_{i=1}^{m} x_{ij}=A_j$$

$$\text{(f)}\ \sum_{i=1}^{m} x_{ij}=0$$

$$\text{(g)}\ x_{ij}\geq 0$$

The coefficients(C_{ij}) in objective function represent the discounted net value over the planning horizon, of each area of age class i assigned to management prescription j. The C_{ij} is computed as follows:

$$C_{ij}=\sum_{k=1}^{5}\frac{P_{ijk}V_{ijk}-C_p(1+R)^{A_{ijk}}}{(1+R)^{(k-1)Y}}+\frac{E_{ijH}-C_p(1+R)^{A_{ijk}}}{(1+R)^{HY}}$$

where x_{ij} = area of age class i assigned to management prescription j

i = number of age classes (i = 1, ⋯, 5)

j = number of prescriptions (j = 1, ⋯, 13)

k = number of periods in planning horizon (k = 1, ⋯, 5)

P_{ijk} = price per unit volume of timber harvested at beginning of period k from age class i under management prescription j (for this study, P_{ijk} = midpoint of each age class: 300-400, 400-500, 500-600, and 600-700 Baht/m³)

V_{ijk} = volume per unit area harvested at beginning of period k from age class i under

Table 2. Tree value and volume for each age class(3 years) of Eucalypt(*E. camaldulensis*) plantations.

Age class (years)	Value* (Baht/m³)	Volume (m³/rai)	Volume (m³/ha)
	unmerchantable		
1(1-3)	0	0	0
	merchantable		
2(4- 6)	300-700	6.04	37.8
3(7- 9)	300-700	9.79	61.2
4(10-12)	300-700	13.54	84.6
5(13-15)	300-700	17.30	108.1
6(16-18)	300-700	21.05	131.6
7(19-21)	300-700	24.80	155.0
8(22-24)	300-700	28.56	178.5
9(25-27)	300-700	32.31	201.9
10(28-30)	300-700	36.07	225.4

* 1 US$ = 25 Baht.

C_p = cost per unit area of establishing forest plantations by planting (for this study C_p = 14,750 Baht/ha)

A_{ijk} = age of stand i at the beginning of period k, in years, when it is managed under prescription j

H = total number of periods (H = 5)

R = discount rate (R = 0.08)

Y = number of years in each planning period (Y = 3)

E_{ijk} = net value per unit area of inventory remaining in age class i under management prescription j at the end of the planning horizon ($E_{ijH} = P_{ijH} V_{ijH}$)

The equality function (a) is an area contraint. The area of each age class (A_i), which are cut by any management prescription in each planning period, should be identical with areas of inventory investigated (Table 1).

The two inequality functions (b) and (c) are harvest flow constraints. The forest landowner, FIO, does not want to the great fluctuation in harvest levels from period to period. For that reason the harvest flow constraints are added in LP model in order to restrict the fluctuation. In the function (b) α is the maximum fractional reduction permitted in harvest level from period to period and in the function (c) β is the maximum fractional increase permitted in harvest level from period to period. For our LP model α is assumed to be 0.2 and β to be 0.2

The inequality function (d) is a ending inventory constraint. For the sustainable management of the forest plantations a reasonable level of merchantable ending inventory would be needed at the end of the planning horizon. That is, a certain amount of merchantable timber should be left standing at least for the next planning horizon. V^t_{ijH} is the volume per unit area of timber left as merchantable ending inventory in age class i under management prescription j. E^t is the level of merchantable ending inventory required. For this problem, E^t is assumed to be at least 20 percent of the merchantable inventory at the beginning of the next period 1, that is, 447,460m^3 $*$0.2 = 89,481m^3.

The equality function (e) is a regulation constraint. In order to produce the stable sustained yield the distribution of each age class should be regulated from period to period. Through the regulation the forest plantation could reach a fully regulated condition at the end of the planning horizon like a normal forest.

To produce a regulated forest consistent with a 15-year planning horizon, the total area of 10,586 ha must be distributed equally among the 5 age classes at the end of period 5 (A_j). Therefore, each of age classes have to occupy 10,586/5 = 2117.2ha. The constraint (f) means that any action of cutting in the management prescription 1 does not occur until the end of period 5. The equation (g) is the non-negativity constraint.

Fuzzy multiobjective linear programming model

As explained in introduction, the tree value was devided to four groups subjectively in order to estimate the value of the coefficients for four objective functions. The present net worth were optimized by the LP model (13) with 4 groups of the tree value, respectively. The LP model (13) could be reformulated with four objective functions following equation (10). This is called a multiple objective linear programming model, and can be thus formulated using the equation (10) as follows :

$$\begin{aligned}&\text{maximize } Z_1 = C_1x \cdots \text{(case of 350Baht/m}^3\text{)}\\&\text{maximize } Z_2 = C_2x \cdots \text{(case of 450Baht/m}^3\text{)}\\&\text{maximize } Z_3 = C_3x \cdots \text{(case of 550Baht/m}^3\text{)}\\&\text{maximize } Z_4 = C_4x \cdots \text{(case of 650Baht/m}^3\text{)}\\&\text{subject to } Ax \le b_o\\&\qquad x \ge 0\end{aligned} \tag{14}$$

Using the membership function (11) and following FMLP model (12), the FMLP model for this forest plantation problem is formulated as MAXMIN problem described below :

$$\begin{aligned}&\text{maximize } \lambda\\&\text{subject to } C_1x - \lambda(f_{01} - f_{11}) \ge f_{11}\\&\qquad C_2x - \lambda(f_{02} - f_{12}) \ge f_{12}\\&\qquad C_3x - \lambda(f_{03} - f_{13}) \ge f_{13}\\&\qquad C_4x - \lambda(f_{04} - f_{14}) \ge f_{14}\\&\qquad Ax \le b_o\\&\qquad x > 0\end{aligned} \tag{15}$$

The value of f_{01} and f_{11} have to be known to find a solution using this FMLP formulation (15). These values

Table 3. The values of f_{oi} and f_{ii} calculated by pay-off method. (unit : 1000 Baht)

objective value	max Z_1	max Z_2	max Z_3	max Z_4	f_{oi}	f_{ii}	$f_{oi}-f_{ii}$
Z_1	-91, 226	-82, 819	-109, 034	-127, 252	-82, 819	-127, 252	44, 432
Z_2	-406	738	-2, 857	-17, 623	738	-17, 623	18, 361
Z_3	88, 131	95, 059	103, 322	100, 143	103, 322	88, 131	15, 191
Z_4	181, 236	189, 174	253, 524	213, 696	253, 524	181, 236	72, 288

may be specified by the decision maker, or some benchmark information may be used if available. Otherwise, these values can be computationally derived using a pay-off table as illustrated in Table 3(Mendoza et al. 1993). Introducing the method of the pay-off calculation, which Mendoza et al. used, let f_k, $k=1, 2, 3, 4$, be the feasible ideal values for the following four LP problems :

$$\max(\min)\ f_k(x)=C_k x\ (k=1, 2, 3, 4)$$
$$\text{subject to } Ax \leq b_o \qquad (16)$$
$$x \geq 0$$

In Table 3, Z_{ij} is the value of the *i*th objective function when the *j*th objective is optimized.

$$f_{ii}=\min\{Z_{ij}\}\ \text{if } i=1, 2, 3, 4,\ j=1, 2, 3, 4$$
$$f_{oi}=\max\{Z_{ij}\}\ \text{if } i=1, 2, 3, 4,\ j=1, 2, 3, 4$$

RESULTS AND DISCUSSIONS

By the fuzzy multiobjective linear programming model a compromise solution could be attained with the value $\lambda=0.386$, which is the highest degree (between 0 and 1) that the desirable levels can be met simultaneously. If the actual objective function values and the desirable and tolerable levels shown in Table 3 and Table 4 are compared, the membership functions for each objective can be calculated as $Z_1=0.415$, $Z_2=0.569$, $Z_3=0.990$ and $Z_4=0.386$. These membership functions mean the measures of the degree of satisfaction of any solution. Therefore, through the equation (15) different membership function values(degree of satisfaction) were attained for each objective. For the objective Z_3 the highest membership function value $\lambda=0.990$ was attained and for the objective Z_4 the lowest membership function value $\lambda=0.386$ was attained. Now, the problem is to choose the "best" compromise solution considering all membership function values of each objective. The FMLP model searches for a solution that yields the highest minimum membership function value. This means a compromise solution where the four objectives are at a minimum overall degree of satisfaction equal to λ.

Table 4. Comparison of four objective values calculated by LP model(unfuzzy) respectively and the compromise solutions by FMLP model(fuzzy). (unit : 1000 Baht)

objective value	Z_1	Z_2	Z_3	Z_4
unfuzzy	-91, 226	738	103, 322	213, 696
fuzzy	-108, 820	-7, 177	103, 173	209, 169

For this case study the minimum degree of satisfaction is 0.386. That is, the fourth objective only has the degree of satisfaction equal to 0.386. Actually, the objective of LP model was to maximize the present net worth(PNW) under uncertainty for four cases of timber values, 350 Baht/m^3, 450 Baht/m^3, 550 Baht/m^3 and 650 Baht/m^3, respectively. By LP model the optimal solutions had to be calculated for four cases respectively, although there was few difference among each age class(Table 2). But, through FMLP model only one compromise solution could be attained for this PNW problem.

Table 5 shows decision variables used for optimal solutions by the objectives Z_1(9 : the number of decision variables), Z_2(11), Z_3(11), Z_4(10), and for the compromise solution by FMLP model(12). It appears here that more decision variables were used for the compromise solution by FMLP model than by LP model.

Woo et al. (1993) described LP model under uncertainty in order to solve the uncertain situations of timber prices. For each age class there were four different subjective probabilities. With the application of different subjective probabilities the uncertain degree of PNW could be reduced. In fuzzy problems also the uncertain or imprecise degree of PNW in this study could be reduced through ranking or prioritization of objectives like goal programming. For such a problem, the methods of combining the membership functions in formulation (15)

Table 5. Decision variables and the corresponding harvest areas(ha) used for 4 optimal solutions by LP model and the compromise solution by FMLP model.

unfuzzy				fuzzy
Z_1	Z_2	Z_3	Z_4	compromise solution
$x_{112}=1089.0$	$x_{112}=1089.0$	$x_{13}=444.0$	$x_{16}=1089.0$	$x_{13}=354.4$
$x_{22}=1028.2$	$x_{23}=1630.0$	$x_{19}=645.0$	$x_{24}=583.4$	$x_{19}=734.6$
$x_{23}=601.8$	$x_{33}=487.2$	$x_{25}=1630.0$	$x_{25}=1046.6$	$x_{24}=23.7$
$x_{33}=1515.4$	$x_{34}=661.0$	$x_{35}=61.6$	$x_{37}=1533.8$	$x_{25}=1606.3$
$x_{35}=2117.2$	$x_{35}=2117.2$	$x_{37}=1472.1$	$x_{33}=2117.2$	$x_{36}=135.9$
$x_{38}=18.4$	$x_{37}=284.9$	$x_{38}=2117.2$	$x_{410}=1016.3$	$x_{37}=1358.9$
$x_{48}=2098.8$	$x_{38}=100.7$	$x_{411}=1544.4$	$x_{411}=112.5$	$x_{38}=2117.2$
$x_{413}=1147.2$	$x_{48}=2016.5$	$x_{415}=1701.6$	$x_{413}=2117.2$	$x_{313}=39.0$
$x_{513}=970.0$	$x_{413}=1229.5$	$x_{510}=425.6$	$x_{510}=54.3$	$x_{410}=510.9$
	$x_{512}=82.3$	$x_{511}=128.9$	$x_{511}=915.7$	$x_{411}=1626.9$
	$x_{513}=887.7$	$x_{513}=415.6$		$x_{413}=1108.2$
				$x_{513}=970.0$
(9)	(11)	(11)	(10)	(12)
10586	10586	10586	10586	10586

could be used.

CONCLUSIONS

To demonsrate the role of fuzzy multiple objective linear programming in the process of forest plantation planning one LP model was at first developed. The objective function was formulated in terms of maximizing the present net worth(PNW) of timber produced from the forest plantations under uncertainty of future timber price during the planning horizon, as required from Forest Industry Organization(FIO) of Thailand. The tree value was devided to four groups subjectively in order to estimate the value of the objective function coefficients. The present net worth should be optimized four times by LP model respectively, because LP model has only one objective function. Therefore, four optimal solutions were obtained through four objective functions.

To reduce the imprecision or uncertainty, which could be caused by the future timber price, a fuzzy approach was applied. A fuzzy multiobjective linear programming model was developed to attain a optimal compromise solution. For this model only the future timber price was considered as the source of uncertainty. Because of that, only the objective function coefficients had to be reformulated under fuzzy environments to accommodate imprecision or uncertainty. As mentioned above, however, there are various sources of uncertainty affecting the timber management problems. That is, the constraint set of model has also imprecise or uncertain parameters and/or elements. Among various sources of uncertainty, the future volume yield is subject to error through the inaccurate growth projection. Therefore, constraints such as evenflow or nondeclining yield could also reflect these inaccuracies.

The FMLP model could be generalized to accommodate such an imprecision in the constraint set, while in the FMLP model developed in this study only the objective function coefficients were considered under fuzzy environments. Hence, the general optimization model that accommodates fuzziness not only in the objection function but also in the constraints could be developed with the same information. While forest planning has uncertainty in the form of a lack of information, imprecision or inaccuracies in estimating model parameters, and inexact or imperfect data, forest planning is also inherently multiple objective, mainly due to the multiple use nature of forest management, for example, besides function of timber production, functions of recreation, protection, erosion control, wild life management and so on. This here developed FMLP model could be further applied to solve the problem of multiple use nature of forest management.

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