A Study on Variation Stack-up Analysis using a Monte Carlo Simulation Method

- 시뮬레이션 방법을 이용한 허용차 해석에 관한 연구 -

Byoungki Lee *

요 지

오늘날 소비자의 요구가 다양해 지면서 여러개의 복잡한 부품들로 이루어진 제품이 많아지게 되었다. 각각의 부품들의 공차가 제대로 결정되지 못함으로 해서 완성된 조립품의 품질이 예상외로 나빠지는 경우가 종종있다. 본 연구는 각각의 부품들의 치수 공차와 생산 공정에서 생길 수 있는 오차들을 모두 포함시켜서 완성된 조립품의 품질을 예상할 수 있는 정확한 공차 해석 방법에 대해서 연구한다. 그 방법으로 몬테카를로 시뮬레이션 방법을 소개하고 그것에 대해서 연구한다. 본 연구를 통해서 완성된 조립품의 품질 및 성능을 저하시키는 요인을 효율적으로 결정할 수 있게 된다.

1. Introduction

One of many problems that the manufacturing industry today is faced with is specification of proper dimensional tolerances on individual components of assembled products so that the performance requirements of the assembled product can be satisfied. Figure 1.1 is a typical example of relationships between dimensional tolerances of each component and manufacturing cost, and between the dimensional tolerances and the performance of an assembled product. It demonstrates that tight tolerances can lead to an expensive product, while loose tolerances may lead to a product with poor performance. Improper tolerance specification may also result in assembly problems and increased waste. Thus, design engineers have increased pressure to specify proper tolerances for manufacturing and assembly efficiency as well as performance requirements.

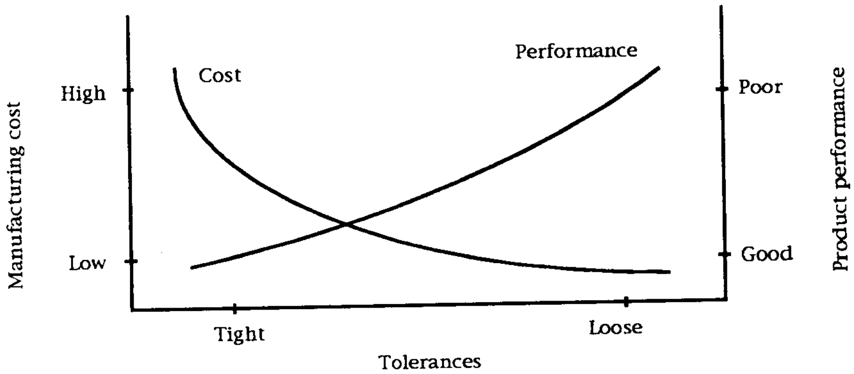

Figure 1.1 **A typical example of tolerances-manufacturing cost and tolerances-product performance relationships.**

* Department of Industrial Engineering, Myong Ji University

At the design stage, engineers set the specification of each dimensional characteristic X by determining its nominal value N and tolerance T. The specification of ith dimensional characteristic can be expressed as: $X_i = N_i \pm T_i$. The nominal of each dimensional characteristic is a target value toward which the operator aims. However, in the real situation, the mean value of the dimensional characteristic may not be the same as its target value because of process mean shifts due to random factors existing in the manufacturing system such as: (1) different materials, people, and machines having worked on the component; (2) tool wear and tool setup errors of the machines; and (3) environmental changes.

The tolerance of each dimensional characteristic is usually given as a range within which the dimension may deviate from its nominal value. It is usually set by design engineers using their experience, by draftsman, or as a part of default routine of a CAD system. The worst situation is where a proper stack-up analysis of dimensional tolerances has not been performed for all of the components in a functional system. When this is not done, all component parts can be within specifications, but the final assembly may not be functional. Thus, in order to improve the performance of the final assembly, a proper stack-up analysis is essential for specifying dimensional tolerances of each component.

2. Variation Stack-Up Analysis

The variation stack-up analysis in Figure 2.1 interprets the tolerance specification range as statistical parameters instead of physical limits; then it computes and statistically analyzes the stack-up variation of the final assembly. It allows engineers to evaluate the effect of dimensional tolerances of each component on the performance of the assembly.

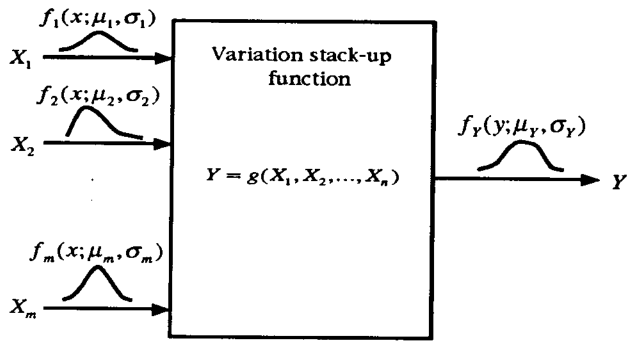

Figure 2.1 Variation stack-up analysis.

Let $X_1, X_2, \cdots, X_n$ represent the component dimensional variations which may affect the performance of the final assembly. Each variation is denoted by the probability density function $f_i(x;\mu_i,\sigma_i)$ with mean μ_i and standard deviation σ_i. The statistical distribution of each variation represented by the probability density function is transmitted through the variation stack-up function $g(X_1, X_2, \cdots, X_n)$ in a statistical distribution of the final assembly performance characteristic Y, denoted by the probability density function $f_Y(y;\mu_Y,\sigma_Y)$.

The problems of the variation stack-up analysis involve the calculation of statistical parameters of the distribution of the final assembly performance Y. Engineers require mean and standard deviation of Y with respect to the probability density function $f_Y(y)$:

$$\mu_Y = \int_{-\infty}^{\infty} y f_Y(y) dy$$

$$\sigma_Y = \left(\int_{-\infty}^{\infty} (y-\mu_Y)^2 f_Y(y) dy \right)^{\frac{1}{2}}$$

We may arrive at an equivalent definition of mean in terms of the input probability density functions $f_i(x)$'s of the component dimensional variations. The mean of Y is the expectation of $g(X_1, X_2, \cdots, X_n)$ with respect to the probability density functions $f_i(x)$'s. If each component dimensional variation X_i is independent of each other, then:

$$\mu_Y = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(X_1, X_2, \cdots, X_n) f_1(x_1) f_2(x_2) \cdots f_n(x_n) dx_1 dx_2 \cdots dx_n$$

More commonly the yield or the reject rate may need to be evaluated. If the performance requirement of Y is $N_Y \pm T_Y$, then for the final assembly to be acceptable, the performance characteristic needs to meet the following constraint:

$$N_Y - T_Y \leq Y \leq N_Y + T_Y$$

Yield P is that proportion of the final assemblies which meet the above constraint:

$$P \equiv \int_{N_Y - T_Y}^{N_Y + T_Y} f_Y(y) dy$$

Similarly, we may arrive at an equivalent definition of yield in terms of the input probability density function $f_i(x)$. First we define an indicating function $I(X_1, X_2, \cdots, X_n)$ such that

$$I(X_1, X_2, \cdots, X_n) \equiv \begin{cases} 1 \text{ if } N_Y - T_Y \leq g(X_1, X_2, \cdots, X_n) \leq N_Y + T_Y \\ 0 \text{ elsewhere} \end{cases}$$

Yield is the expectation of $I(X_1, X_2, \cdots, X_n)$ with respect to the probability density function $f_i(x)$

$$P = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I(X_1, X_2, \cdots, X_n) f_1(x_1) f_2(x_2) \cdots f_n(x_n) dx_1 dx_2 \cdots dx_n$$

We have also assumed that each X_i is independent of each other.

3. Process Mean Change

In many manufacturing processes today, the process means are monitored by a $\bar{x}$ chart using the standard three sigma control limits. However, the $\bar{x}$ chart with the usual sample size of 3, 4, or 5 does not indicate for many samples when a severe mean shift has occurred [14]. Furthermore, many processes are difficult to adjust and hold the process mean accurately. Even for processes where the mean can be easily adjusted, 10 expected numbers of samples are required to detect a shift of one standard deviation of the process natural variation when the sample size is equal to 3. Some processes have permanent bias in the process mean value from the nominal. For example, if a die dimension is deviated from its nominal value in a molding process, then we produce molded parts of which the mean is not equal to the nominal.

In order to perform the variation stack-up analysis properly, we classify variations as the result of part-to-part variation and process mean variation:

(1) The natural variation at a specified time, that is, part-to-part variation σ_P.

(2) The variation in the actual process mean value over time, that is, process mean variation σ_m.

3.1 Part-to-Part Variation

In every manufacturing process, a certain amount of natural variation will always exist. This variation occurs regardless of how well of the process was designed or implemented, or how adequately it is being maintained. The variation is uncontrollable and results from numerous small causes. These variations are referred to as part-to-part variations, and, when they are small, we say that the process is in statistical control.

The part-to-part variation σ_{p_i} of ith dimensional characteristic can be estimated from R control charts:

$$\hat{\sigma}_{p_i} = \frac{\overline{R}_i}{d_2}$$

where $\overline{R}_i$, which is the center line of R control chart of ith dimensional characteristic, is the average range of samples taken when the process is allowed to run with no adjustments, and no lot changes for at least 20 acceptable samples with all ranges outside the control limits of $\overline{R}$ being excluded [4]. The constant d_2 is a function of the sample size, for a sample of 5, $d_2 = 2.326$. Consequently, the estimate of the part-to-part variation σ_{p_i} reflects within-sample variability only.

3.2 Process Mean Variation

A process that is out of control is operating in the presence of process mean change as long as sample range R is within control. This process mean change may come from one or more of a number of sources associated with the machines, the operators, or the materials. The amount of change in the process mean value is commonly expressed in terms of the part-to-part variation σ_p. If a process mean is changed by the amount of $k\sigma_p$ from its nominal value, then the actual process mean is $\mu = N + k\sigma_p$. Figure 3.1 shows an example of process mean change.

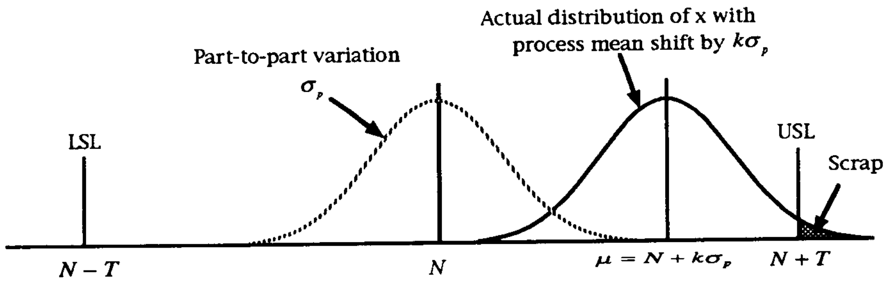

Figure 3.1 An example of process mean change.

In the example of Figure 3.1, if there is no process mean change, then no scrap will be produced, since the distribution is within the tolerance specification limits. However, sifting in the mean value by the amount of $k\sigma_p$ causes scrap.

In some processes, the mean may vary about its nominal value and can be considered as a random variable. If the process mean is random, its variation σ_m is:

$$\sigma_m \equiv \sqrt{\mathrm{var}(\mu)}$$
$$= \sqrt{\mathrm{var}\,(N + k\sigma_p)}$$
$$= \sigma_p \sqrt{\mathrm{var}\,(k)}$$

and cannot be measured instantaneously. It can be only measured over a time period sufficiently wide to permit statistical inferences to be made. Hancock [4, 10] formulated the estimation of the process mean variation.

Because of uncertainty in process mean, we model the process mean variation according to various situations which the process encounters. Given that the process mean has changed, it could have arrived at its new value by any route. It is useful, however, to classify such routes into three categories: bias, shift, and drift changes. A bias change is an intentional or permanent change in process mean from its nominal value. One example of this change is die dimensional bias. A shift change is an instantaneous change in the actual process mean from nominal to some new value where it is assumed to remain. This kind of change is characteristic of the result of suddenly introducing a new material or machine tool change. A drift change occurs when the process mean ceases to be constant over time, and begins to drift in a straight line away from nominal. This is typical of tool wear.

3.2.1 Process Mean Bias

If a process mean is biased by the amount of $k\sigma_p$ from its nominal value, then the actual process mean is $\mu = N + k\sigma_p$ or $\mu = N - k\sigma_p$ and stays permanently at this value. In this case, k is constant and the process mean variation is zero and the actual process mean can be easily estimated from sample data. However, even the process mean change is detected, it cannot be adjusted.

3.2.2 Process Mean Shift

If process mean shifts to a new value, $\mu = N + k\sigma_p$ and stays there for a certain period of time, then k can be assumed to be a constant during that period of time. However, the amount of process mean shift, $k\sigma_p$, is uncertain for a long period of time. We can consider the amount of process mean shift as a random variable. The distribution type of the random variable depends on process characteristic which has mean shift.

If the process mean shift occurs frequently, and can be detected and adjusted easily, then the amount of mean shift, k, has normal distribution with zero mean and σ_k variation because of its natural tendency toward its nominal value. Then the resulting distribution is also normally distributed with the same mean to the nominal value and total variation, σ_T, which combines both variations of part-to-part and process mean change. Since the part-to-part variation and the process mean variation are independent of each other, the total variation can be expressed:

$$\sigma_T = \sqrt{\sigma_p^2 + \sigma_m^2}$$

Since the variation of process mean is:

$$\sigma_m = \sqrt{\mathrm{var}\,(N + k\sigma_p)}$$
$$= \sigma_k \sigma_p$$

the total variation can be expressed as:

$$\sigma_T = \sigma_p \sqrt{1 + \sigma_k^2}$$

3.2.3 Process Mean Drift

In some processes, such as the machining process, the process mean is drifting while the part-to-part variation is kept constant. We can model the mean drifting into a uniform random variable and then the resulting distribution is not normal. In order to compute the reject rates at different specification settings, the cumulative probability function should be derived.

Let the conditional probability density function of X, which is a dimensional characteristic for a given mean value μ, be

$$f_X(x|\mu) = \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left\{ -\frac{1}{2} \left(\frac{x-\mu}{\sigma_p}\right)^2 \right\}, \quad \sigma_p > 0$$

and the density function of the mean μ be

$$g(\mu) = \frac{1}{r}, \quad a_1 < \mu < a_2, \quad a_2 - a_1 = r$$

The marginal density function and the cumulative probability function of X can be written in the form

$$\begin{aligned} f(x) &= \frac{1}{r} \int_{a_1}^{a_2} \frac{1}{\sqrt{2\pi}\sigma_p} \exp\left\{ -\frac{1}{2} \left(\frac{x-\mu}{\sigma_p}\right)^2 \right\} d\mu \\ &= \frac{1}{r} \{\Phi(t_1) - \Phi(t_2)\} \end{aligned}$$

and

$$\begin{aligned} F(x) &= \int_{-\infty}^{x} f(u)du \\ &= \frac{\sigma_p}{r} \{t_1\Phi(t_1) - t_2\Phi(t_2) + \phi(t_1) - \phi(t_2)\} \end{aligned}$$

respectively, where

$$t_i = \frac{x - a_i}{\sigma_p}, \quad i = 1, 2$$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

$$\Phi(t) = \int_{-\infty}^{t} \phi(x)dx$$

4. Monte Carlo Simulation Method for Variation Stack-Up Analysis

The Monte Carlo simulation method is the most recent technique developed for variation stack-up analysis [1, 5, 12, 18]. It is particularly useful when:

(1) A large number of component dimensional characteristics affects the assembly dimension.

(2) The component dimensional characteristics are known to have a probability distribution which is not normal.

(3) The relationship between the component dimensions and the assembly dimension is not linear.

(4) The assembly dimension may be physically impossible to measure, such as a dimension hidden in an assembly or an internal clearance.

Recently, commercial PC-based packages enabling the simulation of tolerances have become available. Pugh [15] briefly describes a software package called PRISM which allows the user to simulate processes. A more comprehensive package, Variation Simulation Analysis software, is also available [18]. This software uses statistical simulation techniques to predict the amount of variation that can occur in an assembly due to specified design tolerances and manufacturing or assembly variation. Additionally, it can determine the locations of the predicted variation, the contributing factors, and their percentage of contribution. This package includes a useful graphics preprocessor for interfacing with CAD packages and interactive simulation model building as well as a simulation language. A package, called GA-2000, by John Deere and Co. is also available [5].

There are two applications of the Monte Carlo simulation method to actual tolerancing problems. Doydum and Perreira [3] presented an analytical method using the Monte Carlo simulation for selecting the dimensions and tolerances of mating parts and precision of assembly equipment where the mating parts possessed simple geometrics such as line and circle. Another application of the Monte Carlo simulation method is to the circuit tolerance analysis problem which, although it has the attraction of insensitivity to the number of toleranced components, is computationally expensive. Soin and Rankin [17] have used variance prediction techniques to improve simulation efficiency.

4.1 Simulation Model

The simulation model determines the relationship between an assembly and its components and computes the expected variation of the assembly based on the components tolerances and variations in manufacturing system. The three key elements in the model which are necessary for the variation stack-up analysis of the assembly are identified as follows:

(1) Input variations have an effect on the performance of a finished assembly product. This includes all the variations from individual component tolerances and variations due to random factors existing in the manufacturing system. These variations are specified with their means, standard deviations, and their statistical distribution types.

(2) Output measurement is a performance characteristic of the final assembly product. It will be expressed as a functional relationship to the dimensional characteristics of each component.

(3) Stack-up function relates the input variations or independent variables and the output measurement or dependent variable.

4.2 Monte Carlo Simulation Analysis

Once the simulation model is developed, a computer program performs the sampling experiments upon the model. The sampling experiment consists of the random selection of a sample value from the random number generator for each of the input variables and substitutes those values into the model to computes the output measurement. This approach repeats until a sufficiently large number of output measurement values are collected.

If we collect m output measurement values and compute an indicating function $I(X_1, X_2, \cdots, X_n)$ defined:

$$I(X_1, X_2, \cdots, X_n) \equiv \begin{cases} 1 \text{ if } N_Y - T_Y \le g(X_1, X_2, \cdots, X_n) \le N_Y + T_Y \\ 0 \text{ otherwise} \end{cases},$$

then the unbiased estimator of reject rate is:

$$\hat{R} = 1 - \frac{1}{m} \sum_{j=1}^{m} I(x_{1j}, x_{2j}, \cdots, x_{nj})$$

Where x_{ij} is the jth random selection of the ith input variation. Since $I(X_1, X_2, \cdots, X_n)$ may have the value 1 or 0, each simulation run and tested against performance constraints constitutes a Bernoulli trial. Therefore, the sampling distribution of the estimator $\hat{R}$ is binomial, with variance

$$\sigma^2_{\hat{R}} = \frac{R(1-R)}{m}$$

which may be approximated by

$$\hat{\sigma}^2_{\hat{R}} = \frac{\hat{R}(1-\hat{R})}{m}$$

If m is large, the normal approximation with the same mean and variance to the binomial can be used, the resulting $100(1-\alpha)\%$ confidence interval for reject rate is determined by

$$\hat{R} - Z_{\alpha/2}\sqrt{\frac{\hat{R}(1-\hat{R})}{m}} \le R \le \hat{R} + Z_{\alpha/2}\sqrt{\frac{\hat{R}(1-\hat{R})}{m}}$$

where $Z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution. This approximation gives reasonable results if mR and $m(1-R)$ are both at least 5 [6]. Similarly, the expected value of the output measurement may be estimated as:

$$\hat{\mu}_Y = \frac{1}{m}\sum_{j=1}^{m} g(x_{1j}, x_{2j}, \cdots, x_{nj})$$

We may obtain the sampling variance of the estimator $\hat{\mu}_Y$ as

$$\sigma^2_{\hat{\mu}_Y} = \frac{1}{m}\sum_{j=1}^{m}\{g(x_{1j}, x_{2j}, \cdots, x_{nj}) - \hat{\mu}_Y\}^2$$

A confidence interval may be constructed around the estimator $\hat{\mu}_Y$ by assuming a particular form of probability density function for the sampling distribution of the estimator $\hat{\mu}_Y$, e.g. normal. For example, if the sampling distribution of $\hat{\mu}_Y$ is normal, then the 99.73% confidence interval is $\hat{\mu}_Y \pm 3\sigma_{\hat{\mu}_Y}$. Note that whatever the form of the sampling distribution of an estimator is, the sampling variance is inversely proportional to the sample size, when the simulation method is employed.

4.3 Number of Samples Required

The number of samples, m , required to estimate a specific reject rate R value with a given precision of a confidence interval at a confidence level of $1-\alpha$ is found using the expression of the confidence interval. First we define the precision ρ as the ratio of the magnitude of the estimator $\hat{R}$ to the confidence interval half-length:

$$\rho \equiv \frac{\hat{R}}{Z_{\alpha/2}\sqrt{\frac{\hat{R}(1-\hat{R})}{m}}}$$

Then the number of samples required is

$$m = \frac{(1-\hat{R})\rho^2}{\hat{R}} Z^2_{\alpha/2}$$

Table 4.1 indicates how the required number of samples increases with precision of the confidence interval. In some practical situations, the reject rate of 0.001 or even 0.000001 is of interest. For these reject rates, we need narrow interval estimates. It requires additional samples, which is further increased with the precision. If a reject rate of 1 part per million (ppm), i.e. $R=0.000001$, is allowed with 99% confidence, then at least 665,639,334 samples are required in the simulation to achieve the precision of $\rho=10$ and this is impractical.

Table 4.1 Number of samples required to estimate a reject rate with precision ρ at 99% confidence level.

$\hat{R}$	ρ = 2	5	10
0.5	27	166	666
0.1	240	1,498	5,991
0.01	2,636	16,475	65,898
0.001	26,599	166,244	664,974
0.0001	266,229	1,663,934	6,655,734
0.00001	2,662,533	16,640,834	66,563,334
0.000001	26,625,573	166,409,834	665,639,334

4.4 Sensitivity Analysis

This method determines the contribution of each input variation to the predicted variation of the output measurement using sensitivity analysis. This information gives engineers a tool to determine which input variations are critical to the variation of the output measurement. Sensitivity analysis is performed separately from random simulations, and varies each input variation to its high, low, and median values, one at a time, while holding all other input variations at their median values. It then notes the effect, if any, on the output measurement. It can be called the High-Low-Median (HLM) analysis.

For each input variation, the range R_Y which is total amount of output measurement changed when the input variation is changed from high to low to median can be calculated:

$$R_Y = Y_{\max} - Y_{\min}$$

where $Y_{\max}$ is the maximum value that the output measurement reached and $Y_{\min}$ is the minimum value that the output measurement reached.

From this range calculation, an approximation of the output measurement variance, σ_Y^2 due to an input variation is determined. The variance is a statistic that defines the amount of spread that a group of data has. Since the group of data generated from the HLM analysis is only two values, output measurement high and low, the variance is approximated. For HLM purposes, simulation assumes that the output measurement behaves normally, and that the range between output measurement high and low is equal to six standard deviations, or:

$$R_Y = 6\sigma_Y$$
$$\sigma_Y^2 = \frac{R_Y^2}{36}$$

where σ_Y^2 is the variance of the output measurement due to an input variation.

HLM analysis calculates σ_Y^2 in this manner for all n input variations and their effect on all output measurements. Then, the results from each output are summed to give the main effect variance:

$$\sigma_{HLM}^2 = \sigma^2_{Y_1} + \sigma^2_{Y_2} + \ldots + \sigma^2_{Y_n}$$

The percentage which each input variation contributes to the overall HLM variation of the output measurement is:

$$\% \text{ contribution} = \left(\frac{\sigma^2_{Y_i}}{\sigma^2_{HLM}} \right) \times 100$$

This process ranks the input variations in order of contribution to the variation of the output measurement based on the HLM analysis and determines the major contributors.

The HLM analysis has a limitation. It assumes that only main effects of tolerances are significant and interactive effects between the tolerances are not present. For example, when tolerance T_i is varied alone, it will cause the output measurement to vary by amount a_i; when tolerance T_j is varied alone, it will cause the output measurement to vary by amount a_j; when both tolerances T_i and T_j are varied, the output variation will be equal to a_i+a_j. This assumption, however, does not always hold true, because of interactive effects. Regression analysis [2, 7, 8, 9, 16] or experimental design [11, 13] can be used to determine the major contributors. By constructing full-factorial experiments, we can assess the significance of interactive affects in addition to main affects. This provision allows for groups of contributors to be studied as a unit alongside any combination of individual and/or grouped input variable sets.

5. Conclusions

The variation stack-up analysis interprets the tolerance specification range as statistical parameters instead of physical limits, then it computes and statistically analyzes the stack-up variation due to not only individual component tolerances, but random factors in the assembly and manufacturing processes. This analysis allows evaluation of the effect of component tolerances and process random factors on the performance of the final assembly.

This research presented a process mean changing model and discussed the effect of process mean change on the final assembly. In this paper, we classified the cases of process mean change according to the types of process variations and for each case, statistically modeled the process mean change. A bias change is an intentional or permanent change in process mean from its nominal value. In this case, process mean variation is zero and actual process mean can be easily estimated from manufacturing data. A shift change is an instantaneous change in the actual process mean from a nominal to some new value where it is assumed to remain. If the process mean shift occurs frequently, and can be detected and adjusted easily, then the amount of mean shift can be modeled as a normal random variable because of its natural tendency toward its nominal value. Then the resulting distribution is also normally distributed. A drift change occurs when the process mean ceases to be constant over time, and begins to drift in a straight line away from nominal. This is typical of tool wear. In this case, we can model the mean drifting into a uniform random variable and then the resulting distribution is not normal.

This research presented a Monte Carlo simulation method for variation stack-up analysis. This method is particularly useful when: (1) input variations affect the performance of the final assembly, (2) some input variations have non-normal distribution, and (3) the stack-up function between input variations and the output measurement is not linear. Sensitivity analysis was also presented as a part of the methodology. Sensitivity analysis calculates the contribution of each input variations to the variation of the final assembly performance. This analysis is a tool for design and manufacturing engineers to help them determine which variations are critical to the final assembly performance.

References

[1] Bjorke, O., Computer-Aided Tolerancing: 2nd Ed., ASME Press, New York (1989).

[2] Chatterjee, S. and A. S. Hadi, Sensitivity Analysis in Linear Regression, John Wiley & Sons, New York, NY (1988).

[3] Doydum, Cemal and N. Duke Perreira, 'Use of Monte Carlo Simulation to Select Dimensions, Tolerances, and Precision for Automated Assembly," Journal of Manufacturing Systems 10, 209-222 (1991).

[4] Hancock, W. M., "Tolerancing Systems and their Quality Interaction," (June, 1991).

[5] Hardy, J. M. and J. J. Hurt, GA-2000 Geometrical Analyzer Technical Overview, Hard, Hurt & Coin Incorporated, Cleveland, Ohio (1991).

[6] Hahn, G. J. and S. Shapiro, Statistical Models in Engineering, John Wiley & Sons, New York (1967).

[7] Iman, R. L. , J. C. Helton, J. E. Campbell, "An Approach to Sensitivity Analysis of Computer Models: Part1-Introduction, Input Variable Selection and Preliminary Variable Assessment," Journal of Quality Technology 13, (1981).

[8] Iman, R. L. , J. C. Helton, J. E. Campbell, "An Approach to Sensitivity Analysis of Computer Models: Part II-Ranking of Input Variables, Response Surface Validation, Distribution Effect and Technique Synopsis," Journal of Quality technology 13, (1981).

[9] Iman, R. L. and J. C. Helton, "An Investigation of Uncertainty and Sensitivity Analysis Techniques for Computer Models," Risk Analysis 8, (1988).

[10] Kelton, W. D. , W. M. Hancock, D. P. Bischak, "Adjustment Rules Based on Quality Control Charts," Technical Report of IOE Dept at the University of Michigan (May 1985).

[11] Kleijnen, J. P. C., and C. R. Standridge, "Experimental Design and Regression Analysis in Simulation: An FMS Case Study," European Journal of Operational Research 33, (1988).

[12] Lehtihet, E. A. and B. A. Dindelli, "Tolcon: Microcomputer-Based Module for Simulation of Tolerances," Manufacturing Review 2, (September, 1989).

[13] Logothetis, N. and H. P. Wynn, Quality Through Design: Experimental Design, Off-Line Quality Control,and Taguchi's Contributions, Oxford University Press, New York (1989).

[14] Montgomery, D. C., Introduction to Statistical Quality Control, John Wiley & Sons, New York (1985).

[15] Pugh, G. A., "PRISM-A Software Review," IE News 22, (Winter, 1988)

[16] Smith, H., Regression Analysis of Variance. In the Design of Computer Simulation Experiments edited by T. H. Naylor, Duke University Press, Durham, North Carolina.

[17] Soin, R.S. and P.J. Rankin, 'Efficient tolerance analysis using control variates," IEE Proceedings 132, 131-142 (1985).

[18] Variation Simulation Analysis Software V2.1, Applied Computer Solutions, St. Clair Shores, MI. (1990).