Technical Paper

Transactions of the Society of
Naval Architects of Korea
Vol. 31, No. 2, May 1994
大 韓 造 船 學 會 論 文 集
第 31 卷 第 2 號 1994年 5月

Application of Optimal Control Techniques to SWATH Motion Control

by

Chan Wook Park*, Bo Hyeon Heo* and Chun Tae Song*

반잠수 쌍동선의 최적 운동제어기 설계

박찬욱*, 허보현*, 송준태*

Abstract

This paper presents a detailed application procedure of the linear quadratic (LQ) theory for a SWATH heave and pitch control.

A time domain model of coupled, linear time-invariant second order differential equations is derived from the frequency response model with the frequency dependent added mass and damping approximated as constant values at the heave natural frequency. Wave exciting forces are modeled as a sum of sinusoids.

A systematic selection procedure of state and control weighting matrices is presented to obtain good transient behavior and acceptable fin movement.

The validity of this controller design process is thoroughly investigated by simulations both in time domain and frequency domain and singular value plots of transfer function matrices.

The finally designed control system shows good overall performances revealing that the applicability of the present study is proved successful.

요 약

본 논문은 SWATH선의 히브 및 피치 제어에 대한 LQ이론의 상세한 응용과정을 제시한다. 부가 질량과 감쇄계수를 상하동요 고유주기에서의 값으로 근사함으로써 선형 시불변 2차 연립 미분방정식이 주파수 응답 모델로 부터 유도된다. 파기진력은 사인파들의 합으로서 모델링 된다.

Manuscript received: December 6, 1993

* Member, Agency for Defense Development

좋은 과도응답(transient response)과 적절한 제어핀 운동을 얻기 위하여 상태 및 제어 가중행렬의 체계적인 선택과정이 제시된다.

본 논문의 제어기 설계과정의 타당성이 시간영역 및 주파수 영역에서의 시뮬레이션과 전달함수행렬의 특이값 선도에 의해 철저히 조사되어 진다.

최종 설계된 제어시스템은 본 연구의 응용이 성공적이었음을 나타내는 좋은 전체 성능을 보여준다.

Introduction

One of the merits of a SWATH ship is its superior seakeeping performance in waves. These ships are normally equipped with fins to improve the vertical stability at high speeds and to further improve their motion characteristics by the additional damping stemming from fin generated lift.

This paper presents an application procedure of the linear quadratic (LQ) theory for a SWATH heave and pitch control.

The LQ theory was first applied to a SWATH ship by Ware et al. [1][2] for heave and pitch control in both the platforming and contouring modes. Zarnick [3] applied the same technique for a SWATH roll control in addition to heave and pitch modes and carried out simulations in frequency domain. In Korea, Lee [4] applied and compared various control techniques including PID, LQ, Linear Quadratic Gaussian (LQG) and self-tuning controller using extended Kalman filter for a SWATH ship, and Park et al. [5] showed that there was no significant difference in efficacy of control between LQ and LQG controller.

The essential differences between the above previous works and the present study are as follows;

1. The time domain model derived from the frequency response model of frequency dependant added mass and damping is resultantly identical to those of the previous works. However, we compared differences of two models and confirmed the applicability of the approximate time domain model for LQ controller design in respect of the close-loop robustness characteristics.

2. The singular value plot, which is a powerful measure of the control performance of a multi-input-multi-output (MIMO) system, was provided and used for analysis of the designed controller in addition to the simulation result.

3. A detailed selection procedure of the state and control weighting matrices Q and R, which are the key to a successful design of an optimal controller, was presented for later use as a guideline.

4. The amount of fin movement was presented quantitatively and compared with the relevant criteria to show the practial applicability of the present study, because the control gains determined by LQ theory assume the availability of an infinite amount of fin angle and fin rate [3].

In developing computer programs for this work, we quoted several subroutines from the reference [4]; the Riccati equation solver, Gaussian white noise generator, and the Runge-Kutta 4th order integration scheme. The frequency dependant added mass and damping were calculated using the program developed by Lee and McCreight [6].

The ADD (Agency for Defense Development) SWATH ship [7], which has been in operation since April 1993, was chosen for calculation.

Modeling and formulation of the LQ problem

Frequency domain equation of motion

The equation of motion for heave and pitch is [8].

$$\left\{-\omega^2(M+A_{33})-j\omega B_{33}+C_{33}\right\}Z(\omega)+\left\{-\omega^2 A_{35}-j\omega B_{35}+C_{33}\right\}\theta(\omega)=F_3(\omega)$$
$$\left\{-\omega^2(A_{53})-j\omega B_{53}+C_{53}\right\}Z(\omega)+\left\{-\omega^2(I+A_{55})-j\omega B_{55}+C_{55}\right\}\theta(\omega)=F_5(\omega) \tag{1}$$

where
$$A_{33}=A_{33}^{\circ} \qquad A_{53}=A_{53}^{\circ}+\frac{u}{\omega^2}B_{33}^{\circ}$$
$$B_{33}=B_{33}^{\circ}+B_{33}^{*}+B_{33}^{f} \qquad B_{53}=B_{35}^{\circ}-uA_{33}^{\circ}+B_{53}^{*}+B_{56}^{f}$$
$$A_{35}=A_{35}^{\circ}-\frac{u}{\omega^2}B_{33}^{\circ} \qquad A_{55}=A_{55}^{\circ}+\frac{u^2}{\omega^2}A_{33}^{\circ}$$
$$B_{35}=B_{35}^{\circ}+uA_{33}^{\circ}+B_{35}^{*}+B_{35}^{f} \qquad B_{55}=B_{55}^{\circ}+\frac{u^2}{\omega^2}B_{33}^{\circ}+B_{55}^{*}+B_{55}^{f}$$
$$C_{33}=\rho g A_w$$
$$C_{35}=\rho g M_w+C_{35}^{*}+C_{35}^{f}$$
$$C_{53}=\rho g M_w$$
$$C_{55}=\rho\nabla g GM+C_{55}^{*}+C_{55}^{f}$$

The coefficients with a zero superscript are added mass(A_{ij})and damping(B_{ij}) at zero speed.

The superscript asterisk and f are associated with the vicous lift on the bare hull and the fin generated lift, respectively.

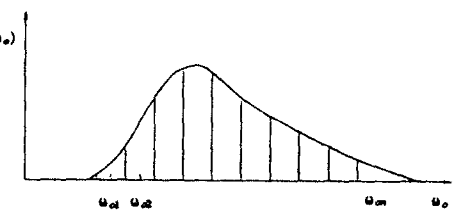
Equation (1) can be written as,

$$Z(\omega)=H_{33}(\omega)F_3(\omega)+H_{35}(\omega)F_5(\omega) \tag{2}$$
$$\theta(\omega)=H_{53}(\omega)F_3(\omega)+H_{55}(\omega)F_5(\omega)$$

where,
$$H_{33}(\omega)=\frac{L_{55}}{L_{33}L_{55}-L_{35}L_{53}} \qquad L_{33}=-\omega^2(M+A_{33})-j\omega B_{53}+C_{53}$$
$$H_{35}(\omega)=\frac{-L_{35}}{L_{33}L_{55}-L_{35}L_{53}} \qquad L_{35}=-\omega^2 A_{35}-j\omega B_{35}+C_{35}$$
$$H_{53}(\omega)=\frac{L_{53}}{L_{33}L_{55}-L_{35}L_{53}} \qquad L_{53}=-\omega^2 A_{53}-j\omega B_{53}+C_{53}$$
$$H_{55}(\omega)=\frac{L_{33}}{L_{33}L_{55}-L_{35}L_{53}} \qquad L_{55}=-\omega^2(I+A_{55})-j\omega B_{55}+C_{55}$$

Exciting force modeling

A pseudorandom seaway is generated by superposing sinusoids.



$$\eta(t)=\sum_{i=1}^{n}\sqrt{2\frac{\sigma^2}{n}}\cos(\omega_i t+\Psi_i) \tag{3}$$

where

$\sigma^2=\int_0^\infty S(\omega_0)d\omega_0$: power of the sea spectrum

Ψ_i : random phase

$\omega_i=\omega_{oi}-\frac{\omega_{oi}^2}{g}u\cos\beta$: encounter frequency

Exciting forces are modeled on the basis of the linear superposition principle.

$$F_{3,5}(t)=\sum_{i=1}^{n}|F_{3,5}(\omega_i)|\sqrt{2\frac{\sigma^2}{n}}\cos(\omega_i t+\phi_{3,5}(\omega_i)+\Psi_i) \tag{4}$$

where $|F(\omega)|$ and $\phi(\omega)$are magnitude and phase of the exciting frequency response function $F(\omega;u,\beta)$ per unit wave amplitude.

Time domain modeling of the equation of motion

The frequency domain equation of motion (1) has the coefficients A_{ij} and B_{ij} which show unnegligible frequency dependancy, resulting in the form of an integro-differential equation or a differential equation of higher order with frequency independant coefficients so as to be transformed into the time domain [9].

In this paper A_{ij} and B_{ij} are assumed constant taking the values at the heave natural frequency, the technique selected by Lee and Curphey [10] and Ware et al. [1,2], deriving the second order differential equation which fits into the LQ framework.

$$(M+A_{33})\ddot{Z}+B_{33}\dot{Z}+C_{33}Z+A_{35}\ddot{\theta}+B_{35}\theta+C_{35}\theta=F_3(t)$$
$$A_{53}\ddot{Z}+B_{53}\dot{Z}+C_{53}Z+(I+A_{55})\ddot{\theta}+B_{55}\dot{\theta}+C_{55}\theta=F_5(t) \tag{5}$$

The limitation embedded in this approximation is to be checked by comparing the transfer function $H_{ij}(j\omega)$ of the above system with that of the reference system, equation(1). Fig. 1 shows excellent closeness between the two results at frequencies above 0.6 rad/s, while the closeness is very poor at low frequencies.

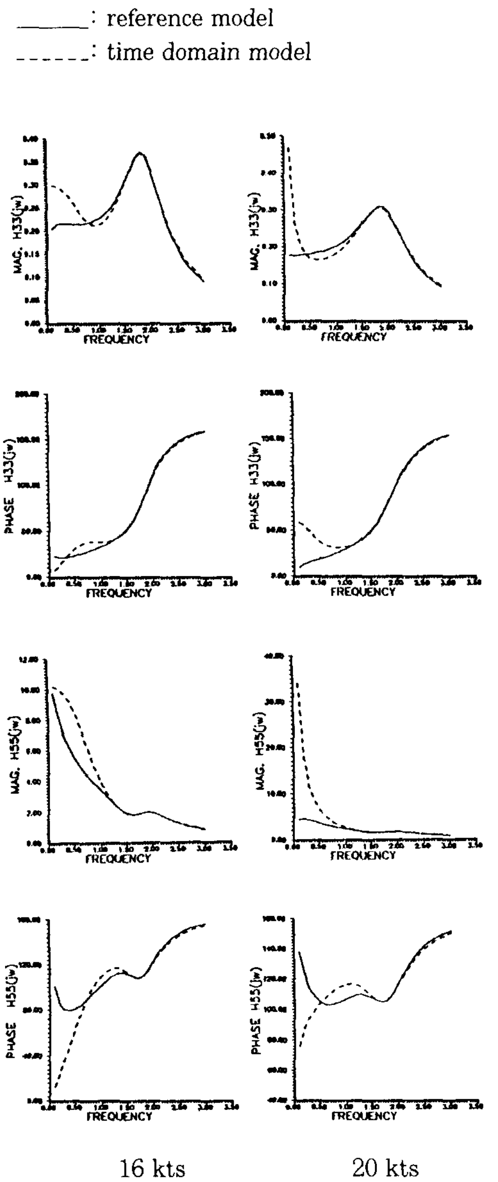


Fig. 1 Comparison of nondimensional transfer functions of the reference model and the time domain approximation (frequency = 1.7 ω)

If most of exciting force is expected in the frequency range above 0.6 rad/s, this will be an acceptable model for the purpose of time domain simulation and controller design. In the case of following seas where the encounter frequency becomes very low, this plays as a poor model articularly for the simulation purpose.

However, we use the approximate model for the design of control gains regardless of ship's heading. The efficacy of control will be displayed through the frequency domain simulation using the reference model (1) in addition to the time domain simulation results of the model (5).

State variable representation

The linear time invariant differential equation (5) with the control force added can be written in the state variable form as follows

$$\begin{bmatrix} M+A_{33} & 0 & A_{35} & 0 \\ 0 & 1 & 0 & 0 \\ A_{53} & 0 & I+A_{55} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{Z} \\ \dot{Z} \\ \ddot{\theta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -B_{33} & -C_{33} & -B_{35} & -C_{35} \\ 1 & 0 & 0 & 0 \\ -B_{53} & -C_{53} & -B_{55} & -C_{55} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{Z} \\ Z \\ \dot{\theta} \\ \theta \end{bmatrix} + \begin{bmatrix} b_{31} & b_{32} \\ 0 & 0 \\ b_{51} & b_{52} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_F \\ \alpha_A \end{bmatrix} + \begin{bmatrix} F_3(t) \\ 0 \\ F_5(t) \\ 0 \end{bmatrix} \tag{6}$$

where α_F: the deflection of the forward fin, positive with leading edge up

α_a : the deflection of the aft fin, positive with leading edge up

$b_{31} = u^2\rho(C_LA)_F$

$b_{32} = u^2\rho(C_LA)_A$

$b_{51} = -u^2\rho(C_LAl)_F$

$b_{52} = -u^2\rho(C_LAl)_A$

This can be written into the proper form for LQ theory application.

$$\dot{X} = T^{-1}\bar{A}X + T^{-1}\bar{B}U + T^{-1}\bar{F} = AX + BU + F \tag{7}$$

The linear quadratic (LQ) control problem

The LQ optimum control problem may be described in the following manner. Given a dynamic system represented by a set of first order differential equations of the form,

$$\dot{X} = AX(t) + BU(t) \tag{8}$$

where X : n by 1 state vector
U : m by 1 control vector
A : n by n system dynamic matrix
B : n by m control effectiveness matrix

then, determine the optimum control input U (t) which minimize the following performance index.

$$J(U) = \int_0^\infty (X^TQX + U^TRU)dt \tag{9}$$

where Q: n by n state weighting matrix, positive semidefinite
R : m by m control weighting matrix, positive definite

The solution to this problem can be found in many text books on optimum control and results in a control law of state feedback.

$$U(t) = -KX(t) = -R^{-1}B^TPX(t) \tag{10}$$

where the n by n matrix P is obtained solving the matrix Riccati equation.

$$A^TP + PA + Q - PBR^{-1}B^TP = 0 \tag{11}$$

As presented above, the determination of the optimum control gain K is a straightforward procedure.

However, the selection of weighting matrix Q and R is a subjective matter, making it necessary to check the validity of these weightings based on the result of simulation.

In this paper we varied these weightings in such a manner that increased the relative weighting of Q vs R and, no significant improvement in motion performance was achieved while excessive control action was required. This is the same phenomenon that was already observed by Ware and Scott [2] and implies an ease in selecting weightings.

The heave and pitch equation (7) contains the additional disturbance F compared with equation (8). If F is Gaussian white noise, the optimum solution to equation (7) is identical to that for equation (8) because the solution to the control problem can be obtained independent of the solution to the estimation problem for a linear system with Gaussian white process and observation noise [11]. Here we assume to obtain all states (heave rate, heave,pitch rate and pitch) and do not deal with the estimation problem.

For the non-white process noise F, as in the case of heave and pitch exciting forces, the optimum control U(t) is dependant upon additional noise shaping states. Fortunately, even in this case the optimum control gain K linked to the usual system states satisfies the same Riccati equation (11) and the gain linked to the noise shaping states is of little importance [2].

Presentation of results

The initial set of weightings Q and R was selected from the following standard deviations for each variable.

$\sigma_\theta = 2^\circ$ (corresponding to the significant pitch 4 degree, the value decided as an upper bound in sea state 5)
$\sigma_z = 0.5\,m$ (equivalent to the pitch deviation 2 degree on our ship of length 30 meter)
$\sigma_{\dot\theta} = 2^\circ/s$ (disturbed response is usually
$\sigma_{\dot Z} = 0.5 m/s$ dominated by motions at a heave natural frequency [10], 1 rad/s for our vessel)
$\sigma_{\alpha F}, \sigma_{\alpha A} = 10^\circ$ (more than one excursion in ten should not exceed the fin angle limit [12], 25 degree for our vessel)

Taking the square of deviation reciprocals, we have

$$Q = \begin{bmatrix} 4[m/s]^{-2} & 0 & 0 & 0 \\ 0 & 4[m]^{-2} & 0 & 0 \\ 0 & 0 & 820[rad/s]^{-2} & 0 \\ 0 & 0 & 0 & 820[rad]^{-2} \end{bmatrix}$$

$$R=\begin{bmatrix} 33[rad]^{-2} & 0 \\ 0 & 33[rad]^{-2} \end{bmatrix}$$

As stated earlier, the choice of Q and R is a subjective matter and to be finally determined on the basis of resultant control performance.

At first we varied relative weights on θ and Z vs those on θ and Z to obtain damping ratios about 0.7 to achieve good transient behavior. Next we increased the weighting Q relative to R to improve motion performance within the acceptable range of fin movements. Here the criteria suggested by Cox and Lloyd [12] was chosen ; more than one excursion in ten should not exceed the fin angle limit and the fin rate should not exceed the rate limit more than once in one hundred excursions. For our vessel, $\sigma_{\alpha F}, \sigma_{\alpha A} < 10°$ and $\sigma_{\alpha F}, \sigma_{\alpha A} < 5°/s$ was selected as the acceptable fin movements.

Fig. 2 shows calculated damping ratios for the vessel advancing at 16 kts, varying Q_{11} and Q_{33} up to a factor of 256.

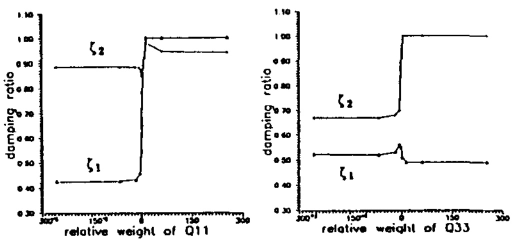


Fig. 2 Change of damping ratios with variation of Q_{11} and Q_{33} (16 kts)

As expected, there appears a clear tendancy that Q_{11}, the relative weighting on Z, changes the damping ratios ζ_1 keeping the other damping ζ_2 constant, while Q_{33}, the relative weighting on θ, changes ζ_2 without significant variation of ζ_1 . The selected Q is,

Q = [8, 4, 205, 820]

Fig. 3 shows the standard deviations of heave Z, pitch θ, fin angle α_F, and fin angle rate $\dot{\alpha}_F$ of the forward fin at 16 kts in sea state 5 head sea, varying R_{11} and R_{22} up to a factor of 16 with Q fixed as above.

The weighting R is chosen as follows,

R = [132, 132]

which results in the acceptable fin angle rate of less than 5 deg/s. The aft fin moves less than the forward fin, so values of the aft fin are not presented in fig. 3.

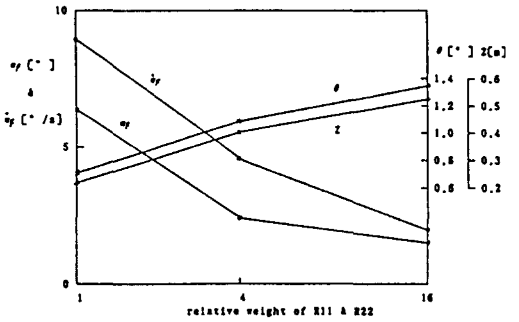


Fig. 3 Change of standard deviations of heave, pitch, forward fin angle, and fin angle rate with variation of R_{11} and R_{22} (16 kts, head sea, sea state 5)

The optimal control gain K is calculated at 16 kts to be,

$$K=\begin{bmatrix} 0.1117 & 0.0308 & -1.520 & -1.192 \\ 0.0646 & 0.00612 & 1.773 & 1.414 \end{bmatrix} \text{ at 16 kts}$$

The gain for speed of 20 kts is calculated using the same sets of weighting matrices Q and R as for 16 kts. With these weightings we obtained reasonable control performance at 20 kts as well as at 16 kts.

$$K=\begin{bmatrix} 0.1287 & 0.0462 & -1.613 & -1.674 \\ 0.0717 & 0.00904 & 1.789 & 1.921 \end{bmatrix} \text{ at 20 kts}$$

Table 1 presents system poles and simulation results with and without the above optimal control.

Inherent low heave damping increases and high pitch damping decreases with control, at 16 and 20 kts in common, making the system more stable and the transient responses more acceptable. Fig. 4 shows step responses of the closed-loop system, i.e., the controlled system, at 16 and 20 kts.

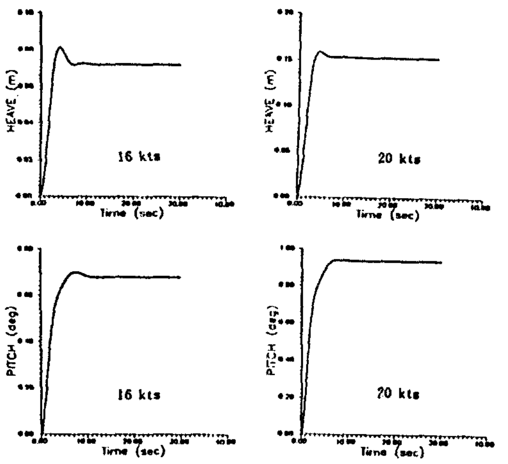


Fig. 4 Step responses of the closed-loop system

System natural frequencies and damping ratios are well reflected upon the transient features such as percent overshoot and settling time. The performance of command following appears to be very poor, particularly in heave mode where the steady state error reaches up to 90 %. We've searched various sets of weighting matrix Q and R to improve this performance and found the fact that the reduction of steady state errors can be achieved only at the sacrifice of acceptable fin movements and transient features. Laying emphasis on the command following of pitch, we'll assume that present results are practical; steady state error of 30 % at 16 kts and of 3 % at 20 kts in pitch mode.

The performance of disturbance rejection can be estimated quantitatively through the degree of reduction of heave and pitch. Fig. 5 shows some time series obtained from time domain simulation.

Table 1 shows that in head sea, heave and pitch decrease to be about 50 to 70 % of the uncontrolled cases. In following sea, results of time domain simulation shows drastic reduction of heave and pitch, particularly at speed of 20 kts. However, frequency domain simulation based on the reference model (1) shows the results which are severely different from those of time domain simulation in case of following sea and without control. As stated earlier, the time domain model (5) approximates the reference model (1) poorly in following sea resulting in such a large difference. Even with these frequency domain results, we can say that the performance of disturbance rejection is also acceptable in following sea where pitch decreases to be 40 to 50 % of values without control.

The closeness of the controlled results between time domain and frequency domain simulation in case of following sea gives some information on the performance-robustness of the closed-loop system. Even with the large modeling error for the open-loop system in following sea, the efficacy of control changes a little between two significantly different models, equation (1) and equation (5).

Table 2 is presented to show the sensitivity of the closed-loop system to measurement noise. The linear Gaussian measurement model is assumed,

$$Z = X + V$$

where V is a Gaussian white noise matrix with covariance R_f.

The measurement model 1 means perfect state feedback, U = -KX.

The model 2 and 3 means measured state feedback, U = -KZ, with R_f as follows.

model 2 :

$$R_f=[0.0025(m/s)^2,\ 0.0025(m)^2,\ 1.77E\text{-}7(rad/s)^2,\ 1.77E\text{-}7(rad)^2]$$

model 3 :

$$R_f=[0.0025,\ 0.0025,\ 1.77E\text{-}6,\ 1.77E\text{-}6]$$

There appears no performance degradation as measurement noise increases, implying that the closed-loop system is excellently insensitive to sensor noise. However the incidental problem of

Table 1 System poles and simulation results with and without control

SPEED		16 KTs				20 KTS			
CONTROL		WITHOUT CONTROL		WITH CONTROL		WITHOUT CONTROL		WITH CONTROL	
POLE		-0.236±1.083j -0.311±0.273j		-0.450±1.081j -0.538±0.499j		-0.281±1.081j -0.780, -0.019		-0.674±1.106j -0.733±0.441j	
DAMPING RATIO		0.21 , 0.75		0.38, 0.73		0.25, 1.00		0.52, 0.86	
HEADING		HEAD SEA	FOLLOW ING SEA	HEAD SEA	FOLLOW ING SEA	HEAD SEA	FOLLOW ING SEA	HEAD SEA	FOLLOW ING SEA
Standard deviation (sea states 5)	$\dot{Z}$ [m/s]	0.65	0.15	0.41	0.09	0.61	0.25	0.33	0.11
	Z [m]	0.61 (0.59)*	0.66 (0.53)*	0.39 (0.37)*	0.37 (0.45)*	0.54 (0.51)*	2.40 (0.41)*	0.28 (0.28)*	0.46 (0.49)*
	$\dot{\theta}$ [°/s]	1.90	1.14	1.34	0.39	1.88	1.70	1.20	0.39
	θ [°]	1.57 (1.61)*	6.13 (5.25)*	1.05 (1.07)*	1.88 (2.12)*	1.45 (1.49)*	19.64 (3.55)*	0.85 (0.87)*	1.50 (1.70)*
	α_F [°]	-	-	3.80	2.83	-	-	3.76	3.74
	α_A [°]	-	-	2.85	2.75	-	-	2.59	2.80
	$\dot{\alpha}_F$ [°/S]	-	-	4.92	0.59	-	-	5.49	1.35
	$\dot{\alpha}_A$ [°/S	-	-	3.81	0.69	-	-	3.92	0.78

* : results from frequency domain simulation

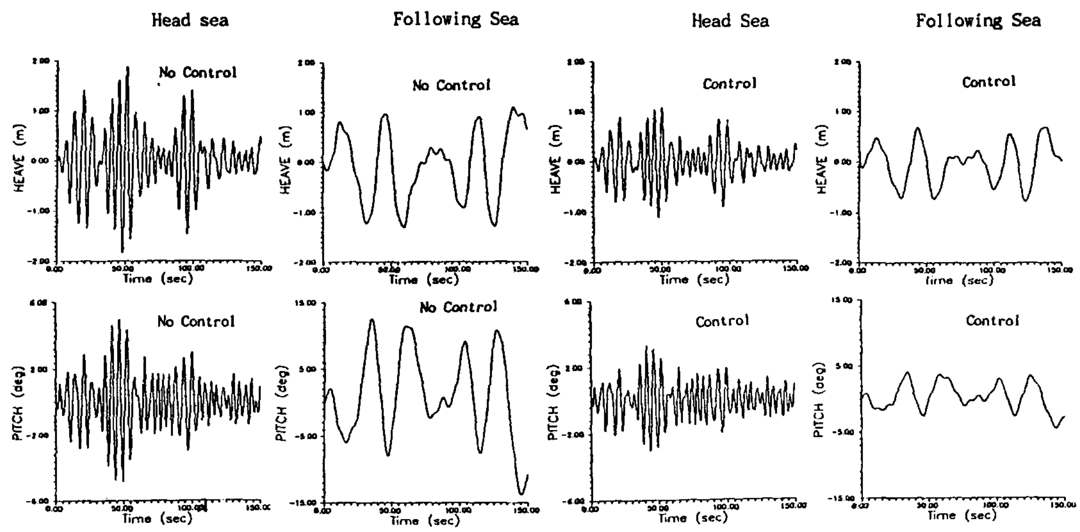


Fig. 5 Time domain simulation results(16 kts, sea state 5, head and following sea)

excessive fin rate increase should be solved. The simplest way may be to introduce a low-pass filter to remove high frequency components from sensor signal.

The following figure shows a typical feedback system.

The plant is G(s) and K(s) is the feedback compensator which is designed by LQ technique in this work. The plant output y(t) is $[Z(t)\ \theta(t)]^T$ the plant control input u(t) is $[\alpha_F(t),\ \alpha_A(t)]^T$

Table 2 Control performance with and without measurement noise

SPEED		16 KTs						20 KTS					
HEADING		HEAD SEA			FOLLOWING SEA			HEAD SEA			FOLLOWING SEA		
MEASUREMENT MODEL		1	2	3	1	2	3	1	2	3	1	2	3
Standard deviation (sea states 5)	$\dot{z}$ [m/s]	0.41	0.41	0.41	0.09	0.09	0.09	0.33	0.33	0.33	0.11	0.11	0.11
	z [m]	0.39	0.38	0.38	0.37	0.37	0.37	0.28	0.28	0.28	0.45	0.45	0.45
	$\dot{\theta}$ [°/s]	1.34	1.34	1.34	0.39	0.39	0.39	1.20	1.20	1.20	0.39	0.39	0.39
	θ [°]	1.05	1.05	1.05	1.88	1.88	1.88	0.85	0.85	0.85	1.50	1.50	1.50
	α_F [°]	3.80	3.81	3.94	2.83	2.86	3.04	3.76	3.78	3.95	3.74	3.76	3.96
	α_A [°]	2.85	2.86	2.93	2.75	2.75	2.80	2.59	2.60	2.68	2.80	2.81	.2.88
	$\dot{\alpha}_F$ [°/S]	4.92	6.86	15.9	0.59	4.82	15.15	5.49	7.90	18.81	1.35	5.85	18.05
	$\dot{\alpha}_A$ [°/S]	3.81	4.72	9.57	0.69	2.85	8.79	3.92	5.01	10.63	0.78	3.22	9.91

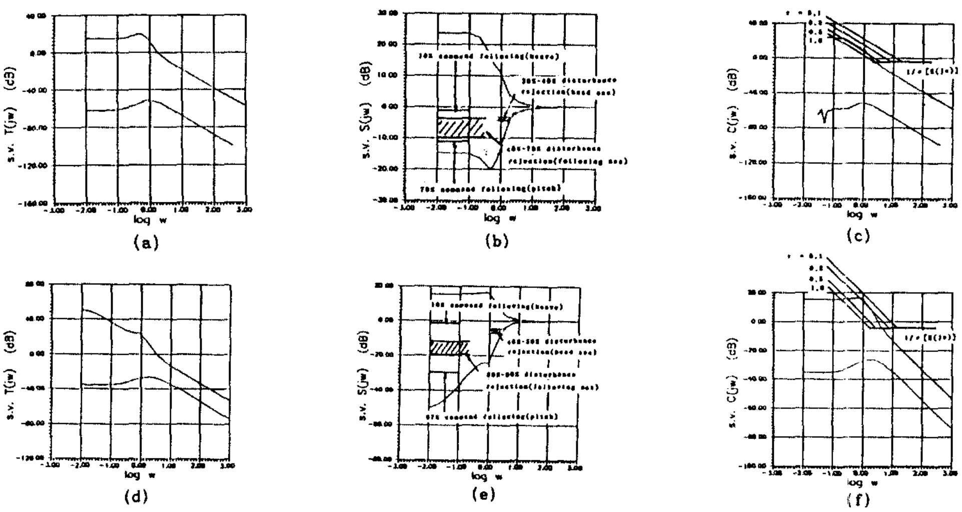
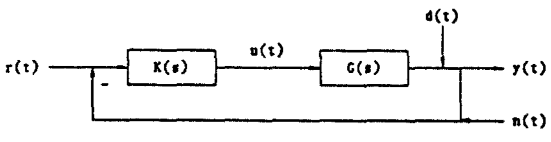


Fig. 6 Singular value plots of $T(j\omega)$, $S(j\omega)$ and $C(j\omega)$



and the reference input r(t) is heave and pitch commands $[Z_c(t)\ \theta_c(t)]^T$. The signal n(t) represents a sensor measurement noise and the disturbance d(t) means heave and pitch exciting forces reflected at the plant output. The 2 by 2 transfer function matrix (TFM) G(s) and K(s) of our system are.

$$G(s) = S[sI - A]^{-1}B, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K(s) = \begin{bmatrix} K_{11}s + K_{12} & K_{13}s + K_{14} \\ K_{21}s + K_{22} & K_{23}s + K_{24} \end{bmatrix}$$

where A(4 by 4) and B(4 by 2) matrices are those in equation (7) and K_{ij} is the component of the LQ optimal control gain matrix K.

To perform a frequency domain analysis, let's introduce the error e(t) = r(t) - y(t) and the following well known relationship.

$$y(s) = C(s)[r(s) - n(s)] + S(s)d(s)$$
$$e(s) = S(s)[r(s) - d(s)] + C(s)n(s)$$

where $S(s) = [I + T(s)]^{-1}$: sensitivity TFM
$C(s) = [I + T(s)]^{-1}T(s)$: closed-loop TFM
$T(s) = G(s)K(s)$: loop TFM

Fig. 6 shows the singular value plots of T(s), S(s) and C(s), $S=j\omega$, for speed of 16 and 20 kts. The abscissa means circular frequency ω in log scale and the singular value is presented on a longitude in decibel unit. The singular value of a matrix A means the square root of the eigenvalue of the matrix AA^H, which gives a measure of the magnitude of the TFM of a MIMO system.

From fig. of C(s),, fig. 6(c) and 6(f), we can see that σ_{max} crosses 0 dB at a very low frequency around 2 ~ 3 rad/s, consequently showing excellent insensitivity to sensor noise, which we've already observed through time domain simulation.

Fig. 6(b) and 6(e) contain the information on disturbuance rejection and command following characteristics of the system. However, the large gap between σ_{max} and σ_{min} makes it almost impossible to catch any features about these performances. So the results obtained by time domain simulation are overlaped to see that they are bounded by σ_{max} and σ_{min} at least. No discrepancy can be detected between the two results.

This large gap of singular values of S(s) also prevents from analyzing performance - robustness of the system. Introducing the multiplicative modeling error reflected at the plant output E(s), the degree of performance-robustness can be measured by the magnitude of S(s), since the portion of plant output due to this modeling error takes the following formula,

$$\Delta y = [I + T + ET]^{-1}Ey$$
$$\cong [I + T]^{-1}Ey = SEy$$

where y(t) is the closed-loop output of the modeled plant G(s).

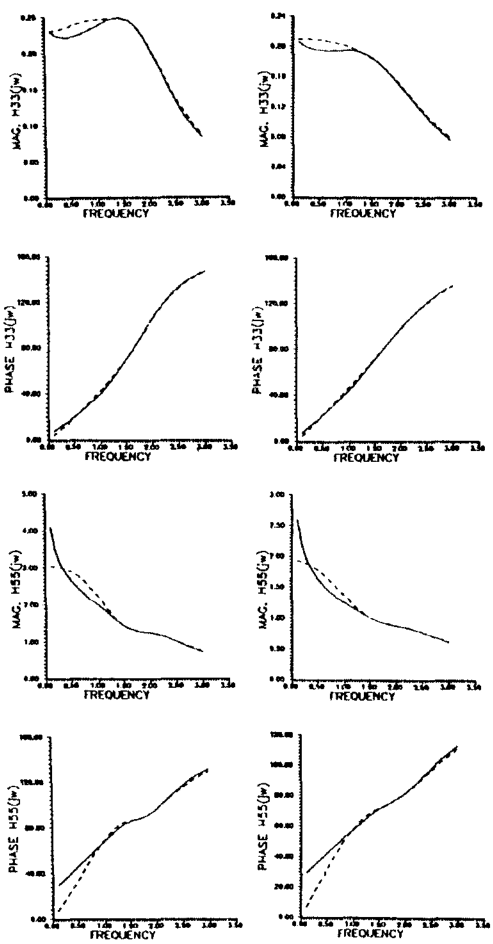
Only with fig. 6(b) and 6(e), we would be seriously concerned about the possibility of misfortune that severe performance degradation may appear in the real plant even with small modeling error E(s).

In this paper we've estimated the performance - robustness through frequency domain simulation by comparing the performance result of the modeled plant, equation (5), with that of the reference model, equation (1), assuming the latter is the real plant.

As mentioned earlier, the two results are almost identical showing good performance - robustness. Fig. 7, which shows the transfer functions H_{ij}(s) of the closed-loop system with plant models equation (1) and (5), reveals the reason for the above identity. The LQ control gain makes the closed-loop transfer functions approach to each other inspite of the large difference of the plant transfer functions between these two models, particularly at a low frequency zone.

In fig. 6(c) and 6(f), the Bode plot of a typical high frequency modeling error $E(s)=e^{-sz}-1$ is depicted to estimate stability - robustness of the system. The significant modeling error between equation (1) and (5) causes no adverse effects on stability - robustness, and thus is discarded from consideration, because the large error appears only at a low frequency zone where the phases of transfer functions are nearly zero.

______: reference model
_ _ _ _ _ _: time domain model



16 kts 20 kts

Fig. 7 Comparison of nondimensional closed - loop transfer functions of the reference model and the time domain approximation (frequency = 1.7(ω)

that is, never approach to the value of 180 degrees.

To guarantee the stability - robustness the following requirement is to be satisfied,

$\sigma_{max}[C(s)] < \sigma_{max}^{-1}[E(s)]$

Fig. 6(c) and 6(f) shows that if the retarding time λ becomes shorter than 0.2 second this system will be stable - robust. Our work neglects the dynamics of fin actuators. If this dynamics can be modeled by a second order differential equation, the above bound for λ is approximately equivalent to the requirement that the natural frequency of the actuators should be higher than 5 Hz if the damping ratio fixed at 0.7.

Investigating fig. 6, we can detect some abnormal results: the DC gains of S(s) and C(s) are greater than 0 dB which means that the ratio of response to command can be even greater than unity. This is very unusual and undesirable situation. So, we've surveyed various sets of command vector examining step responses, and found that there appears some equilibrium state of nonzero heave even with zero heave command when pitch command is not zero, and that this phenomenon is unavoidable consequence of large C_{35} and small heave feedback gain K_{12} , K_{22}. Specifically, when command vector is $[0m\ ,\ 1°]^T$, the bias of heave becomes -0.1 m, showing unlimited amplification of heave.

We agree that this unusual large DC gain and large gap between σ_{max} and σ_{min}, mentioned before, should be thoroughly investigated and avoided if possible. But within the work of our trade-off search we couldn't find any practical controller showing good performance, simultaneously satisfying the restrictions on fin movement.

Here, we'll consider the amount of the above heave bias as acceptable in the practical point of view, assuming heave control is of little importance relative to pitch control.

Summary and conclusions

This paper presents a detailed application procedure of the linear quadratic (LQ) theory for a SWATH heave and pitch control.

A time domain model of coupled, linear time-invariant second order differential equations is derived from the f quency response model of Lee [8] with the frequency dependant added mass and damping approximated as constant of the values at the heave natural frequency. This approximated model shows excellent agreement with the frequency response model at high frequency, whereas the closeness is very poor at a low frequency zone.

Wave exciting forces are modeled as the sum of sinusoids on the basis of the linear superposition principle.

The LQ problem is formulated with the state variable representation of the time domain model, introducing a proper performance index.

The solution to the LQ problem is the optimal control gain K, which is uniquely determined by the weighting matrix Q and R appearing in the performance index. A systematic selection procedure of weighting matrices is presented to obtain good transient responses and acceptable fin movements.

The validity of this controller design process is thoroughly investigated in several ways ; examination of system poles and step responses, simulations in a seaway both in time domain and frequency domain,and singular value plots of transfer function matrices.

The finally designed control system shows good transient features like percent overshoot and settling time, less than 30 % steady state error to pitch command, acceptable level of disturbance rejection, remarkable insensitivity to sensor noise, and reasonable performance - robustness and stability - robustness.

The important results acquired from the present study are;

1. The present time domain model ignoring frequency dependancy of added mass and damping is acceptable as a practical approximation for application of the LQ technique to SWATH heave and pitch control, providing that the designed control system shows reasonable performance - robustness.

2. The present systematic selection procedure of weightings Q and R can be used as a guideline for achieving desirable transient behavior and acceptable actuator movement.

3. The singular value plot of transfer function matrices, which shows a great deal of aspect on control performance for a MIMO system in usual cases, does not reveal quantitative information on command following, disturbance rejection and performance - robustness in this case, due to the large gap between σ_{max} and σ_{min} at low frequencies.

4. There appears no discrepancy between simulation results and singular value plots, implying that these techniques can be used complementarily in examining overall control performance, in spite of the abnormal large DC gain and large gap between σ_{max} and σ_{min} of transfer function matrices.

5. Though not fully investigated, the large C_{35} and small heave feedback gain K_{12} ,K_{22} are found to be the main causes for the above abnormality and no practical solution was found within the work of our trade-off search, implying that the requirement for heave control performance should be mitigated substantially relative to that for pitch performance.

6. Various important specifications for hardware implementation can be extracted such as requirements on sensor noise level and on actuator dynamics, for instance, through the present study, appealing that the applicability of the proposed technique is proved successful.

7. We will continue to search for the way to remove the abnormality since it is unusual and undesirable, though at present it is found to cause no problem in the practical point of view.

References

[1] Ware,J.R., Best,J.F. and Scott,V.A., "Application of Optimal Control Techniques to SWATH Vertical Plane Control",

ORI Technical Report 1682, March1980.

[2] Ware,J.R. and Scott,V.A., " Comparison of Platforming and Contouring Automatic Control Systems for SWATH Craft - An Optimal Approach ",*ORI Technical Report* 1790, September 1980.

[3] Zarnick,E.E., "Vertical Plane and Roll Motion Stabilization of SWATH Ships", *DTNSRDC/SPD* 1199-01,September 1986.

[4] Lee,J.G., " Controller Design in the Presense of Wave Exciting Force", *ADD SENP-413-86007*, December 1985.

[5] Park,S.H., Park,C.W. and Park,J.H., " SWATH Controller Design ", *Journal of SNAK*, Vol.22, No 4 , December 1985.

[6] McCreight,K.K. and Lee,C.M., "Manual for Mono-Hull or Twin-Hull Ship Motion Prediction Computer Program", DTNS RDC Report SPD-686-02, 1976.

[7] Song, C.T. et al, " Development of the SWATH Type Underwater Test Support Ship",at *the '93 conference of SNAK* (to be published).

[8] Lee,C.M., " Theoretical Prediction of Motion of Small-WaterPlane-Area, Twin-Hull(SWATH) Ships in Waves", DTNS RDC Report 76-0046, 1976

[9] Tick,L.J. , " Differential Equations with-Frequency - Dependant Coefficients", *Journal of Ship Research*, Vol.3, No.2, October 1959.

[10] Lee, C.M. and Curphey,R.M., " Prediction of Motion, Stability, and Wave Load of Small Waterplane-Area, Twin - Hull Ships", *SNAME Annual Meeting*, November 1977

[11] Kwakernaak,H. and Sivan,R., Linear Optimal Control Systems, Wiely-Interscience Division of John Wiley and Sons, Inc., 1972.

[12] Cox,G.G. and Lloyd,A.R., " Hydrodynamic Design Basis for Navy Ship Motion Stabilization", *SNAME Transactions*, Vol. 85, 1977.