Wave Response and Ship Motion in a Harbor Excited by Long Waves

Il-Hyoung Cho* and Hang S. Choi†

(From T.S.N.A.K., Vol. 29, No. 2, 1992)

Abstract

Herein the surge-heave-pitch motion of a ship in harbor has been analyzed within the framework of linear potential theory. The ship is assumed to be slender and moored at an arbitrary position in a rectangular harbor with a constant depth. The coast line is assumed to be straight. The ship and harbor responses to incident long waves are represented in terms of Green's function, which is the solution of the Helmholtz equation satisfying necessary boundary conditions. An integral equation is obtained from matching condition between harbor and ocean solutions, and it is replaced by an equivalent variational form. Numerical results show that the ship motion can be highly amplified at the frequencies, where the harbor is resonated by the incident wave. At the resonant frequencies, the added mass for vertical motions becomes negative and the damping force changes abruptly.

1. INTRODUCTION

As the sea transportation is being ever intensified in recent time, ships often have to wait in harbor for a long period of time until they are allowed to offload at piers due to the limited cargo-handling capacity of harbor. It is well known that harbors are excited by long waves. Harbor oscillations are the typical example. The harbor response in such a situation has long been investigated. On this topic, there are ample sources of literature, for example, a comprehensive description is given in the textbook of Mei[1]. Ships moored in harbor are also excited and they undergo motions of six-degrees of freedom. As a consequence, the ship in turn excites the harbor. In extreme cases, collisions with neighboring ships or pier fenders occur resulting in severe damages. In spite of its practical importance, ship motions in harbor have been seldom investigated. As a step to this direction, we studied in this paper the ship motion in a harbor.

The primary concern here is to clarify the physics involved rather than immediate applications. Under this aspect, we have simplified the problem as much as possible. The configuration of the harbor considered is a rectangle with constant depth and has a straight coastline. Lateral boundaries are vertical and perfectly reflective throughout the water depth.

*Member, Korea Research Institute of Ships and Ocean Engineering
†Member, Seoul National University

Waves are assumed to be small in amplitude and long compared to the depth, which implies that we are dealing with waves in shallow water. The width of the harbor entrance is assumed to be small compared to the wavelength. To an observer who views with the length scale of wave length, a ship looks like a shrunken line and its transverse dimensions are ineffective to disturb the incoming waves. It suggests us to assume the ship to be slender. Therefore a ship is represented by line sources along its centerline.

The leading-order modes of ship motions are horizontal, since the ship moves more or less as does a water particle and the horizontal component of the water particle motion is much greater than its vertical component in shallow water. The lateral motions such as sway and yaw are associated with strong blockage of the flow underneath the keel and thus the motion displacements are limited to some extent. In addition, the lateral motions are to be drastically reduced in real fluid because of flow separation. Here the surge motion is the primary mode of practical concern, which normally couples with heave and pitch, even though their displacements have magnitudes of one order smaller than that of the surge. Accordingly it is reasonable to confine our analysis to the coupled surge-heave-pitch motion of a slender ship.

The occurrence of negative added mass has been reported frequently in the literature. Examples, in which such phenomena have been observed, are followings: a ship close to a quay, a ship with a moon pool, catamaran problem, moving bodies close to the free surface(see Vinje[2]). The common feature of the above cases is that the structural configuration allows localized hydrodynamic resonance in the fluid domain adjacent to the bodies. The ship motion in harbor with which we are concerned falls into this category.

2. HARBOR RESPONSE

In order to formulate the problem, a right-handed Cartesian coordinate system together with polar coordinates are used. Let it be understood that in the ocean region the coordinate system (x, y) has its origin at the center of the entrance. To describe the interior of the harbor, however, it is convenient to use a different coordinate system(x_1, y_1) where the origin is located at a corner of the basin(see Fig.1). The positive z-axis points vertically upward. Under the usual assumptions of potential flow, the velocity potential is introduced as follows.

$$\Phi(x, y, z, t) = Re\{\phi(x, y, z)e^{-i\omega t}\} \tag{1}$$

$$\phi(x, y, z) = -\frac{ig}{\omega}\eta(x, y)\frac{\cosh\ k(z+h)}{\cosh\ kh} \tag{2}$$

$$\eta(x, y) = \frac{i\omega}{g}\phi(x, y, z)\mid_{z=0} \tag{3}$$

where g denotes the gravitational acceleration, h the constant depth of water and $\eta(x, y)$ the wave elevation. The wave frequency ω satisfies the following linearized dispersion relation for shallow water

$$\omega^2 = gk^2h \quad with \quad kh = O(kA) = o(1) \tag{4}$$

where A stands for the incident wave amplitude coming from the ocean. In the ocean

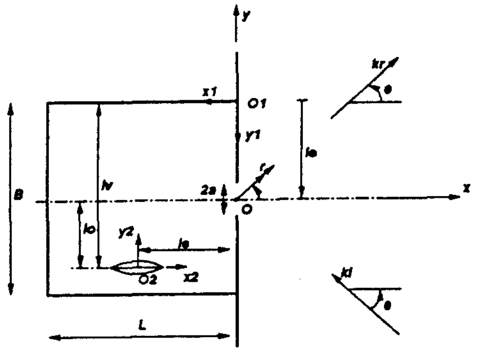

Figure 1: Definition sketch

region($x > 0$), the total wave system consists of incident wave, reflected wave from the straight coastline, and disturbances along the gap.

$$\eta^o = \eta_i + \eta_r + \eta_s \tag{5}$$

$$\eta_i + \eta_r = A\{e^{-ik(x\cos\theta - y\sin\theta)} + e^{ik(x\cos\theta + y\sin\theta)}\} \tag{6}$$

where the superscript o means the solution valid in the ocean region and θ is the incident angle as depicted in Fig. 1. The radiated wave η_s can be formally written(Ünlüata & Mei[3]) as

$$\eta_s = \frac{i\omega}{g}\int_{-a}^{a} U(y_o)[-\frac{i}{2}H_o^{(1)}(kr)]dy_o \tag{7}$$

where $r^2 = x^2 + (y - y_o)^2$, $U(y)$ denotes the velocity across the harbor entrance, and $H_o^{(1)}$ the Hankel function of the 1st kind.

Inside of the harbor, the wave function is merely the radiated wave from the harbor entrance.

$$\eta^h = \frac{i\omega}{g}\int_{l_e-a}^{l_e+a} U(y_o)G^h(x_1, y_1/y_o)dy_o \tag{8}$$

where the superscript h means the solution in the harbor and l_e denotes the distance between the origin and the harbor entrance. G^h represents the solution for a point sink of unit discharge and it is a kind of Green's function. For a detailed derivation, it is referred to Mei[1]. Here we only cite the result:

$$\begin{aligned} G^h(x_1, y_1/y_o) &= -\sum_{n=0}^{\infty} X_n(x_1)Y_n(y_1)Y_n(y_o) \\ X_n(x_1) &= \frac{\varepsilon_n \cos k_n(x_1 - L)}{k_n B \sin k_n L} \\ Y_n(y_1) &= \cos\frac{n\pi}{B}y_1 \\ k_n &= [k^2 - (\frac{n\pi}{B})]^{\frac{1}{2}} \end{aligned} \tag{9}$$

where ε_n is the Jacobi symbol, namely $\varepsilon_o = 1$, $\varepsilon_n = 2, n \geq 1$. An integral equation for the unknown velocity U can be constructed based on the continuity of wave function across the entrance. We require $\eta^o = \eta^h at\ x = 0, \mid y \mid < 0$.

$$\frac{i\omega}{g}\int_{-a}^{a} U(y_o)M(y/y_o)dy_o = 2A\cos(ky\sin\theta) \tag{10}$$

where $M(y/y_o) = G^h(0, l_e - y/l_e - y_o) + \frac{i}{2}H_o^{(1)}(k \mid y - y_o \mid)$ It can be shown that solving the integral equation is equivalent to finding the extreme of the following functional.

$$\begin{aligned} J(U(y)) &= \int_{-a}^{a} 4AU(y)\cos(ky\sin\theta)dy \\ &- \frac{i\omega}{g}\int\int_{-a}^{a} U(y)M(y/y_o)U(y_o)dy_o dy \end{aligned} \tag{11}$$

To simplify the problem, a uniform velocity distribution is assumed. A gross global error is not likely to occur due to this approximation. For J to be stationary, U is chosen such that $dJ/dU = 0$.

$$U = \frac{4A\sin(ka\sin\theta)/k\sin\theta}{\frac{i\omega}{g}\int\int_{-a}^{a} M(y/y_o)dy_o dy} \tag{12}$$

To enhance the convergence of a series under the assumption of narrow entrance, the double integral expressed in Eqn. (12) is reduced to the following form.

$$\begin{aligned} \frac{1}{4a^2}\int\int_{-a}^{a} M dy_o dy &\simeq \frac{i\omega}{g}(\frac{i}{2} + F - X - Y) \\ &+ O(k^2a^2\ln(ka)), \end{aligned} \tag{13}$$

where

$$\begin{aligned} F &= -(\ln\frac{\pi\gamma ka^2}{4B} + \ln 16 - 3)/\pi, \\ X &= \frac{\cot kL}{kB} + 2\sum_{n=1}^{\infty}\{\frac{\cot k_n L}{k_n B} + \frac{1}{n\pi}\}(\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})^2, \\ Y &= \frac{l_e}{2\pi a}\ln\frac{l_e + a}{l_e - a} + \frac{1}{2\pi}\ln(l_e^2 - a^2) + \frac{1}{\pi}(\ln\frac{\pi}{B} - 1) \\ &+ \frac{B}{2a\pi^2}\sum_{k=1}^{\infty}(-1)^k\frac{B_{2k}}{(2k+1)!4k}(\frac{2\pi}{B})^{2k+1}[(l_e + a)^{2k+1} - (l_e - a)^{2k+1}] \end{aligned} \tag{14}$$

Details of the derivation are given in Appendix. From Eqns (8) and (12), the response at a point (x_1, y_1) in the harbor yields to

$$\begin{aligned} \eta^h(x_1, y_1) &= - \frac{2A}{\frac{i}{2} + F - X - Y}\frac{\sin(ka\sin\theta)}{ka\sin\theta} \\ &\times [\frac{\cos k(x_1 - L)}{kB\sin kL} + 2\sum_{n=1}^{\infty}\frac{\cos k_n(x_1 - L)}{k_n B\sin k_n L} \\ &\times (\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})\cos\frac{n\pi}{B}y_1] \end{aligned} \tag{15}$$

The harbor response given above is to be utilized as the incident wave function to the ship in a rectangular basin. The normalized mean square response takes the form,

$$\sigma^2 = \frac{1}{2BL}\int_0^L dx_1 \int_0^B dy_1 \mid \frac{\eta^h}{2A} \mid^2 \tag{16}$$

3. OUTER PROBLEM FOR A SLENDER SHIP

Now we turn to the disturbed waves by the ship, of which slenderness is defined by

$$\delta = \frac{d}{2l} \tag{17}$$

where $2l$ and d denote the ship's length and draft, respectively. Without loss of generality, we may assume $\delta = O(kA)$, which implies that the draft is comparable with the water depth and small in comparison with the wavelength. Consequently we are dealing with the ship motion in shallow water confined by the harbor boundary. Beck & Tuck[4] analyzed the ship motion problem in unbounded shallow water rigorously based on the matched asymptotic method. We shall utilize their work by modifying it for different incident waves and Green's function. The velocity potential induced by each mode of ship motion may be defined as

$$\begin{aligned} \eta_j(x,y) &= \frac{i\omega}{g}\phi_j(x,y) \\ \phi_j(x,y) &= -i\omega\zeta_j\varphi_j(x,y) \end{aligned} \tag{18}$$

where ζ_j is the complex displacement for the j-th motion ($j = 1,3,6$). Let $O_2x_2y_2z_2$ be a fixed coordinate system attached to the ship. The wave which radiates to the ocean through the entrance can be represented by source distribution.

$$\eta_j^o(x,y) = \frac{i\omega}{g}\int_{-a}^{a} u_j(y_o)[-\frac{i}{2}H_o^{(1)}(kr)]dy_o \tag{19}$$

As far as the outer region is concerned, the ship has shrunk down to a ribbon $y_2 = 0$, $|x_2| < l$ and the boundary condition has to be applied to the ribbon. The disturbed wave field η_j^h is expressed by invoking Green's theorem.

$$\begin{aligned} \eta_j^h(x_2,y_2) &= \frac{i\omega}{g}\{\int_{-l}^{l}\Delta v_j(x_o)H(x_2,y_2,x_o,0)dx_o \\ &\quad - \int_{l_o-a}^{l_o+a} u_j(y_o)H(x_2,y_2,l_s,y_o)dy_o\} \end{aligned} \tag{20}$$

$$\text{with} \quad \Delta v_j(x) = \frac{\partial\phi_j(x,o^+)}{\partial y} - \frac{\partial\phi_j(x,o^-)}{\partial y}$$

where l_o denotes the distance between the centerline of the ship and the harbor entrance, l_s between the midship and the harbor entrance. In a similar manner as for the Green function

G^h given in Eqn. (9), the Green function H can be readily derived by satisfying the Helmholtz equation and appropriate boundary conditions.

$$\begin{aligned} H(x_2,y_2,x_o,y_o) &= \sum_{n=0}^{\infty} \frac{\varepsilon_n \cos k_n(x_2-l_s)\cos k_n(x_o+L-l_s)}{k_n B \sin k_n L} \\ &\quad \times \cos\frac{n\pi}{B}(y_2-l_v)\cos\frac{n\pi}{B}(y_o-l_v), \quad x_2>x_o \qquad (21) \\ H(x_2,y_2,x_o,y_o) &= \sum_{n=0}^{\infty} \frac{\varepsilon_n \cos k_n(x_o-l_s)\cos k_n(x_2+L-l_s)}{k_n B \sin k_n L} \\ &\quad \times \cos\frac{n\pi}{B}(y_2-l_v)\cos\frac{n\pi}{B}(y_o-l_v), \quad x_2<x_o \qquad (22) \end{aligned}$$

with $k_n = [k^2 - \frac{n\pi}{B}]^{\frac{1}{2}}$ where x_o and y_o denote the source point in the (O_2, x_2, y_2) coordinate system. As $x_2 \to x_o$ and $y_2 \to y_o$, the asymptotic form of the Green function H is obtained after some tedious arithmatic operations (see Collin[5]).

$$\begin{aligned} H(x_2,y_2,x_o,y_o) &\simeq \frac{\cos k(x_2-l_s)\cos k(x_o+L-l_s)}{kB\sin kL} \\ &+ 2\sum_{n=0}^{\infty}\left[\frac{\cos k_n(x_2-l_s)\cos k_n(x_o+L-l_s)}{k_n B\sin k_n L} + \frac{e^{-\frac{n\pi}{B}(x_2-x_o)}}{n\pi}\right] \\ &\times \cos\frac{n\pi}{B}(y_2-l_v)\cos\frac{n\pi}{B}(y_o-l_v) \\ &+ \frac{1}{2\pi}\ln\{\frac{2\pi}{B}\sin\frac{\pi}{B}(y_o-l_v)\} \\ &+ \frac{1}{2\pi}\ln\{(x_2-x_o)^2+(y_2-y_o)^2\}^{1/2}, \quad x_2>x_o \qquad (23) \\ H(x_2,y_2,x_o,y_o) &\simeq \frac{\cos k(x_o-l_s)\cos k(x_2+L-l_s)}{kB\sin kL} \\ &+ 2\sum_{n=0}^{\infty}\left[\frac{\cos k_n(x_o-l_s)\cos k_n(x_2+L-l_s)}{k_n B\sin k_n L} + \frac{e^{-\frac{n\pi}{B}(x_o-x_2)}}{n\pi}\right] \\ &\times \cos\frac{n\pi}{B}(y_2-l_v)\cos\frac{n\pi}{B}(y_o-l_v) \\ &+ \frac{1}{2\pi}\ln\{\frac{2\pi}{B}\sin\frac{\pi}{B}(y_o-l_v)\} \\ &+ \frac{1}{2\pi}\ln\{(x_2-x_o)^2+(y_2-y_o)^2\}^{1/2}, \quad x_2<x_o \qquad (24) \end{aligned}$$

For matching with the outer expansion of the inner solution, we need an inner expansion of the outer solution, which is in the present case simply a Taylor series expansion of η_j^h for small y_2 (see Tuck[6]).

$$\begin{aligned} \eta_j^h(x_2,y_2) &\sim \frac{i\omega}{g}\{W_j(x_2) + \frac{\Delta v_j(x_2)}{2} \mid y_2 \mid + V_j(x_2)y_2\} \\ W_j(x_2) &= \int_{-l}^{l} \Delta v_j(x_o) H(x_2,0,x_o,0)dx_o \\ &- \int_{l_o-a}^{l_o+a} u_j(y_o) H(x_2,0,l_s,y_o)dy_o \end{aligned}$$

$$V_j(x_2) = \int_{-l}^{l} \Delta v_j(x_o) \frac{\partial \tilde{H}(x_2, 0, x_o, 0)}{\partial y_2} dx_o - \int_{l_o-a}^{l_o+a} u_j(y_o) \frac{\partial H(x_2, 0, l_s, y_o)}{\partial y_2} dy_o \tag{25}$$

where $\tilde{H}$ denotes the remaining regular function subtracting the log function from the asymptotic forms given in Eqns (23) and (24).

4. INNER PROBLEM FOR A SLENDER SHIP

The inner region near the ship has the following order of magnitude

$$x_2 = O(1), \qquad y_2, z_2 = O(\epsilon) \tag{26}$$

The governing equation and the appropriate boundary conditions are thus

$$(\frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2})\phi_j^s = 0 \tag{27}$$

$$\frac{\partial \phi_j^s}{\partial N} = -i\omega\zeta_j n_j \quad \text{on ship} \tag{28}$$

$$\frac{\partial \phi_j^s}{\partial z_2} = 0 \quad \text{at} \quad z_2 = 0, -h \tag{29}$$

$$\phi_j^s \to W_j(x_2) + \frac{\Delta v_j(x_2)}{2} \mid y_2 \mid + V_j(x_2) y_2 \quad \text{as} \quad y_2 \to \infty \tag{30}$$

where the superscript s denotes the solution valid in the region close to the ship and N normal to the cross section C at station x_2. Since the inner solution behaves as shown in Eqn. (30) for large $\mid y_2 \mid$, we split the boundary-value problem into two parts, so as that the radiation condition is symmetric with respect to y_2 in one part and asymmetric in the other part.

$$\begin{aligned}
&(\frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2})\phi_j^{s1} = 0, \\
&\frac{\partial \phi_j^{s1}}{\partial N} = -i\omega\zeta_j n_j \quad \text{on ship}, \\
&\frac{\partial \phi_j^{s1}}{\partial z_2} = 0 \quad \text{at} \quad z_2 = 0, -h \\
&\phi_j^{s1} \to W_j(x_2) + \frac{\Delta v_j(x_2)}{2} \mid y_2 \mid \quad as \quad y_2 \to \infty
\end{aligned} \tag{31}$$

$$\begin{aligned}
&(\frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial z_2^2})\phi_j^{s2} = 0 \\
&\frac{\partial \phi_j^{s2}}{\partial N} = 0 \quad \text{on ship} \\
&\frac{\partial \phi_j^{s2}}{\partial z_2} = 0 \quad \text{at} \quad z_2 = 0, -h \\
&\phi_j^{s2} \to V_j(x_2) y_2 \quad \text{as} \quad y_2 \to \infty
\end{aligned} \tag{32}$$

ϕ_{s1} represents symmetric outgoing waves for y_2 due to the symmetric motion of the ship and ϕ_{s2} asymmetric part due to the reflection from lateral boundaries and the radiation from harbor entrance. Therefore, by solving two boundary-value problems, the inner solution becomes completed. The term $\Delta v_j(x_2)$, which we need in outer field, reflects the flux across the hull section C. Thus the source strength can be immediately obtained in terms of body geometric functions

$$\begin{aligned}\Delta v_j(x_2) &= -\frac{i\omega\zeta_j}{h}A_j(x_2)\\ A_j(x_2) &= \int_S n_j dl\end{aligned} \tag{33}$$

The integration is performed along the contour of transverse section with the normal being approximated to the plane. For symmetric modes(surge, heave, and pitch), these are as follows:

$$\begin{aligned}A_1(x_2) &= -S'(x_2)\\ A_3(x_2) &= -b(x_2)\\ A_5(x_2) &= x_2 b(x_2)\end{aligned}$$

where $S'(x_2)$ means the longitudinal variation of the sectional area at the station x_2, $b(x_2)$ the breadth. Substituting source strength into Eqn. (20), we get the wave field in the harbor generated by the ship motion

$$\begin{aligned}\eta_j^h(x_2, y_2) &= \frac{\omega^2}{gh}\zeta_j\int_{-l}^{l} A_j(x_o)H(x_2, y_2, x_o, 0)dx_o\\ &- \frac{i\omega}{g}\int_{l_o-a}^{l_o+a} u_j(y_o)H(x_2, y_2, l_s, y_o)dy_o\end{aligned} \tag{34}$$

The unknown function $u_j(y)$ is to be determined by invoking the matching condition that the surface height must be continuous at every point along the harbor entrance.

$$\eta_j^o(x, y)\mid_{x=0} = \eta_j^h(x_2, y_2)\mid_{x_2=l_s} \quad \mid y \mid < a \tag{35}$$

Following the similar procedure as the previous section, we obtain the uniform velocity u_j generated by ship motion at the harbor entrance.

$$\begin{aligned}u_j &= \frac{1}{\frac{i}{2}+F-X-Y}\left(\frac{i\omega\zeta_j}{2ah}\right)\\ &\times \int_{-l}^{-l} A_j(x_o)\left[\frac{\cos k(x_o+L-l_s)}{kB\sin kL}\right.\\ &+ 2\sum_{n=1}^{\infty}\frac{\cos k_n(x_o+L-l_s)}{k_n B\sin k_n L}\\ &\times \left.\left(\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha}\right)\cos\frac{n\pi}{B}l_v\right]dx_o\end{aligned} \tag{36}$$

Substituting u_j into Eqn. (34), we get the complete expression for symmetric modes.

$$\eta_j^h(x_2, y_2) = \frac{\omega^2}{gh}\zeta_j[\int_{-l}^{l} A_j(x_o)H(x_2, y_2, x_o, 0)dx_o + D_j] \quad (37)$$

where

$$\begin{aligned} D_j &= \frac{1}{\frac{i}{2} + F - X - Y}\int_{-l}^{-l} A_j(x_o)[\frac{\cos k(x_o + L - l_s)}{kB\sin kL} \\ &+ 2\sum_{n=1}^{\infty}\frac{\cos k_n(x_o + L - l_s)}{k_nB\sin k_nL} \\ &\times (\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})\cos\frac{n\pi l_v}{B}]dx_o \times [\frac{\cos k(x_2 + L - l_s)}{kB\sin kL} \\ &+ 2\sum_{n=1}^{\infty}\frac{\cos k_n(x_2 + L - l_s)}{k_nB\sin k_nL} \\ &\times (\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})\cos\frac{n\pi}{B}(y_2 - l_v)] \end{aligned} \quad (38)$$

In order to estimate the motion responses, we have to solve the equation of motion

$$\sum_j(-\omega^2 M_{ij} - T_{ij} + C_{ij})\zeta_i = F_i \quad (i, j = 1, 3, 5) \quad (39)$$

Here M_{ij} is a generalized mass matrix.

$$\begin{bmatrix} M & 0 & Mz_G \\ 0 & M & -Mz_G \\ Mz_G & -Mx_G & I_{55} \end{bmatrix}$$

where M is referred to the ship mass, $(x_G, 0, z_G)$ to the center of gravity, I_{55} to the moment of pitching inertia. C_{ij} is a matrix of restoring force coefficients, including all hydrostatic effects and mooring force, of which non-zero components are

$$\begin{aligned} C_{11} &= K_m \\ C_{33} &= \rho g A_w \\ C_{35} &= C_{53} = -\rho g x_F A_w \\ C_{55} &= \rho g K_w^2 A_w \end{aligned}$$

where K_m means the horizontal component of mooring stiffness, A_w the waterplane area, x_F the longitudinal center of floatation and K_w the radius of gyration of the waterplane. T_{ij} is the hydrodynamic coefficients defined by

$$\begin{aligned} T_{i,j} &= -\rho\omega^2 \int_S n_i\varphi_j dS = \omega^2 a_{ij} + i\omega b_{ij} \\ &= -\tfrac{\rho\omega^2}{h}\{ \int\int_{-l}^{l} A_j(x_o)A_i(x_2)H(x_2, 0, x_o, 0)dx_o dx_2 + \\ &\qquad \int_{-l}^{l} A_i(x_2)D_j(x_2, 0)dx_2\} \end{aligned} \quad (40)$$

where φ_j denotes the normalized radiation potential for $j-th$ mode, a_{ij} and b_{ij} added mass and damping coefficients, respectively. The exciting force and moment are evaluated in terms of radiation potential by invoking the Haskind relation.

$$F_i = (T_{io} - T_{oi})2A = (T_{io} + T_{i7})2A \tag{41}$$

where the subscript o corresponds to the incident wave to the ship, which is provided by solving the harbor response without the ship. T_{i7} can be evaluated from Eqn. (4.15). The body geometric function needed in T_{i7} is readily determined by utilizing the long wave approximation

$$n_7 = -n_o \approx -[n_1\frac{\partial\phi_o}{\partial x_2} + n_2\frac{\partial\phi_o}{\partial y_2} + n_3\frac{\partial\phi_o}{\partial z_2}] \tag{42}$$

where ϕ_o is given from the previous section. Substituting ϕ_o into the above equation and integrating along the section contour for constant x_2, we immediately have

$$\begin{aligned} A_7(x_2) = & - \frac{1}{\frac{i}{2}+F-X-Y}\frac{\sin(ka\sin\theta)}{ka\sin\theta} \\ \times & [S'(x_2)\{\frac{\sin k(x_2+L-l_s)}{B\sin kL} + 2\sum_{n=1}^{\infty}\frac{\sin k_n(x_2+L-l_s)}{B\sin k_nL} \\ \times & (\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})\cos\frac{n\pi l_v}{B}\} \\ + & k^2\{S(x_2)-hb(x_2)\}\frac{\cos k(x_2+L-l_s)}{kB\sin kL} \\ + & 2\sum_{n=1}^{\infty}\{k_n^2S(x_2)-k^2hb(x_2)\}\frac{\cos k_n(x_2+L-l_s)}{k_nB\sin k_nL} \\ \times & (\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})\cos\frac{n\pi l_v}{B}] \end{aligned} \tag{43}$$

Using the incident wave potential, T_{io} is easily calculated by the integral below

$$\begin{aligned} T_{io} = \rho\ \ g & \frac{1}{\frac{i}{2}+F-X-Y}\frac{\sin(ka\sin\theta)}{ka\sin\theta}\int_{-l}^{l}A_i(x_2) \\ \times & [\frac{\cos k(x_2+L-l_s)}{kB\sin kL} + 2\sum_{n=1}^{\infty}\frac{\cos k_n(x_2+L-l_s)}{k_nB\sin k_nL} \\ \times & (\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})\cos\frac{n\pi}{B}l_v]dx_2 \end{aligned} \tag{44}$$

With help of Eqn. (40)-(44), the motion response ζ_i can be estimated from Eqn. (39).

5. RESULTS & DISCUSSION

As a numerical example, we consider a S-175 container ship, which is moored by a single-point linear cable at the center of a rectangular harbor, The length and width of the harbor are both $2000m$. Since the block coefficient of the ship is 0.57, the slender body approximation may be justified. The specifications are listed in Table 1. To examine the asymmetric

Table 1: Particulars of S-7-175 container ship

Ship Length($2l$)	175.00(m)
Beam at Midship(b)	25.40(m)
Draft at Midship(d)	9.50(m)
Displacement	24742(tons)
Center of Gravity(KG^*)	9.52(m)
Center of gravity(LCG^*)	90.20(m)
Center of Buoyancy(KB^*)	5.19(m)
Center of Floatation(LCF^{**})	94.31(m)
Pitch Gyration(K_W)	42.00(m)
Waterplane Area(A_W)	3155(m^2)
Block Coefficient(C_b)	0.57
Design Ship Speed(V)	22.13(knots)
Mooring Stiffness(K_m)	99640(N/m)

*) Keel reference
**) F.P. reference

contribution to the responses, the harbor entrance is located at the one quarter of the harbor breadth.

Fig.2 shows the normalized flux intensity at the entrance for $2a/B$=0.03, which clearly indicates sharp tunings at the natural frequencies of the harbor. Compared to the case of the symmetric harbor(Choi & Cho[7]), the overall response looks similar but the third peak at kL=4.6 is responsible for the asymmetry. It is not to mention that the peak values are of qualitative meaning only, because real fluid effects are not included herein.

Fig.3 shows the wave response ratio at the harbor boundary (x=1000m, y=0m), which is amplified exactly at the natural frequencies. It is to note that the Helmholtz mode gives the largest response and the asymmetric mode is rather insignificant. The root mean square of the wave response without the presence of a ship is depicted in Fig.4. From this result, we may conclude that the harbor response in an average sense differs only slightly according to the modes except the Helmholtz mode.

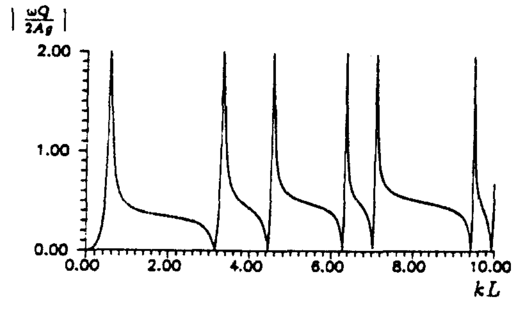

Figure 2: Normalized flux intensity

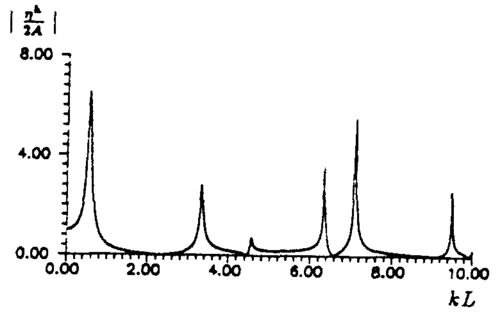

Figure 3: Amplification factor($x_1 = 2000m, y_1 = 0m$)

The heaving added mass is illustrated in Fig.5. It is to note that its value becomes negative at kL=0.5,6.2 and 8.8. The first two corresponds to the first and the fourth mode of the harbor response, but the third one is independent of it. At these frequencies, the wave damping force changes sharply as shown in Fig. 6.

The heave exciting force and the resulting heave amplitude ratio are given in Fig.7 and Fig.8, respectively. It is recognized that the two curves show the same trend and the most peak values occur when added mass becomes negative. A similar behavior can be observed in the case of the pitch. The added moment of inertia for pitch also becomes negative at kL=0.5,6.2 and 8.8(see Fig.9). The pitch response is shown in]ig.10. The only difference from the heave is an additional tuning at kL=9.2 beside the scale in magnitude.

The surging added mass contains sharp changes, but the corresponding frequencies are different from those for heave and pitch, and the magnitude is always positive as shown in Fig.11. Fig.12 illustrates the surge amplitude ratio. The peak value at kL=9.2 is attributed to the resonance caused by the mooring cable, while that at kL=9.5 to the exciting force. Although these values are based on potential theory, it indicates that an extremely large horizontal excursion may happen and collisions with neighboring ships are possible.

As a closing remark, it is emphasized that mathematical models for the problem of the ship motions in harbor must include the wave radiation outside the harbor, since these models enable us to predict the added mass and wave damping correctly.

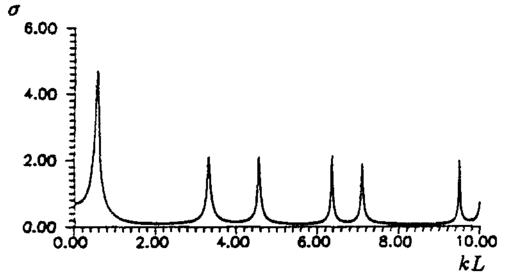

Figure 4: Root mean square response

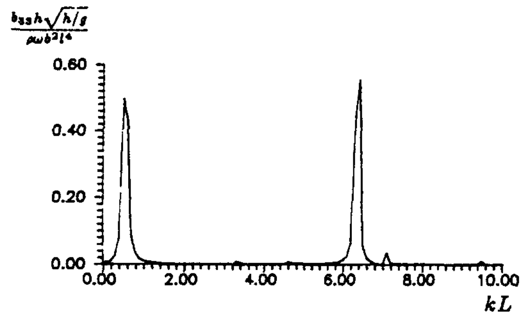

Figure 6: Damping coefficient for heave

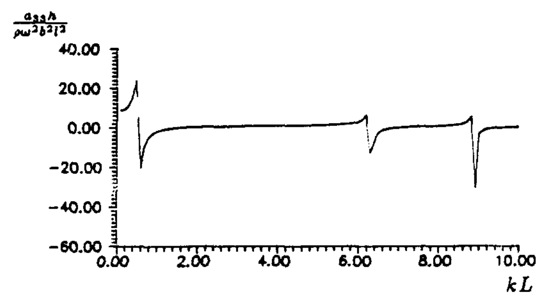

Figure 5: Added mass for heave

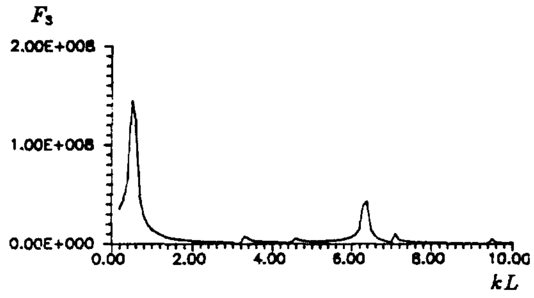

Figure 7: Heave exciting force

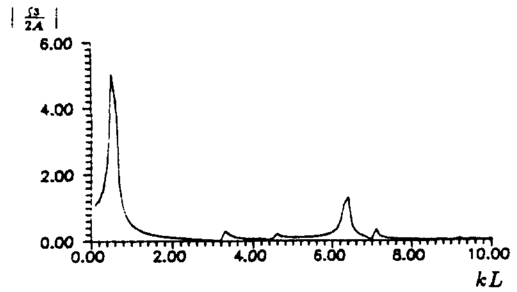

Figure 8: Heave amplitude ratio

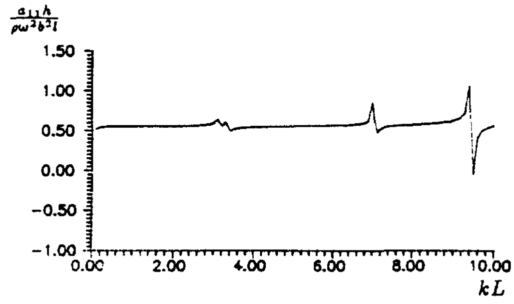

Figure 11: Added mass for surge

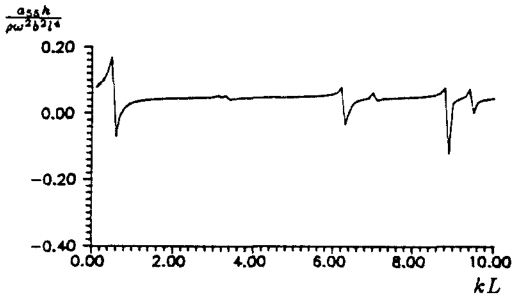

Figure 9: Added mass for pitch

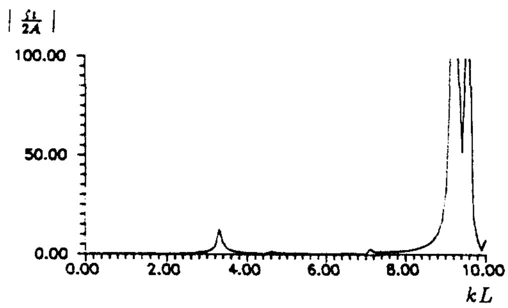

Figure 12: Surge amplitude ratio

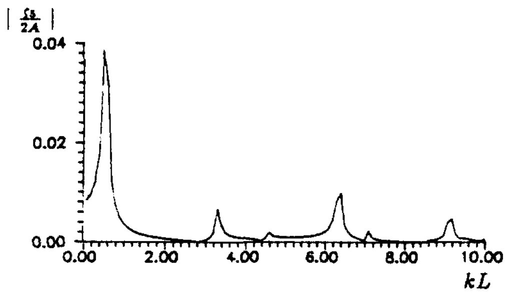

Figure 10: Pitch amplitude ratio

References

[1] Mei, C.C., *The Applied Dynamics of Ocean Surface Waves,* John Wiley & Sons, 1983.

[2] Vinje, T., "On the significance of negative added mass," 8th OMAE Symposium, 1985, pp.53-61.

[3] Üntüata, U. and Mei, C.C., "Effects of entrance loss on harbor oscillations," J. Waterways Harbors and Coastal Eng. Div. ASCE 101, 1975, pp.161-180.

[4] Beck, R.F. and Tuck, E.O., "Computation of shallow water ship motion," Proc. 9th Symp. on Naval Hydrodyn., 1972 pp.1543-1587.

[5] Collin, R.E., *Field Theory of Guided Waves*, McGraw-Hill, New York, 1960.

[6] Tuck, E.O., "Ship motions in shallow water," J. Ship Res., Vol.14, 1970, pp.317-328.

[7] Choi, H.S. and Cho, I.H., "On the surge motion of a ship in a rectangular harbor," J. Korea Society of Coastal and Ocean Engineers, Vol.1, 1989, pp.81-86(in Korean).

[8] Gradshteyn, I.S. and Ryzhik, I.M., *Table of Integrals, Series, and Products,* Academic Press, Inc., 1965.

Appendix A:

The integral in hand Eqn. (13) is

$$\begin{aligned} & \frac{1}{4a^2}\int\int_{-a}^{a} M(y/y_o)dydy_o \\ = & \frac{1}{4a^2}\int\int_{-a}^{a}[G^h(0,l_e-y/l_e-y_o)+\frac{i}{2}H_o^{(1)}(k\mid y-y_o\mid)]dydy_o \\ = & \Upsilon_h+\Upsilon_o \end{aligned} \tag{A.1}$$

It follows

$$\begin{aligned} \Upsilon_o &= \frac{1}{4a^2}\frac{i}{2}\int\int_{-a}^{a} H_o^{(1)}(k\mid y-y_o\mid)dydy_o \\ &= \frac{i}{2}\int\int_{-1/2}^{1/2} H_o^{(1)}(2ak\mid \xi-\xi'\mid)d\xi d\xi' \end{aligned} \tag{A.2}$$

after the transformation $y=2a\xi$,$y_o=2a\xi'$. Approximating $H_o^{(1)}$ for small ka, we get

$$\begin{aligned} \Upsilon_o &= \frac{i}{2}(1+\frac{2i}{\pi}\ln\gamma ka)-\frac{1}{\pi}\int\int_{-1/2}^{1/2}\ln\mid\xi-\xi'\mid d\xi d\xi' \\ &= \frac{i}{2}-\frac{1}{\pi}\ln\gamma ka+\frac{3}{2\pi} \end{aligned} \tag{A.3}$$

where the following identity has been used

$$\int\int_{-1/2}^{1/2} \ln \mid \xi - \xi' \mid d\xi d\xi' = \frac{3}{2} \tag{A.4}$$

and $\ln\gamma(=0.5772157)$ means Euler's constant. In a similar manner as for Υ_o, it follows

$$\begin{aligned} \Upsilon_h &= \frac{1}{4a^2}\int\int_{-a}^{a} G^h(0, l_e - y/l_e - y_o) dy dy_o \\ &= -\frac{1}{4a^2}\int\int_{-a}^{a} \sum_{n=0}^{\infty} \frac{\varepsilon_n \cos k_n L}{k_n B \sin k_n L} \\ &\times \cos\frac{n\pi}{B}(y - l_e)\cos\frac{n\pi}{B}(y_o - l_e) dy dy_o \end{aligned} \tag{A.5}$$

For large n, we separate the series into singular and regular parts(see Mei[1]). The singular part takes the form, which dies out as fast as $1/n$

$$S = -\sum_{n=1}^{\infty} \frac{2}{n\pi} \cos\frac{n\pi}{B} \cos\frac{n\pi}{B}(y - l_e)\cos\frac{n\pi}{B}(y_o - l_e) \tag{A.6}$$

The regular part

$$\begin{aligned} R &= \frac{\cot kL}{kB} + \sum_{n=1}^{\infty}\left(\frac{2\cot k_n L}{k_n B} + \frac{2}{n\pi}\right) \\ &\times \cos\frac{n\pi}{B}(y - l_e)\cos\frac{n\pi}{B}(y_o - l_e) \end{aligned} \tag{A.7}$$

converges much faster than the singular part. The singular series can be converted into the following form

$$S = -\sum_{n=1}^{\infty} \frac{1}{2\pi}\left[\left(\frac{e^{-nZ_s}}{n} + \frac{e^{-nZ'_s}}{n}\right) + *\right] \tag{A.8}$$

where $Z_s = j\frac{\pi}{B}(y - y_o)$, $Z'_s = j\frac{\pi}{B}(y + y_o - 2l_e)$ and $*$ denotes the complex congugate. The final result is

$$S = \frac{1}{2\pi}\ln\{\mid 1 - e^{-Z_s} \mid^2 \mid 1 - e^{-Z'_s} \mid^2\} \tag{A.9}$$

where the following formula is used

$$\sum_{n=1}^{\infty} \frac{e^{-ns}}{n} = -\ln(1 - e^{-s}) \tag{A.10}$$

According to the assumption that the harbor dimension is much larger than the entrance width, we must have $\mid Z_s \mid \ll 1$. Since

$$\begin{aligned} Z'_s &= Z_s + \frac{2i\pi}{B} y_o - \frac{2i\pi}{B} l_e \\ 1 - e^{-Z_s} &\simeq Z_s(1 + O(Z_s)) \\ 1 - e^{-Z'_s} &\simeq (1 - e^{2i\pi(l_e - y_o)/B})(1 + O(Z_s)) \end{aligned} \tag{A.11}$$

it follows that

$$| 1 - e^{-Z_s} |^2 = (\frac{\pi}{B})^2 (y - y_o)^2$$
$$| 1 - e^{-Z'_s} |^2 = 4 \sin^2 \frac{\pi}{B}(l_e - y_o) \quad \text{(A.12)}$$

Substituting these formulas into Eqn. (A.9), we get the leading order solution

$$S = \frac{1}{\pi} \ln\{\frac{\pi}{B} \mid y - y_o \mid 2 \sin \frac{\pi}{B}(l_e - y)\} \quad \text{(A.13)}$$

which is logarithmically singular as $y \to y_o$ as expected. Taking the leading order solution, we get the following result(see Gradshteyn & Ryzhik[8]).

$$\Upsilon_h = -\frac{1}{\pi} \ln(\frac{4a\pi}{B}) + \frac{3}{2\pi} - X - Y \quad \text{(A.14)}$$

where

$$X = \frac{\cot kL}{kB} + 2 \sum_{n=1}^{\infty} \{\frac{\cot k_n L}{k_n B} + \frac{1}{n\pi}\} (\frac{\sin(n\alpha)\cos(n\beta)}{n\alpha})^2$$
$$Y = \frac{l_e}{2\pi a} \ln \frac{l_e + a}{l_e - a} + \frac{1}{2\pi} \ln(l_e^2 - a^2) + \frac{1}{\pi}(\ln \frac{\pi}{B} - 1)$$
$$+ \frac{B}{2a\pi^2} \sum_{k=1}^{\infty} (-1)^k \frac{B_{2k}}{(2k+1)!4k} (\frac{2\pi}{B})^{2k+1} [(l_e + a)^{2k+1} - (l_e - a)^{2k+1}]$$

with $\alpha = \frac{a\pi}{B}$ and $\beta = \frac{l_e \pi}{B}$ where B_{2k} denotes Bernoulli's number. Therefore the integral given in Eqn. (A.1) is finally

$$\frac{1}{4a^2} \int \int_{-a}^{a} M(y/y_o) dy dy_o = \frac{i}{2} + F - X - Y \quad \text{(A.15)}$$

where

$$F = -(\ln \frac{\pi \gamma k a^2}{4B} + \ln 16 - 3)/\pi \quad \text{(A.16)}$$