Hydroelastic Vibration Analysis of Structures in Contact with Fluid

Kie-Tae Chung *, Young-Bok Kim † and Ho-Seung Kang ‡

(From T.S.N.A.K., Vol. 29, No. 1, 1992)

Abstract

In the vibration analysis of submerged or floating bodies such as ships and offshore structures, the coupled system between fluid and structure should be considered using the compatibility conditions on the wetted surface.

It is well known that the hydroelastic vibration analysis of structures in contact with fluid can be done by applying the finite element method(FEM) to structures and the boundary element method(BEM) to the fluid domain. However, such an approach is impractical due to the characteristics of the fully coupled added mass matrix of fluid on the entire wetted surface.

To overcome this difficulty, an efficient approach based on a reanalysis scheme is proposed in this paper. The proposed method can be applied for cases of higher local modes and beam-like modes for which three-dimensional reduction factors are not known. The three dimensional reduction factors are not needed and thus the restrictions can be removed in the analyses of non-beam like modes or local vibration modes by considering fluid-structure interaction. The validity and calculation efficiency of the proposed method are proved through numerical examples.

1. INTRODUCTION

In the vibration analysis of ship structures, the decoupled analysis between fluid and structure has been conventionally employed by applying two dimensional sectional added mass multiplied by the predetermined three dimensional correction factors for the ellipsoid of revolution or a finite cylinder with similar dimensions. In this conventional method, it is assumed that the structures are in motion only with the global beam-like modes without any sectional local modes and any non-beam like modes.

But in today's weight optimized ship structures, it is common to find the non-beam like modes of vibration such as the double bottom vibration or the local vibration in ships with large openings. However, it is not easy to find the added mass and the corresponding three dimensional reduction factors. In this case, thorough insight into the fluid-structure interaction is needed to analyze the vibration of the structures in water, which leads to the

*Member, Department of Naval Architecture & Ocean Engineering, Chungnam National University
†Member, R & D Center, Korean Register of Shipping
‡Member, R & D Center, Korean Register of Shipping

conclusion that it is considered most reasonable to solve this problem by regarding the fluid and the structure as one coupled system considering the compatibility conditions on the wetted surface.

In general the motion of a structure is described by a set of algebraic equations by applying the finite element discretization. Also the fluid can be analyzed using Lagrangian formulation or Eulerian formulation according to the independent variables employed in describing fluid motion[1]. In case of Lagrangian formulation, the fluid field can be idealized into the same finite elements with the independent variables such as stresses and strains as in the case of the structure. But it can be pointed out as disadvantages to require too large degrees of freedom and to create some imaginary radiating boundaries for bounding the infinite fluid domain, which are normal in the vibration analysis of ships and offshore structures.

The boundary element method in Eulerian formulation is based on the inverse formulation of the weighted residual integrations of governing equations and boundary conditions. Therefore, the degrees of freedom of the equilibrium equations can be restricted on the wetted structural surface by applying the weighting function such as Green function, which satisfies the remaining boundary conditions on free surface, sea bottom or radiating boundary, etc. But in this case, apart from the case of finite element method, the algebraic equations is fully coupled on the whole degrees of freedom on the wetted surface because of the Green function employed. Therefore, if this shortcoming can be overcome, the vibration analysis by finite/boundary element method will be very efficient and practical.

In this paper an efficient finite/boundary element method is proposed for the vibration analysis of the ships and offshore structures considering the fluid-structure interaction, in which the modal reanalysis scheme is used in order to overcome the above mentioned shortcoming of large coupled degrees of freedom originated from the Green function.

2. THE VIBRATION ANALYSIS OF A STRUCTURE IN WATER

2.1 Coupled Analysis of a Fluid-Structure Interaction

The equations of motion representing a fluid-structure interaction can be described as follows:

$$M^s\ddot{X} + C\dot{X} + KX = F^e + F^i \tag{1}$$

where M^s, C, K are the mass, damping and stiffness matrix of the structure, respectively. And F^e and F^i denote the pure external force and the force acting on the wetted structural surface by the fluid-structure interaction, respectively. From above equation (1), it can be said that the problem is to obtain F^i, which is dependent on the unknown displacements. Assuming that the fluid is ideal, the tangential components of F^i are negligible on the wetted surface of the structure and F^i can be described by integrating the pressure P on the wetted surface S_i, that is

$$F^i = -\int_{S_i} N^T \bar{n} P dS \tag{2}$$

where N is a shape function for approximation of the pressure on any surface element, P is the pressure vector to be solved in the coupled fluid-structure system, and $\bar{n}$ is the unit normal vector on the wetted surface into the fluid. To obtain the pressure P, there are two different methods, Lagrangian and Eulerian methods depending on the independent variables in the fluid domain. The finite element method based on the weak formulation or the boundary element method based on the inverse formulation can be used in Eulerian method. In this study, as mentioned in the Introduction, the boundary element method is used in combination with the modal reanalysis scheme. Following the procedure in [2], if the pressure P obtained by BEM is substituted into the equation (2), the force F^i can be obtained as a function of acceleration of the structure, namely

$$F^i = -M^f \ddot{X} \tag{3}$$

where M^f is the added mass matrix. This matrix is asymmetric and fully coupled because the shape function N for approximation and the Green function used for residual weighting are different. However, as illustrated in [2], this asymmetry is rather weak and can be symmetrized by using the energy conservation principle. Substituting equation (3) into the equation (1), the equations of motion for the fluid-structure interaction problem can be formulated as follows:

$$(M^s_{ij} + M^f_{ij})\ddot{X}_j + C_{ij}\dot{X}_j + K_{ij}X_j = F^e_i, \quad i,j = 1,2,\cdots,n \tag{4}$$

where n is the total degrees of freedom of the structure.

Since the added mass matrix has very wide coupled bandwidth due to the coupled terms with the whole degrees of freedom on the wetted surface, it is not economical to solve these equations by using the general purpose structural analysis package program and furthermore practically impossible to apply to the huge model such as a ship structure. Therefore, the modal reanalysis method as illustrated in the Appendix is applied. The fluid-structure interaction problem in the equation (4) can be solved using by substituting into the equation (A.3) in Appendix the relations as follows:

$$\begin{aligned} M_{ij} &= M^s_{ij}, \\ \delta M_{ij} &= M^f_{ij}, \quad i,j = 1,2,\cdots,n \\ \delta C &= \delta K = 0 \end{aligned} \tag{5}$$

Using the n_f pairs of the natural frequency ω and the natural modal function ϕ of the structure in air, the modal equations of motion can be described as follows:

$$\begin{gathered} (I_{kk} + \phi_{ik}M^f_{ij}\phi_{jk})\ddot{Z}_k + 2\zeta_k\omega_k\dot{Z}_k + \omega_k^2 Z_k = \phi_{ik}F_i, \\ i,j = 1,2,\cdots,n \text{ and } k = 1,2,\cdots,n_f \end{gathered} \tag{6}$$

where n_f is the number of modes considered. These equations have the same forms as the equation (A.4). In case of ships, the structural damping is so small that the effect on the natural vibration characteristics would be negligible. Therefore, after neglecting the damping and the external forces, the natural frequencies ω's of the structure in water are obtained from the equation (6), and the modal functions ϕ's from the equation (A.5). Hereafter, this method is called the Coupled Reanalysis Method.

2.2 Decoupled Analysis of a Fluid-Structure Interaction

Two dimensional added mass m^{f2D} of a structure in water with arbitary cross sections and large slenderness ratio like ships, can be obtained by the Lewis mapping function[3], the close-fit-method of Frank[4] or the boundary element method[2]. The added mass matrix M^f is obtained by distributing m^{f2D} on the wetted surface. Thus,

$$M^f_{ij} = \begin{cases} m^{f2D} & \text{(on the surface in water)} \\ 0 & \text{(on the surface in air)} \end{cases} \tag{7}$$

Above results are limited to two dimensional behaviour of fluid along the girth of the section. However, M^f should be reduced by some factor to take into account three dimensional flow effect along the length. In the conventional ship vibration analysis, this effect has been accounted for in terms of $J-$factors, the three dimensional correction factors, which are derived theoretically for the ellipsoid of revolution or the finite cylinder[5,6,7,8]. Then, the added mass matrix for the $k-$th vibration mode can be written as:

$$\begin{aligned} M^{f,k}_{ij} &= M^f_{ii} J_k, \\ & i,j = 1,2,\cdots,n \text{ and } k = 1,2,\cdots,n_f \end{aligned} \tag{8}$$

The equations of motion of the structure in water are

$$\begin{gathered} (M^s_{ij} + M^{f,k}_{ij})\ddot{X}_j + C_{ij}\dot{X}_j + K_{ij}X_j = F_i, \\ i,j = 1,2,\cdots,n \text{ and } k = 1,2,\cdots,n_f \end{gathered} \tag{9}$$

It is clear that the equations of motion should be solved separately for each mode and the orthogonality between each vibration mode can not be sustained, which makes it impractical and time-consuming to solve these equations directly. Therefore, the following two methods, the conventional Decoupled Reference-mode Method and the Decoupled Reanalysis Method, are employed in practice[9].

2.3 Decoupled Reference-mode Method

The equation (9) can be rewritten for an arbitrary reference mode among the n_f modes as follows:

$$\begin{gathered} (M^s_{ij} + M^f_{ii} J_{ref})\ddot{X}_j + C_{ij}\dot{X}_j + K_{ij}X_j = F_i, \\ i,j = 1,2,\cdots,n \end{gathered} \tag{10}$$

Solving these equations of motion, the other eigenpairs can be derived in terms of the reference eigenpairs as follows:

$$\omega_j = \sqrt{\frac{M_s + M_f J_{ref}}{M_s + M_f J_j}}\,\omega_j^{ref} \tag{11}$$

$$\phi_{ij} = \phi_{ij}^{ref}, \quad i = 1,2,\cdots,n \text{ and } j = 1,2,\cdots,n_f \tag{12}$$

$$\begin{aligned} M_S &= \sum_i M^s_{ii} \\ M_f &= \sum_i M^f_{ii} \end{aligned} \tag{13}$$

In equation (12), it is assumed that the change of the modal added mass has almost no effect on the eigenmode shapes. It is to be noted that equation (10) can not be solved at one time, because the different reference mode must be used for each case of independent modes as vertical or horizontal-tortional modes.

2.4 Decoupled Reanalysis Method

To overcome the above-mentioned difficulty, the Decoupled Reanalysis Method can be used economically and accurately by considering the added masses multiplied by the three-dimensional correction factors for every required mode simultaneously[9]. The three dimensional correction factor is represented as the following diagonal matrix, namely

$$J_{kk} = J_k, \qquad k = 1, 2, \cdots, n_f \tag{14}$$

The following relations are substituted into the equation (A.3), to make use of the reanalysis methods in the Appendix, that is

$$\begin{aligned} M_{ij} &= M^s_{ij} \\ \delta M_{ij} &= M^f_{ij}, \qquad i,j = 1,2,\cdots,n \\ \delta C &= \delta K = 0 \end{aligned} \tag{15}$$

The equations of motion can be written as follows:

$$\begin{aligned} (M^s_{ij} + M^f_{ii})\ddot{X}_j + C_{ij}\dot{X}_j + K_{ij}X_j = F_i, \\ i,j = 1,2,\cdots,n \end{aligned} \tag{16}$$

Here, it is remarkable that the M^f_{ii} in the above equations is estimated from the two dimensional added mass. Substituting $X_i = \phi_{ij} z_j$ into equation (16), followed by premultiplying ϕ^T on both sides and using the ϕ's and ω's determined from the mode analysis for the structure in air leads to

$$\begin{aligned} (I_{kk} + \phi_{ik} M^f_{ii} \phi_{ik})\ddot{z}_k + 2\zeta_k \omega_k \dot{z}_k + \omega_k^2 z_k = \phi_{ik} F_i, \\ i = 1,2,\cdots,n \ \text{ and } \ k = 1,2,\cdots,n_f \end{aligned} \tag{17}$$

where the second term in the parenthesis of equation (17) should be the added mass matrix corresponding to the $k-$th mode. Thus,

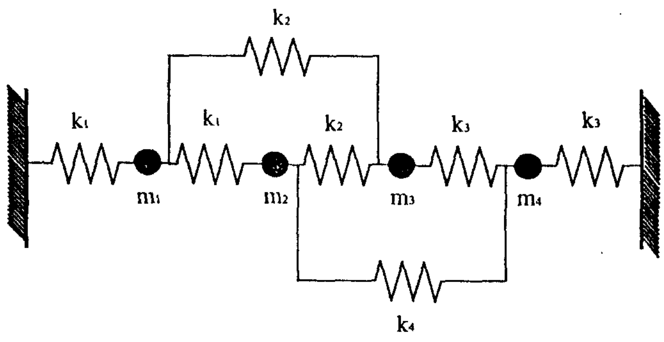
$$\begin{aligned} (I_{kk} + \phi_{ik} M^f_{ii} \phi_{ik} J_{kk})\ddot{z}_k + 2\zeta_k \omega_k \dot{z}_k + \omega_k^2 z_k = \phi_{ik} F_i, \\ i = 1,2,\cdots,n \ \text{ and } \ k = 1,2,\cdots,n_f \end{aligned} \tag{18}$$

From these equations, ω' and ϕ' of the structure in water can be deduced easily by use of the reanalysis method. In comparison with the Decoupled Reference-mode Method, it is the advantage that the vertical mode of vibration and the horizontal-torsional coupling mode vibration can be analyzed simultaneously and the modal functions are orthogonal between modes[9], although this still requires the three dimensional reduction factors.

3. EXAMPLES AND RESULTS

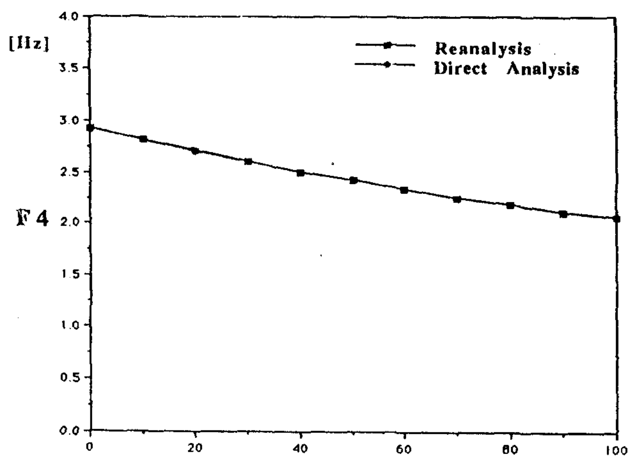
3.1 Spring Mass System

Firstly, in order to investigate the effectiveness and accuracy of the reanalysis method, the natural vibration analysis was carried out for the spring-concentrated mass system shown in Figure 1(a), by both the reanalysis method and the direct analysis method by increasing δM from 10 % to 100 % of the original structural mass. Figure 1(b) shows the results of the above analyses from which the effectiveness and accuracy of the former can be confirmed.



$$k_1, k_2, k_3, k_4 = 60, 80, 100, 20\text{kg/m}$$
$$m_1, m_2, m_3, m_4 = 3, 1, 2, 1\text{kg}$$

(a) Modeling



(b) The variation of 4-th natural frequency

Figure 1: Spring-mass system

Secondly, the applicability of the reanalysis method to the vibration analysis of the structure in water is investigated. Since the added mass matrix deduced from BEM is fully coupled and its magnitude is larger than or of the same order with the original structural mass, the numerical value of δM is taken as the following additional fully coupled mass matrix.

$$\delta M = \begin{bmatrix} 2.0 & -0.5 & -0.5 & -0.5 \\ -0.5 & 2.0 & -0.5 & -0.5 \\ -0.5 & -0.5 & 2.0 & -0.5 \\ -0.5 & -0.5 & -0.5 & 2.0 \end{bmatrix}$$

In Table 1, the natural frequencies by each method are summarized and compared. It is shown that the results by the reanalysis method agree well with those by the direct analysis, even for the fully coupled additional mass matrix δM.

Table 1: Comparison of natural frequencies by various analyses(in Hz)

Mode	Original Structure	Modified Structure	
		Reanalysis	Direct analysis
1	0.69766	0.61305	0.61305
2	1.25165	0.87372	0.87372
3	2.39007	1.40254	1.40254
4	2.93504	1.60264	1.60264

3.2 Semi-Submerged Cylindrical Shell Structures

As an example of the structural vibration in water, the vibration analysis for the semi-submerged cylindrical shell as shown in Figure 2 was carried out. This model has dimensions of length L = 18 m, diameter d = 2 m and shell thickness t = 0.001 m.

The Young's modulus E and the material density ρ are 2.06E11 N/m^2 and 236950 kg/m^3, respectively. The circumferencial fluid density is 1000 kg/m^3. As shown in Figure 2, the external shell is modeled with 36 6-noded shell elements and the cross sections with the 8-noded membrane elements whose nodes are connected to the outer-shell nodes. The natural vibration analyses were performed by means of the three methods.

- Coupled Reanalysis Method
- Decoupled Reference-mode Method
- Decoupled Reanalysis Method

In Table 2, the natural frequencies calculated and the CPU times taken by a VAX 6210 computer are tabulated for comparison. The vibration modes are not very sensitive to the mass variations and analysis methods. Only the representative vibration modes by the above three methods are depicted in Figure 2. Here the abbreviations V, H,T and L mean vertical, horizontal, torsional and longitudinal mode, respectively. And the subscript denotes the number of vibration nodes. In conventional decoupled methods, the accuracy of the solutions depends on the three dimensional factors of the vertical and horizontal modes. The J-factors

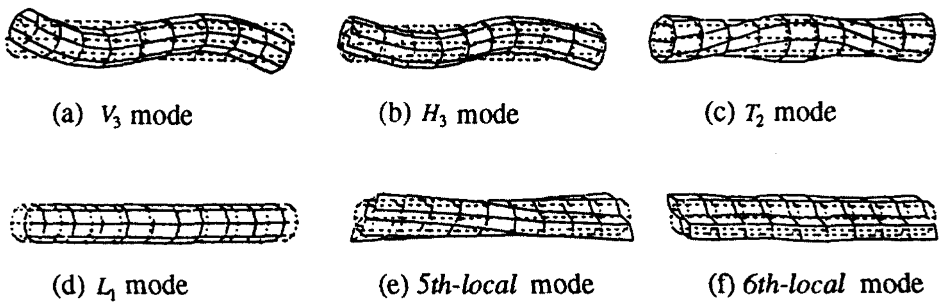


Figure 2: Some vibration modes of a floating structure by Coupled Reanalysis

Table 2: Comparison of frequency analysis results for a floating structure (in Hz)

Mode	Freq. in air	Freq. in water		
	f_{air}	f_{2D}^{Ref}	f_{2D}^{Rea}	f_{3D}^{Ref}
H	7.39	6.604	6.727	6.524
V	7.39	5.964	5.848	5.718
T	15.88	15.88	15.88	15.88
V	17.93	15.24	15.27	14.40
H	17.93	16.09	16.27	16.05
L	25.70	-	-	23.24
H	30.56	20.61	27.93	27.52
V	30.56	26.81	26.67	25.51
T	31.25	31.25	31.25	31.25
V	43.67	38.63	37.67	35.54
H	43.67	39.59	40.00	39.35
1st-local	45.40	-	-	43.36
T	45.61	45.61	45.61	45.61
2nd-local	45.67	-	-	43.54
3rd-local	49.80	-	-	46.24
4th-local	49.81	-	-	47.27
L	50.16	-	-	46.67
5th-local	51.31	-	-	47.75
6th-local	51.96	-	-	47.93
CPU time (sec.)	840.76	903.87	937.74	895.05

f_{air} : Natural frequencies in air
f_{2D}^{Ref} : Natural frequencies by Decoupled Reference-mode Method
f_{2D}^{Rea} : Natural frequencies by Decoupled Reanalysis Method
f_{3D}^{Ref} : Natural frequencies by Coupled Reanalysis Method
\- : J-value not available

estimated theoretically by Kumai[5,6] are used and furthermore the J-factor for the four noded vertical mode is taken as reference in the Decoupled Reference-mode Method. In torsional modes, there appears the inertia moment due to the difference between the horizontal centre of the added mass and the structural shear center. Thus in the analysis of the horizontal and torsional mode, the J_k of the corresponding mode should be considered. While the idealized model by Kumai is a beam with a cylindrical section, in which case the theoretical approach is easy, the adopted model in the present study is the shell structure which can be deformed differently from the Kumai's model. It must be noted that the results obtained by the present model are affected by the fluid-structure interaction and those are dependent on the mesh sizes employed as opposed to the Kumai's model. From the analysis results, there is little or no difference in the frequencies of the horizontal and torsional modes regardless of the analysis methods. However, there is a slight difference in the frequencies of the vertical modes by analysis methods used. The reason for no difference in torsional eigen frequencies between modes in air and in water is that there is no coupling effect in the horizontal-torsional mode due to the fact that the wetted cross section of this model is circular[8]. The slight deviation in the frequencies of the vertical modes is due to the difference in the model itself and the analysis methods employed.

4. CONCLUSION

The Coupled and Decoupled Reanalysis Methods are applied for the fluid-structure interaction problem. The applicability and reliability of the reanalysis method are confirmed through the numerical examples of a spring-concentrated mass system.

From the analyses of a semi-submerged cylindrical shell structure, it is shown that in the Decoupled Reference-mode Method, independent vibration modes such as the vertical and the horizontal-torsional modes should be analyzed separately, because the different reference mode must be taken for each mode. On the other hand, all the vibration modes can be solved simultaneously by the Decoupled Reanalysis Method using two dimensional added mass and three dimensional correction factor matrix, although it is limited to the case of hull girder vibration modes with known J factors only.

When compared with the conventional decoupled methods, the Coupled Reanalysis Method proposed in this paper can handle the fluid-structure interaction problem very effectively without introducing three dimensional reduction factors and regardless of the vibration modes. Furthermore, from the practical point of view, it takes the CPU time only of the same order with conventional decoupled methods.

The advanced approach will be presented in the future, in which free surface effects, side wall effects and bottom effects for real ship are investigated and the calculated results are compared with the measurements.

References

[1] Zienkiewicz, O. C. and Bettes, P., "Fluid-Structure Dynamic Interaction and Wave Forces," *An Introduction to Numerical Treatment, Numerical Methods in Engineering*, Vol. 13, 1978, pp.1-16.

[2] Chung, K.T., "On the Vibration of the Floating Elastic Body Using Boundary Integral Method in Combination with Finite Element Method," *Journal of the Society of Naval Architects of Korea,* Vol. 24, No. 4, 1987.

[3] Lewis, F.M., "The Inertia of the Water Surrounding a Vibrating Ship," *Transactions of Society of Naval Architects and Marine Engineers,* Vol. 37, 1929.

[4] Frank, W., "On the Oscillation in or below the free Surface of Deep Fluids," Report 2375, NSRDC, Washington, DC., 1967.

[5] Kumai, T., "On the Three - dimensional Correction Factor for the Vertical Inertia Coefficient in the Vertical Vibration of Ships (1st Report, *J*- Value of elliptical cylinder)," *Journal of the Society of Naval Architects of Japan,* Vol. 112, 1967.

[6] Kumai, T., "Some Correction Factors for Virtual Inertia Coefficient on the Horizontal Vibration of a Ship," *Journal of the Society of Naval Architects of Japan,* Vol. 108, 1960.

[7] Kim, K.C., "A Note on the Three - dimensional Correction Factor for the Virtual Inertia Coefficient of Ships in Vertical Vibrations," *Journal of the Society of Naval Architects of Korea,* Vol. 12, 1975.

[8] Kim, K.C., "Three - dimensional Correction Factors for the Added Mass in the Horizontal Vibration of Ships," *Journal of the Society of Naval Architects of Korea,* Vol. 11, 1974.

[9] Hakala, M.K. and Matusiak, J., "A Practical Approach for the Estimation and Verification of Ship Vibration Response," The 2nd Int'l. Symp. on Practical Design in Shipbuilding, PRADS 83, Tokyo & Seoul, 1983.

[10] Wang, B.P., "Reanalysis of Structural Dynamic Models," *American Society of Mechanical Engineers,"* AMD - Vol. 76, 1986.

Appendix A: The Reanalysis Method of the Dynamic Characteristics of Structural Behavior

The equations of motion of an arbitrary structure by FEM are described as follows:

$$M_{ij}\ddot{X}_j + C_{ij}\dot{X}_j + K_{ij}X_j = F_i, \quad i,j = 1,2,\cdots,n \tag{A.1}$$

where M, C,K and F represent the structural mass, damping, stiffness matrix and the external force vector, and n is the total structural degrees of freedom. Letting ω and ϕ be the eigen frequencies and the normalized modal functions from equation (A.1) by neglecting the structural damping, the structural displacement vector X can be transformed into a generalized coordinate:

$$X_j = \phi_{jk}z_k, \quad j = 1,2,\cdots,n, \text{ and } k = 1,2,\cdots,n_f \tag{A.2}$$

where z_k is the generalized coordinate of the k-th mode and n_f is the number of modes considered.

In case of the additional structural modifications such as δM, δC and δK, the modified equations of motion can be written as follows:

$$(M_{ij} + \delta M_{ij})\ddot{X}_j + (C_{ij} + \delta C_{ij})\dot{X}_j + (K_{ij} + \delta K_{ij})X_j = F_i, \quad i, j = 1, 2, \cdots, n \tag{A.3}$$

Substituting equation (A.2) into equation (A.3) and premultiplying ϕ^T on both sides leads to

$$m_{lk}\ddot{z}_k + c_{lk}\dot{z}_k + k_{lk}z_k = f_k, \quad l, k = 1, 2, \cdots, n_f \tag{A.4}$$

where

$$\begin{aligned} m_{lk} &= I_{kk} + \phi_{il}\delta M_{ij}\phi_{jk}, \qquad i, j = 1, 2, \cdots, n \\ c_{lk} &= 2\zeta_l\omega_{ll} + \phi_{il}\delta C_{ij}\phi_{jk} \quad (\text{where, } \phi_{il}C_{ij}\phi_{jl} = 2\zeta_l\omega_l) \\ k_{lk} &= \omega_{ll}^2 + \phi_{il}\delta K_{ij}\phi_{jk}, \\ \zeta_k &= k - \text{th modal damping ratio}, \\ f_k &= \phi^T f_i = \phi_{ik} f_i, \\ I_{kk} &= \text{identity matrix} \end{aligned}$$

After solving the eigenvalues ω' and modal functions Ψ from the equation (A.4), the generalized coordinate z of the original structure can be transformed into z' for the modified structure by using following equations.

$$z_k = \Psi_{kp}z'_p, \qquad k, p = 1, 2, \cdots, n_f \tag{A.5}$$

Substituting the equation (A.5) into the equation (A.2), the eigenvalues ω' and modal matrix ϕ' of modified structure can be obtained[10]. That is,

$$X_j = \phi_{jk}\Psi_{kp}z'_p = \phi'_{jp}z'_p, \qquad j = 1, 2, \cdots, n \quad \text{and} \quad k, p = 1, 2, \cdots, n_f \tag{A.6}$$