

# 모드정합을 결합한 UTD에 의한 큰 평행도파관의 고주파간섭 해석

## (UTD-Supplemented Mode-matching Method Analysis of High-Frequency Wave Coupling into Large Parallel Plate Waveguides)

權度熏\*, 宣永植\*, 明魯勳\*

(Do-Hoon Kwon, Young-Seek Sun, Noh-Hoon Myung)

### 要約

본 논문에서는 한쪽으로 무한히 긴 평행도파관에 입사되는 평면파를 해석하였다. 도파관 내부의 전계는 입사되는 평면파를 도파관의 모드로 변환시켜 구하였다. 도파관 입구에서의 전계를 구하기 위해 보통 Kirchhoff 근사식을 이용하는데, 이 논문에서는 Uniform Geometrical Theory of Diffraction (UTD)에 근거한 근사방법을 이용하였다. 본 논문에서 제안한 방법과 UTD 와의 수치결과를 비교 하였는데 매우 좋은 결과를 보였다.

### Abstract

The problem of a plane wave impinging upon a semi-infinite parallel-plate waveguide is investigated. The interior fields can be analyzed by converting the initial field into waveguide modes. Kirchhoff approximation is usually made at the waveguide aperture in the literature. In this paper, a modified approximation is made using the Uniform Geometrical Theory of Diffraction(UTD). Numerical results show excellent agreement between UTD-supplemented mode-matching solution and UTD solution. <sup>[1]</sup>

### 1. INTRODUCTION

The problem of wave scattering by and wave coupling into open-ended waveguides and cavities are of great interest in a variety of applications. The problems are solved by many methods. Exact solutions are obtained by Wiener-Hopf technic and asymptotic

solutions can be obtained through UTD, hybrid ray-mode analysis, mode-matching method(MMM), etc. However, Wiener-Hopf technic entails quite a mathematical complexity and UTD analysis fails for the high incident angle such as  $75^\circ$  or above that because the greater part of interior regions are immersed in the transition regions.

When the waveguide has no variation of geometry in the guided direction, the interior field can be described in terms of guided modes whose excitation coefficients are determined from the initial field at the opening.

\*正會員, 韓國科學技術院 電氣 및 電子工學科  
(Dept. of Elec. Eng., Korea Advanced Institute of Science and Technology)  
接受日字 : 1994年 4月 19日

We know from electromagnetic theory that if we know the exact total field at the opening we can calculate the exact interior fields using the modal expansion (although there are some practically negligible truncation errors in actual computations). In case the waveguide separation is large compared to the wavelength, many former works employ Kirchhoff approximation on the aperture field. Although Kirchhoff approximation is a reasonably good one, it is the crudest approximation we can make on the total aperture field. A modified approximation is suggested in this paper and it will be shown it yields better solutions for the interior fields.

II. FORMULATION

The problem under consideration is shown in Fig. 1. The parallel plate waveguide separation is given by  $a$  and the angle of the incident plane wave is given by  $\theta_i$ .

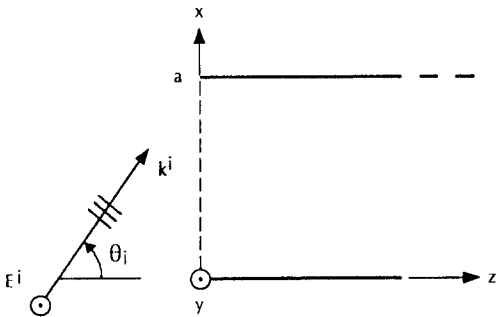


그림 1. 한면이 열린 완전도체로 된 평행도파관에 평면파 입사

Fig. 1. Plane-wave excitation of an open-ended perfectly-conducting parallel plate waveguide.

The waveguide walls are perfectly-conducting and since they are so thin, it is assumed that we can treat them as half planes. An electromagnetic plane wave with transverse electric polarization (TE case) is incident on the open end. The normalized incident wave is given with suppressed  $e^{j\omega t}$  time convention by

$$E^i(x, z) = \hat{y} E_0 e^{-jk_0(x \sin \theta_i + z \cos \theta_i)}, \tag{1}$$

where  $k_0 = \omega \sqrt{\mu \epsilon}$  is the free-space wavenumber. The field coupled into the waveguide can be described by the modal expansion:

$$E_y = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{a} x\right) e^{-jk_z z} \tag{2}$$

Here,  $c_n$  is the modal coefficient for the  $n$ th modal function  $\sin(n\pi x/a)$  and  $k_z = [k_0^2 - (n\pi/a)^2]^{1/2}$ .  $k_z$  is either positive real for propagating modes or pure imaginary with negative sign for attenuating modes.

$c_n$ 's are obtained by matching the modal field in (2) to the total field at  $z=0$ :

$$c_n \frac{2}{a} \int_0^a E_y^{tot}(z=0) \sin\left(\frac{n\pi}{a} x\right) dx \tag{3}$$

For the wide waveguide separation  $ka \gg 1$ , some previous works<sup>[2] [4]</sup> employ Kirchhoff approximation. However, Kirchhoff approximation treats nothing about the wave scattering effect introduced by two sharp discontinuities located at  $(x, z) = (0, 0)$  and  $(a, 0)$ . Here we employ UTD diffracted fields to account for these effects. Thus the total field at the aperture is approximated as

$$E_y^{tot}(z=0) = E_y^i(z=0) + E_y^d(z=0) \tag{4}$$

The superscript  $d$  denotes diffractions by two sharp edges. With  $E_y^{tot}$  in (3) replaced by (4), we have two integral terms and let's denote them by  $K_n$  and  $D_n$ , which are for Kirchhoff-approximated and edge-diffracted terms, respectively, such that

$$c_n = K_n + D_n \tag{5}$$

Then  $K_n$  is represented by<sup>[3]</sup>

$$K_n = \frac{2}{a} \int_0^a e^{-jk_0 x \sin \theta_i} \sin\left(\frac{n\pi}{a} x\right) dx \tag{6}$$

$$= \begin{cases} \frac{2n\pi [e^{-jk_0 a \sin \theta_i} (-1)^{n+1} + 1]}{(n\pi)^2 - (k_0 a \sin \theta_i)^2} & \text{if } k_0 a \sin \theta_i \neq n\pi \\ -j & \text{if } k_0 a \sin \theta_i = n\pi \end{cases}$$

Kirchhoff-approximated MMM analysis uses as its modal expansion coefficient this  $K_n$  only:

$$E_y^{Kir} = \sum_{n=1}^{\infty} K_n \sin\left(\frac{n\pi}{a} x\right) e^{-jk_n z} \quad (7)$$

The geometry for diffraction by two-dimensional conducting wedge is shown in Fig. 2. We can treat two half planes as perfectly-conducting two-dimensional wedges with zero wedge angles.

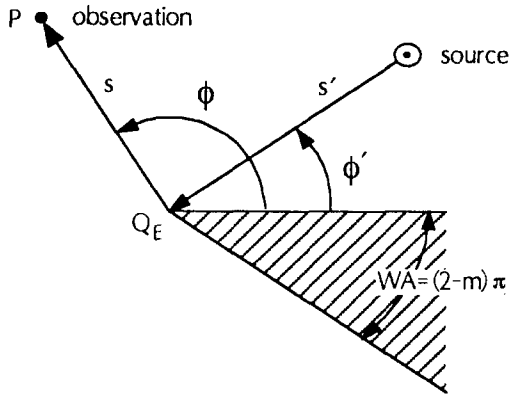


그림 2. 2차원의 도체쐐기에 의한 회절  
Fig. 2. Diffraction by a two-dimensional conducting wedge.

Two-dimensional edge diffracted fields have cylindrical wavefronts originating at the edge, and thus diffracted fields of this soft polarization case are given by<sup>[5]</sup>

$$E_y^{diff} = E_y'(Q_E) D_s \frac{e^{-jk_0 s}}{\sqrt{s}} \quad (8)$$

where  $Q_E$  is the diffraction point on the edge and  $s$  is the distance between the diffraction point and the observation point. The subscript  $s$  in  $D_s$  stands for soft polarization.

Two-dimensional diffraction coefficient for wedges with wedge angle  $(2-m)\pi$  is given by<sup>[5]</sup>

$$D_s(L, \phi, \phi', m) = D_1 + D_2 - (D_3 + D_4) \quad (9)$$

where

$$D_1 = \frac{-e^{-j\pi/4}}{2m\sqrt{2\pi k_0}} \cot\left[\frac{\pi + (\phi - \phi')}{2m}\right] F[k_0 L a^+(\phi - \phi')] \quad (10)$$

$$D_2 = \frac{-e^{-j\pi/4}}{2m\sqrt{2\pi k_0}} \cot\left[\frac{\pi - (\phi - \phi')}{2m}\right] F[k_0 L a^-(\phi - \phi')] \quad (11)$$

$$D_3 = \frac{-e^{-j\pi/4}}{2m\sqrt{2\pi k_0}} \cot\left[\frac{\pi + (\phi + \phi')}{2m}\right] F[k_0 L a^+(\phi + \phi')] \quad (12)$$

$$D_4 = \frac{-e^{-j\pi/4}}{2m\sqrt{2\pi k_0}} \cot\left[\frac{\pi - (\phi + \phi')}{2m}\right] F[k_0 L a^-(\phi + \phi')] \quad (13)$$

and the distance parameter  $L$  is

$$L = \frac{s's}{s' + s} \quad (14)$$

Here  $s'$  is the distance from the source point to the edge point.

The functions  $a^\pm$  are defined as

$$a^\pm(\phi \pm \phi') = 2 \cos^2\left(\frac{2m\pi N^\pm - (\phi \pm \phi')}{2}\right) \quad (15)$$

and  $N^\pm$  are integers that most nearly satisfies the equations

$$2\pi m N^+ - (\phi \pm \phi') = \pi, \quad 2\pi m N^- - (\phi \pm \phi') = -\pi \quad (16)$$

The function  $F'$  is the transition function given by<sup>[5]</sup>

$$F(x) = 2j\sqrt{x} e^{jx} \int_{-\sqrt{x}}^{\infty} e^{-ju^2} du \quad (17)$$

Fig. 3 describes 1st-order and 2nd-order diffractions graphically.

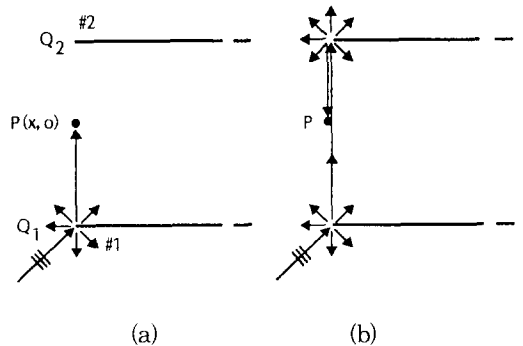


그림 3. (a) 1차 회절 (b) 2차 회절  
Fig. 3. (a) 1st-order and (b) 2nd-order diffractions.

We can write the 1st-order diffracted field at observation point  $P(x,0)$  directly from Fig. 3. Here note that since the incident field is a plane wave  $s'$  in (14) is infinity and thus the distance parameter  $L$  becomes  $s$ .

$$\begin{aligned} E_y^{d1} &= E^i(Q_1)D_{11P} \frac{e^{-jk_0s_1}}{\sqrt{s_1}} + E^i(Q_2)D_{12P} \frac{e^{-jk_0s_2}}{\sqrt{s_2}} \\ &= 1 \cdot D_s(x, \pi/2, \pi + \theta, 2) \frac{e^{-jk_0x}}{\sqrt{x}} + e^{-jk_0a \sin \theta} \\ &\quad \cdot D_s(a-x, 3\pi/2, \pi + \theta, 2) \frac{e^{-jk_0(a-x)}}{\sqrt{a-x}} \end{aligned} \quad (18)$$

2nd-order and higher-order diffracted fields which originates from the plane wave incident upon the edge #1 can be obtained through the following procedure. In evaluating 2nd-order diffracted field, we regard the edge #1 as a new cylindrical wave source and 1st-order diffracted field as the incident wave to the edge #2. We can determine 2nd-order diffracted field as we did to evaluate 1st-order diffracted field. In a similar way, higher-order diffracted fields are calculated. In this way, we obtain the following infinite series:

$$\begin{aligned} E_{\#1} &= E^i(Q_1)D_{11P} \frac{e^{-jk_0x}}{\sqrt{x}} \\ &+ \left\{ E^i(Q_1)D_{112}D_{21} \left( \frac{e^{-jk_0a}}{\sqrt{a}} \right)^2 \right\} \cdot D_{1P} \frac{e^{-jk_0x}}{\sqrt{x}} \\ &\quad \cdot \sum_{i=0}^{\infty} \left[ D_{12}D_{21} \left( \frac{e^{-jk_0a}}{\sqrt{a}} \right)^{2i} \right] + \left( E^i(Q_1)D_{112} \frac{e^{-jk_0a}}{\sqrt{a}} \right) \\ &\quad \cdot D_{2P} \frac{e^{-jk_0(a-x)}}{\sqrt{a-x}} \cdot \sum_{i=0}^{\infty} \left[ D_{12}D_{21} \left( \frac{e^{-jk_0a}}{\sqrt{a}} \right)^{2i} \right], \end{aligned} \quad (19)$$

where

$$D_{11P} = D_s \left( x, \frac{\pi}{2}, \pi + \theta, 2 \right) \quad (20)$$

$$D_{112} = D_s \left( a, \frac{\pi}{2}, \pi + \theta, 2 \right) \quad (21)$$

$$D_{1P} = D_s \left( \frac{a \cdot x}{a+x}, \frac{\pi}{2}, \frac{\pi}{2}, 2 \right) \quad (22)$$

$$D_{2P} = D_s \left( \frac{a \cdot (a-x)}{a+(a-x)}, \frac{3\pi}{2}, \frac{3\pi}{2}, 2 \right) \quad (23)$$

$$D_{12} = D_s \left( \frac{a \cdot a}{a+a}, \frac{\pi}{2}, \frac{\pi}{2}, 2 \right) \quad (24)$$

$$D_{21} = D_s \left( \frac{a \cdot a}{a+a}, \frac{3\pi}{2}, \frac{3\pi}{2}, 2 \right) \quad (25)$$

Similarly, diffracted fields which originate from the plane wave incident upon the edge #2 can be obtained and they are

$$\begin{aligned} E_{\#2}^d &= E^i(Q_2)D_{12P} \frac{e^{-jk_0(a-x)}}{\sqrt{a-x}} \\ &+ \left\{ E^i(Q_2)D_{121}D_{12} \left( \frac{e^{-jk_0a}}{\sqrt{a}} \right)^2 \right\} \cdot D_{2P} \frac{e^{-jk_0(a-x)}}{\sqrt{a-x}} \\ &\quad \cdot \sum_{i=0}^{\infty} \left[ D_{12}D_{21} \left( \frac{e^{-jk_0a}}{\sqrt{a}} \right)^{2i} \right] + \left( E^i(Q_2)D_{121} \frac{e^{-jk_0a}}{\sqrt{a}} \right) \\ &\quad \cdot D_{1P} \frac{e^{-jk_0x}}{\sqrt{x}} \cdot \sum_{i=0}^{\infty} \left[ D_{12}D_{21} \left( \frac{e^{-jk_0a}}{\sqrt{a}} \right)^{2i} \right], \end{aligned} \quad (26)$$

where

$$D_{12P} = D_s \left( a-x, \frac{3\pi}{2}, \pi + \theta, 2 \right) \quad (27)$$

$$D_{121} = D_s \left( a, \frac{3\pi}{2}, \pi + \theta, 2 \right), \quad (28)$$

and the other diffraction coefficients are the same as before.

The total diffracted field  $E_y^d$  is obtained by summing up the fields diffracted by the edges #1 and #2, and given by

$$E_y^d(x, z=0) = E_{\#1}^d + E_{\#2}^d \quad (29)$$

Modal coefficient  $D_n$  is obtained from

$$D_n = \frac{2}{a} \int_0^a E_y^d(x, z=0) \sin \left( \frac{n\pi}{a} x \right) dx \quad (30)$$

The above integral can be evaluated numerically using digital computers. The total modal coefficient  $c_n$  is obtained through (5) and the field inside the waveguide can be calculated using (2).

Analysis for the TM case follows exactly the same procedure as for the TE case. The only differences are that the mode function should be replaced by  $\cos(n\pi x/a)$  and that diffraction coefficients must be changed to  $D_h$  (diffraction coefficients for hard polarization) for the TM case.

### Ⅲ. NUMERICAL RESULTS

Shown in Fig. 4 is calculated aperture fields using aforementioned two approximations for the case  $a=10\lambda$ ,  $\theta=20^\circ$ . For unit-amplitude plane wave incidence, the Kirchhoff-approximated aperture field shows the uniform amplitude of unity across the aperture.

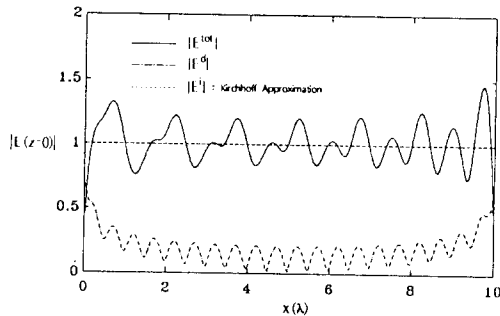


그림 4. 개구부의 전계 분포 : TE 경우  
Fig. 4. Aperture field distribution for the case  $a=10\lambda$ ,  $\theta=20^\circ$  : TE case.

Kirchhoff approximation fails evidently at two edges, where the total electric field must vanish because of the presence of conductor. Shown also is the diffracted field  $E^d$  and the total field approximated by  $E^{tot}=E^i+E^d$ . Note that the total field nears zero at  $x=0$  and  $x=a$ . Interior fields calculated by three methods are shown in Fig. 5 for TE case. The methods are UTD, Kirchhoff-approximated mode matching method, and UTD-supplemented mode matching method, respectively. The graphs show excellent agreement between UTD-supplemented mode-matching solution and UTD solution. However, although Kirchhoff-approximated mode-matching solution has similar shape to the other solutions, it shows some slight differences.

Since UTD analysis is valid for high-frequency waves, that is, valid for scatterers of large size compared to wavelength, UTD-supplemented mode matching method analysis fails for very small waveguide

separation such as  $2\lambda$  or  $3\lambda$ .

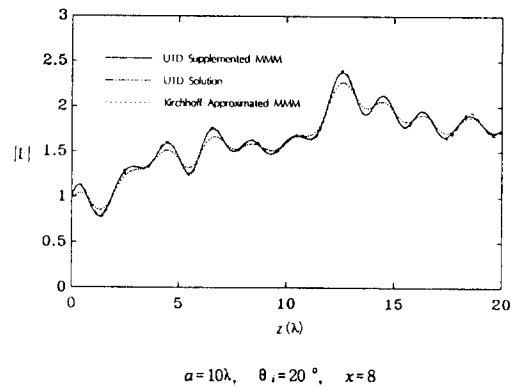


그림 5. 도파관내의 전계 : TE 경우  
Fig. 5. Fields inside the waveguide : TE case.

### Ⅳ. CONCLUSION

For waveguide coupling problems, the total field at the aperture is critical in calculating the field quantities using MMM. Although many former works employ Kirchhoff approximation, we can better approximate the aperture field with the aid of UTD and thus obtain better solution for the fields coupled into the waveguide. And UTD-supplemented MMM analysis for the higher incident angle can show the better results than those of the UTD solutions.

### REFERENCES

- [1] N. H. Myung and Y. S. Sun, "Simple High-Frequency Solution for Interior Fields of Open-Ended Parallel Plate Waveguide," *Electronics Letters*, vol. 28, pp.1285-1286, Jul. 1992.
- [2] L. B. Felsen and H. Shirai, "Hybrid Ray-Mode Analysis of High-Frequency Wave Coupling into Large Waveguides and Cavities," *Optics Letters*, vol. 12, pp.7-9, Jan. 1987.
- [3] Hao Ling, Ri-Chee Chou, and Shung-

Wu Lee, "Rays Versus Modes: Pictorial Display of Energy Flow in an Open-Ended Waveguide," *IEEE Trans. Antennas Propagat.*, vol. AP-35, pp. 605-607, May 1987.

[4] C. A. Balanis, *Advanced Engineering Electromagnetics*, New York : John

Wiley & Sons 1989.

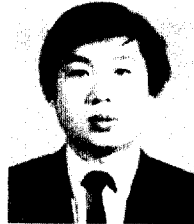
[5] D. A. McNamara, C. W. I. Pistorius, and J. A. G. Malherbe, *Introduction to The Uniform Geometrical Theory of Diffraction*, Boston : Artech House 1990.

— 著 者 紹 介 —



權 度 熏 (正會員)

1973年 2月 7日生. 1994年 2月 한국과학기술원 과학기술대학 전기 및 전자공학과 졸업. 1994年 10月 현재 Ohio State University 석사과정. 주관심 분야는 전파 전파 및 산란 해석, 마이크로파 공학, 이동 및 위성통신, EMI/EMC 등임.



明 魯 勳 (正會員)

1976年 2月 서울대학교 전기공학과 졸업(학사). 1982年 12月 Ohio State Univ. 전기공학과(석사). 1986年 8月 Ohio State Univ. 전기공학과(박사). 1986年 9月 이후 한국과학기술원 전기 및 전자공학과 재직. 주관심 분야는 전파 전파 및 산란 해석, 마이크로파 공학, 이동 및 위성통신, EMI/EMC/EMS 등임.



宣 永 植 (正會員)

1967年 7月 17日生. 1990年 2月 동국대학교 공과대학 전자공학과 졸업(학사). 1992年 2月 한국과학기술원 전기 및 전자공학과 졸업(석사). 1994年 4月 이후 금성정보통신 재직. 1994年 10月 현재 한국과학기술원 전기 및 전자공학과 박사과정. 주관심 분야는 전파 전파 및 산란해석, 마이크로파 공학, 이동 및 위성통신, EMI/EMC 등임.