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Fractals in the Spreading of Drifters: Observation and Simulation

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표류부표 분산의 프랙탈 성질: 관측 및 시뮬레이션

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We examined the temporal characteristics of the oceanic eddy diffusion at 5 coastal regions of Korea by measuring the separation distances of multiple drifters released simultaneously at the same initial position. The Lagrangian trajectories of drifters are tracked for periods between 2 and 6 hours by the GPS and Decca Trisponder System. The observed variance of separation distance, for the time scales from minutes to hours, is proportional to t^m with scaling exponent m between 1.2 and 2.0. The observed Lagrangian trajectories of drifters show fractal characteristics instead of random walk or Brown motion. As an effort toward a development of a realistic model of the oceanic eddy diffusion, we simulated the Lagarangian trajectories of drifters by fractional Brown motion (FBM) model. The observed variances of drifter separations can be generated by the FBM process provided the Hurst exponent is the same as the observed one. We further showed that the observed power law in the variance of drifter separations cannot be simulated with an ordinary Brown motion or random walk process.

동일지점에서 동시에 투하된 여러개 표류부표들 사이의 분산거리를 측정하므로써, 한반도 연안역 5개 해역에서 해양난류확산의 시간적 특성을 조사하였다. 각각의 실험에서 GPS와 Decca Trisponder를 사용하여 표류부표의 궤적을 2~6시간 동안 추적하였다. 실측된 부표간 분리거리의 분산(거리의 제곱)은 시간 t에 따라 t^m에 비례하는 지수관계을 나타내었으며, 수분 내지 수시간의 시간 스케일에서, 승수 m은 1.2와 2.0의 범위였다. 실측된 부표의 궤적은 임의행보나 브라운운동의 특성을 나타내지 않고, 프랙탈 브라운운동의 특성을 나타내었다. 해양에서의 실제적인 상황에 부합되는 확산모델 개발을 위한 일차적인 시도로서 표류부표의 운동을 프랙탈 브라운운동 모델에 의해 재현하였다. 본 논문에서는 실측된 표류부표 궤적에 해당되는 허스트(Hurst) 지수를 적용하여 프랙탈 브라운운동 모델로 표류부표의 이동을 재현하면 부표간 분리거리 분산의 지수관계가 실측치와 일치함을 보였다. 또한, 통상적인 브라운운동 또는 임의행보 모델로 표류부표의 이동을 모델링하면 현장관측과 일치하는 표류부표 분리거리의 지수관계가 재현되지 못함을 보였다.

INTRODUCTION

Transports of pollutants in the ocean are governed by advection and diffusion processes. The molecular diffusion by Brown motion can be successfully represented by a diffusion model with a constant diffusivity or equivalently by random walk process. The variance of dispersion in the Brown motion or random walk process is linearly proportional to time.

In analogy to the molecular diffusion, by Boussinesq assumption, the eddy diffusion in the ocean is usually simulated by diffusion models with constant diffusivity. The spreading area of pollutant patches in a model with constant diffusivity increases linearly with time. The observations in the ocean, however, show that the variances of the spreading of patches increase with time with scaling exponent greater than unity. Okubo's (1971) analysis of various dye experiments off east coast of United State shows that the variance of dye patches for the time scales of hours to weeks can be fitted to an exponential curve

$$\sigma^2 = 0.0108\ t^{2.34}$$

where the variance σ^2 is in units of m^2 and the time t is in sec. Consequently, the apparent eddy diffusivity is not constant but depends on the 'scales' of diffusion.

The exponent m of the relation

$$\sigma^2 = a\ t^m$$

differs depending on the scales of turbulence. For example, logarithmic least squares fit of the available data by Talbot (1974) showed that the exponent m is 0.96 in shallow seas, 1.35 in American estuaries, 1.65 in fjords, 1.79 in the Baltic coast, 1.86 in English estuaries, 2.19 in American coastal waters, and 2.87 in the open sea. These figures suggest that the exponent m is as small as 1 in shallow seas and as large as 3 in deep open seas (Pasmanter, 1988).

Recently, many investigators (Osborne *et al.*, 1986, 1989; Sanderson *et al.*, 1990) reported the fractal characteristics of the drifter trajectories in the sea. Osborne *et al.* (1986, 1989) reported that the drifter trajectories associated with the mid-oceanic eddies of 20~50 km in the Kuroshio extension show fractal behavior with approximate fractal dimension D of 1.3 (D=1.15~1.44). Sanderson *et al.* (1990) showed that the separation distances between pairs of drifters deployed in Lake Erie, the Atlantic Equatorial Undercurrent and coastal waters off the Long Island manifest fractal behavior with average fractal dimension of 1.3 (D=1.12~1.59) at space scales between 10 m and 4000 m.

In this paper we report the exponents m obtained in the experiments at 5 coastal water of Korea. Previous similar studies (eg. Osborne *et al.*, 1986, 1989; Sanderson *et al.*, 1990) were focused in computing fractal dimension of trajectories or that of relative separation. In this study we are interested in developing an algorithm to simulate the observed features of drifter trajectories and separation distances of drifters. This algorithm is expected to be successfully adoped to a 'realistic' diffusion model. We found that a simulation of Lagrangian drifter motions by fractional Brown motion (FBM) yields the observed exponent of the temporal evolutions of variance. In this paper we further show that the ordinary random walk model cannot yield the observed power laws, in the variance of separation distance of drifters, regardless of the 'step size' of random walks.

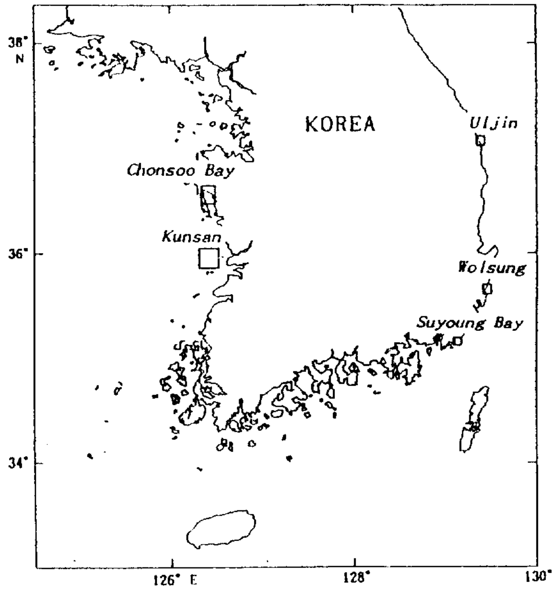

Fig. 1. Regions of our drifter tracking experiments.

OBSERVATION AND ANALYSIS

Field experiment

In each experiment we tracked Lagrangian trajectories of 3 drifters which were released simultaneously at the same starting position. The drifters with vanes of the shape 'X' were designed to follow currents at 1.5m depth. Our experiments were carried out in 1992 and 1993 in Chonsoo Bay and off the coast of Kunsan in the west coast of Korea,

Table 1. Summary of drifter tracking experiments

Exp. name	Region of experiment	Date of experiment (lunar date)	Duration of tracking	Travel distance	Positioning equipment
A	Chonsoo Bay	Mar. 12, 1993 (Feb. 21)	2.5 hr	2 km	GPS*
B1	Off Kunsan	Nov. 12, 1992 (Oct. 18)	4.5 hr	14 km	GPS
B2	Off Kunsan	Nov. 13, 1992 (Oct. 19)	5.4 hr	14 km	GPS
C	Suyoung Bay	Nov. 1, 1992 (Oct. 7)	2.8 hr	3 km	GPS
D	Off Wolsung	Aug. 19, 1993 (Jul. 2)	2.2 hr	4 km	DTS*
E	Off Uljin	Aug. 17, 1993 (Jun. 30)	2.2 hr	1.5 km	DTS

*GPS=Global Positioning System, DTS=Decca Trisponder System

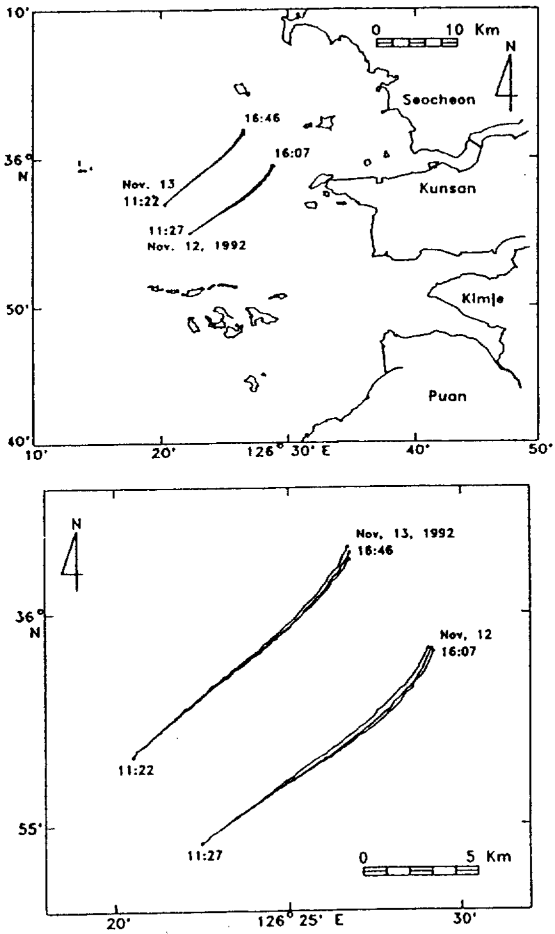

Fig. 2. Trajectories of 3 drifters off the coast of Kunsan (*upper figure*) and their enlarged views (*lower figure*).

Suyoung Bay in the south coast, and off the coasts of Wolsung and of Uljin in the east coast (Fig. 1).

In each experiment the trajectories of drifters were tracked for a period between 2 and 6 hours by a boat equipped with the Global Positioning System (GPS) or the portable Decca Trisponder System (DTS). The nominal accuracies in positioning by GPS and DTS are claimed to be a few tens meters and a few meters, respectively. Dates, durations of drifter tracking, travel distances and the equipments for positioning for each experiment are summarized in Table 1. Sampling interval of positioning is between 2 and 5 minutes. Trajectories of drifters off the coast of Kunsan during our two days experiment are shown in Fig. 2. Trajectories of drifters off the coasts of Wolsung and of Uljin are shown in Fig. 3.

Variance of separation

From the data of Lagrangian trajectories of drifters, we computed the separation distances between each pair of drifters as a function of time. The average variance, which is computed by an arithmetic average of variances of 3 pairs, are plotted on logarithmic scale in Fig. 4. The variances σ^2 of separation distance in each experiment are fitted to a curve

$$\sigma^2 = a\ t^m$$

by the least-squares fitting, where a and m are constants. The results of the estimated variance and eddy diffusivity are listed in Table 2. Magnitudes of the exponent m in our experiments were between 1.25 and 1.99, and their average is 1.62.

Eddy diffusivity

The eddy diffusivity K defined by the time rate

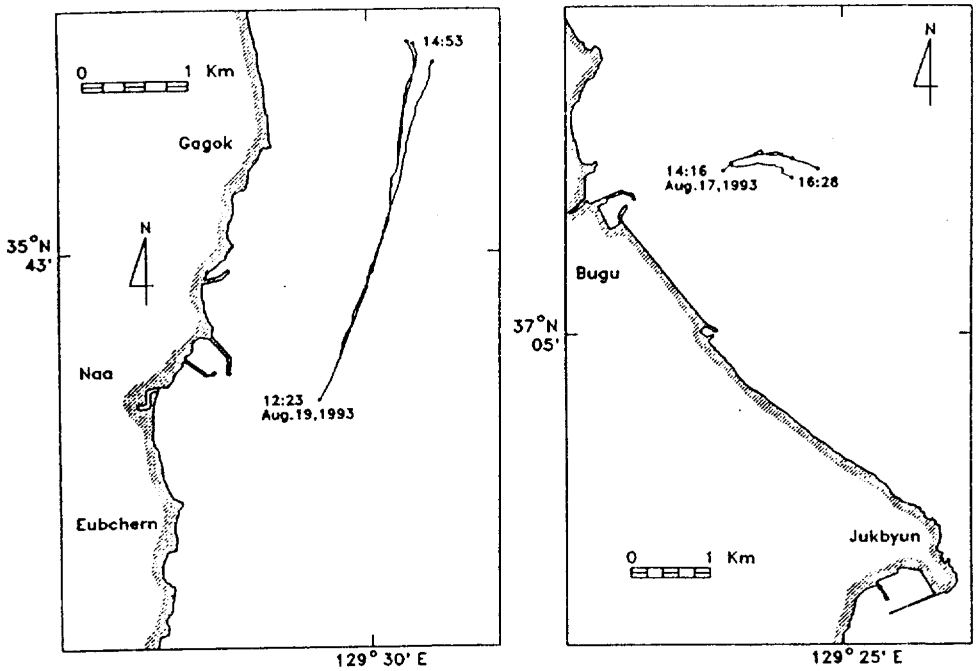

Fig. 3. Trajectories of 3 drifters off the coast of Wolsung (*left figure*) and off the coast of Uljin (*right figure*).

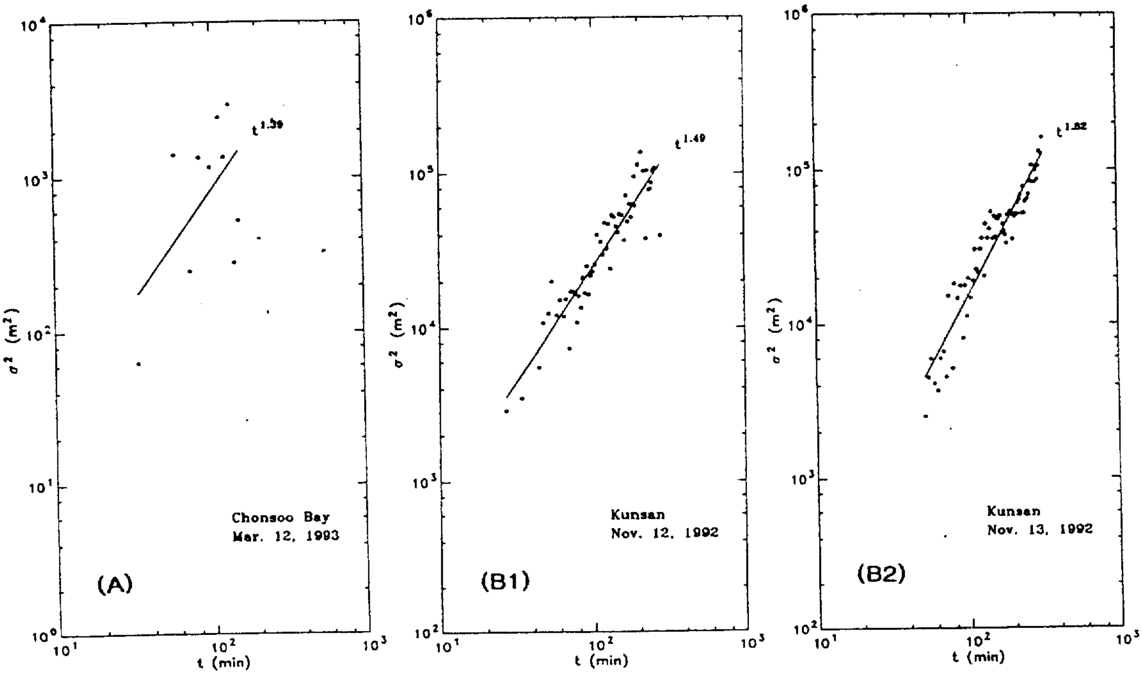

Fig. 4. Variograms of separation distances of 3 drifters in Chonsoo Bay (A), off the coast of Kunsan (B1 and B2), in Suyoung Bay (C), off the coast of Wolsung (D), and off the coast of Uljin (E).

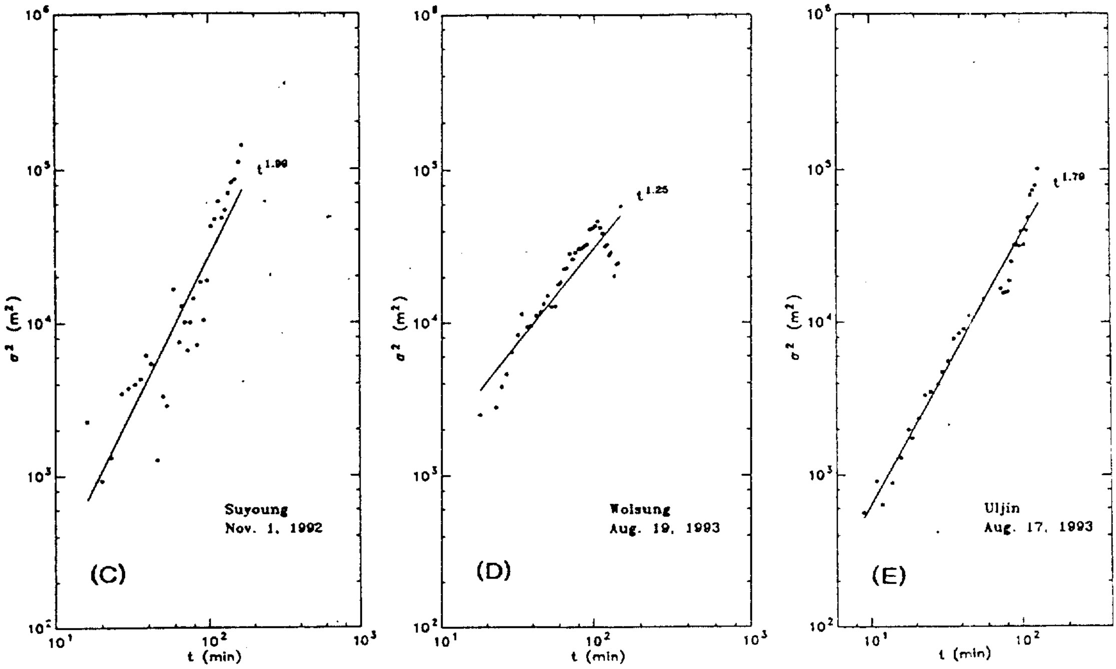

Fig. 4. Continued.

Table 2. Variance (σ^2) and eddy diffusivity (K) associated with separation distances of drifters (σ^2 in units of m^2, K in m^2/sec)

Experiment name	Region of experiment	Variance of separation (σ^2)	Eddy diffusivity (K)
A	Chonsoo Bay	$1.38\ t^{1.39}$	$0.96\ t^{0.39}$
AB1	Off Kunsan	$26.42\ t^{1.49}$	$19.68\ t^{0.49}$
AB2	Off Kunsan	$3.58\ t^{1.82}$	$3.26\ t^{0.82}$
AC	Suyoung Bay	$2.75\ t^{1.99}$	$2.74\ t^{0.82}$
AD	Off Wolsung	$97.05\ t^{1.25}$	$60.66\ t^{0.25}$
AE	Off Uljin	$10.26\ t^{1.79}$	$9.18\ t^{0.79}$

of change of variance is

$$K=\frac{1}{2}\frac{d\sigma^2}{dt}.$$

When the variance is given by $\sigma^2=at^m$, the eddy diffusivity can be represented by

$$K=\frac{1}{2}a\ m\ t^{m-1}$$

or

$$K=b\ \sigma^n,$$

where b is a constant and $n=(2m-1)/m$ (Bowden *et al.*, 1974; Bowden, 1983). Eddy diffusivity estimated from the separations of drifters is not constant but increases with time with the scaling exponent between 0.25 and 1.00 (Table 2).

SIMULATIONS

We numerically simulated Lagrangian motions of 3 drifters in an ocean with northeasterly mean currents of $(U, V)=(0.1, 0.1)$ m/sec by two different models. The first one is the random walk model which corresponds to an ordinary Brown motion. The second one is the fractional Brown motion (FBM) model of which details will be discussed later. In each case the drifter motions are simulated for 9.1 hours (2^{15} sec).

Random walk model

Following a 'classic' approach toward diffusion process, we simulated Lagrangian trajectories of drifters by the random walk process. The position (X_t, Y_t) of each drifter at time t are simulated by

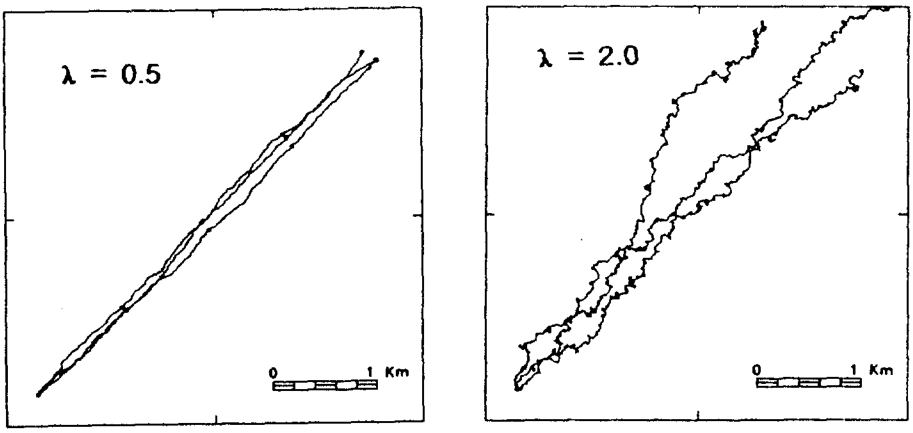

Fig. 5. Trajectories of 3 drifters generated by a random walk model with bandwidth parameter $\lambda=0.5$ and $\lambda=2.0$.

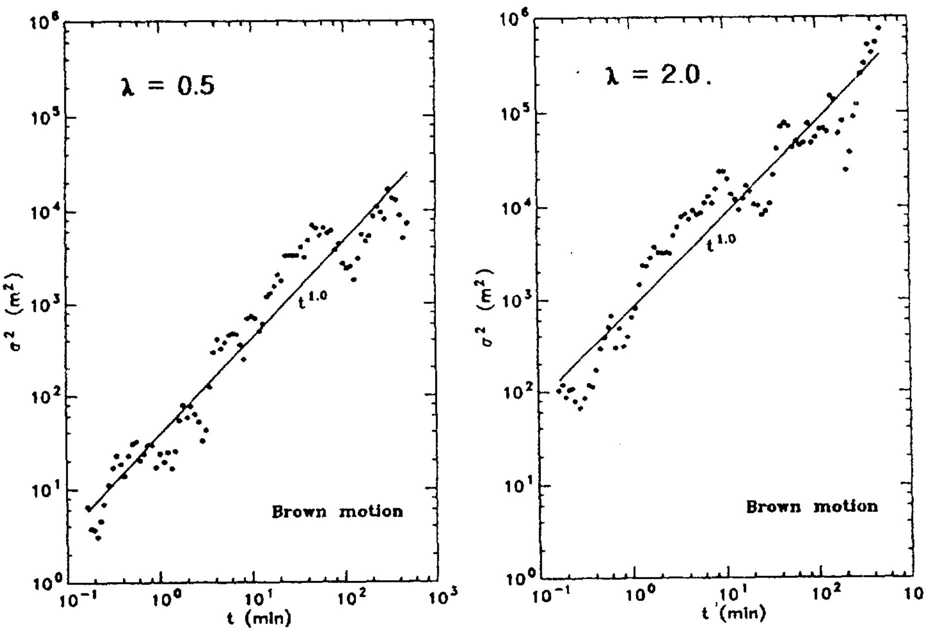

Fig. 6. Variograms of separation distances of 3 drifters generated by a random walk model with bandwidth parameter $\lambda=0.5$ and $\lambda=2.0$.

$$X_t = X_{t-1} + U\delta t + \lambda\xi_t$$
$$Y_t = Y_{t-1} + V\delta t + \lambda\eta_t$$

where U and V are the mean currents in the x and y directions, respectively, δt is time interval ($\delta t=1$ sec in our experiment), λ is a constant multiplier, and ξ_t and η_t are normalized Gaussian random number with mean 0 and standard deviation 1. The Gaussian random number is generated by Box-Muller algorithm (Press *et al.*, 1992). The terms $U\delta t$ and $V\delta t$ represent displacements associated with mean currents and those $\lambda\xi_t$ and $\lambda\eta_t$ represent displacements associated with random motion. The multiplier λ specifies the statistical bandwidth of 'random' displacement at each movement. A large value of λ corresponds to a large eddy diffusivity in the diffusion equation.

Fig. 5 shows the trajectories of 3 drifters of which positions were simulated by random walk model with $\lambda=0.5$ and $\lambda=2.0$. Lagrangian motions of drifters with larger values of bandwidth parameter λ have more wiggliness in their trajectories and larger separation distance than those with smaller bandwidth parameter. The variograms of the separation of drifters, simulated by random walk model for two case, are shown in Fig. 6. Although the variance of separation with larger bandwidth is greater than that with smaller bandwidth, the variances in both cases are linearly proportional to time. The observed exponents between 1.25 and 1.99 cannot be generated by the random walk model. In fact, it can be shown that the variance associated with random walk is always linearly proportional to time (Berg, 1983; Barnsley, 1989).

FBM model

As mentioned before, the variance of random walk process or Brown motion increases linearly with time. In general, the variance can be written in a form

$$\mathrm{Var}(t) \propto t^{2H}$$

where H is Hurst scaling exponent. For Brown motion, magnitude of H is 0.5. A time series with Hurst exponent other than 0.5 can be described by fractional Brown motion (FBM) defined by

$$X_t = X_{t-1} + G_t$$

where G_t is fractional Gaussian noise (Mandelbrot and van Ness, 1968; Mandelbrot and Wallis, 1969).

The FBM time series with H between 0.0 and 0.5 is an anti-persistent series which shows high-frequency oscillatary fluctuations. The FBM with $H=0.5$ is an ordinary Brown motion. The FBM time series with H between 0.5 and 1.0 is a persistent series which have a tendency to maintain its past values. Geophysical time series usually have Hurst exponent greater than 0.5, and those persistent feature of geophysical time series is sometimes referred to, following the stories of floods and droughts in the Bible, as 'Noah' phenomena or 'Joseph' phenomena (Mandelbrot, 1983).

Table 3. Hurst exponents (H) and fractal dimensions (D) associated with separation distances of drifters

Experiment Name	Region of experiment	Hurst exponent (H)	Fractal dimension (D)
A	Chonsoo Bay	0.69	1.31
B1	Off Kunsan	0.74	1.26
B2	Off Kunsan	0.91	1.09
C	Suyoung Bay	0.99	1.01
D	Off Wolsung	0.62	1.38
E	Off Uljin	0.79	1.21

The FBM time series possess fractal characteristics of which dimension is not an integer. The fractal dimension D of a FBM series can be estimated from Hurst exponent H by (Voss, 1985)

$$D = 2 - H.$$

The Hurst exponents and fractal dimensions of the drifter separations of our experiments on coastal regions of Korea are shown in Table 3. The fractal dimensions of observed drifter separations are between 1.0 and 1.4.

We simulated the Lagrangian trajectories of 3 drifters by the FBM model. The position of each drifter is simulated by

$$X_t = X_{t-1} + U\delta t + F_t$$
$$Y_t = Y_{t-1} + V\delta t + G_t$$

where F_t and G_t are fractional Gaussian noises. We made the time series of the fractional Gaussian noise by the difference of the FBM series generated by the mid-point displacement method (Voss, 1985, 1988; Saupe, 1988).

Fig. 7. shows trajectories of 3 drifters simulated by the FBM model with Hurst exponents 0.5, 0.6, 0.75 and 0.9. In the numerical simulations of drifter motion, we adjusted the standard deviations of the mid-point displacement method. This adjustment yields almost the same wiggliness in the trajectories of drifters with different Hurst exponents. The separation distances of drifters with large Hurst exponent at a given time is greater than those with small Hurst exponent.

Variograms of the simulated drifter separations with $H=0.5$, 0.6, 0.75 and 0.9 are shown in Fig.

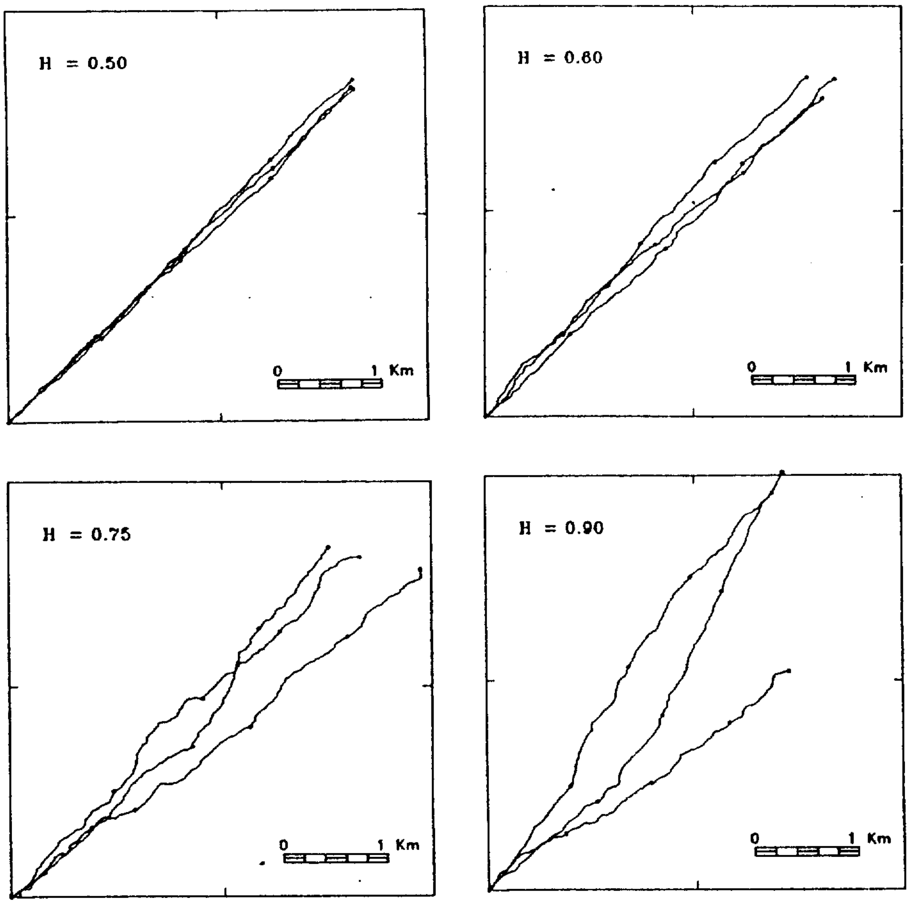

Fig. 7. Trajectories of 3 drifters generated by the FBM model with Hurst exponents of 0.5, 0.6, 0.75 and 0.9.

8. In fact, as shown in Fig. 8, a simulation of the drifter motion by FBM model yields desired scaling exponent in the temporal changes of variance.

DISCUSSION

In this paper, based on the observed trajectories of drifters in coastal regions of Korea, we showed that the variance of separation distance between drifters increase with time with scaling exponent between 1.25 and 1.99. These figures of exponent imply that the Lagragian trajectories of drifter can be understood as FBM process superimposed on the mean currents. The corresponding Hurst exponents of separation distances are between 0.62 and 0.99. The fractal dimension of separation distances are between 1.01 and 1.38, and these figures are in accordance with the approximate mean value of 1.3 reported by Osborne *et al.* (1986) and Sanderson *et al.* (1990).

We demonstrated that the observed scaling properties in the separations of drifters cannot be simulated by an ordinary Brown motion or random walk process. The variance of patch spreading associated with an ordinary Brown motion is always lineary proportional to time. Diffusions associated with ordinary Brown motion is equivalent to the solution of a diffusion equation with a constant diffusivity.

Our observations of drifter separations, of which variance increases with scaling exponent between 1.25 and 1.99, suggest that a realistic simulation of eddy diffusion processes in the ocean cannot

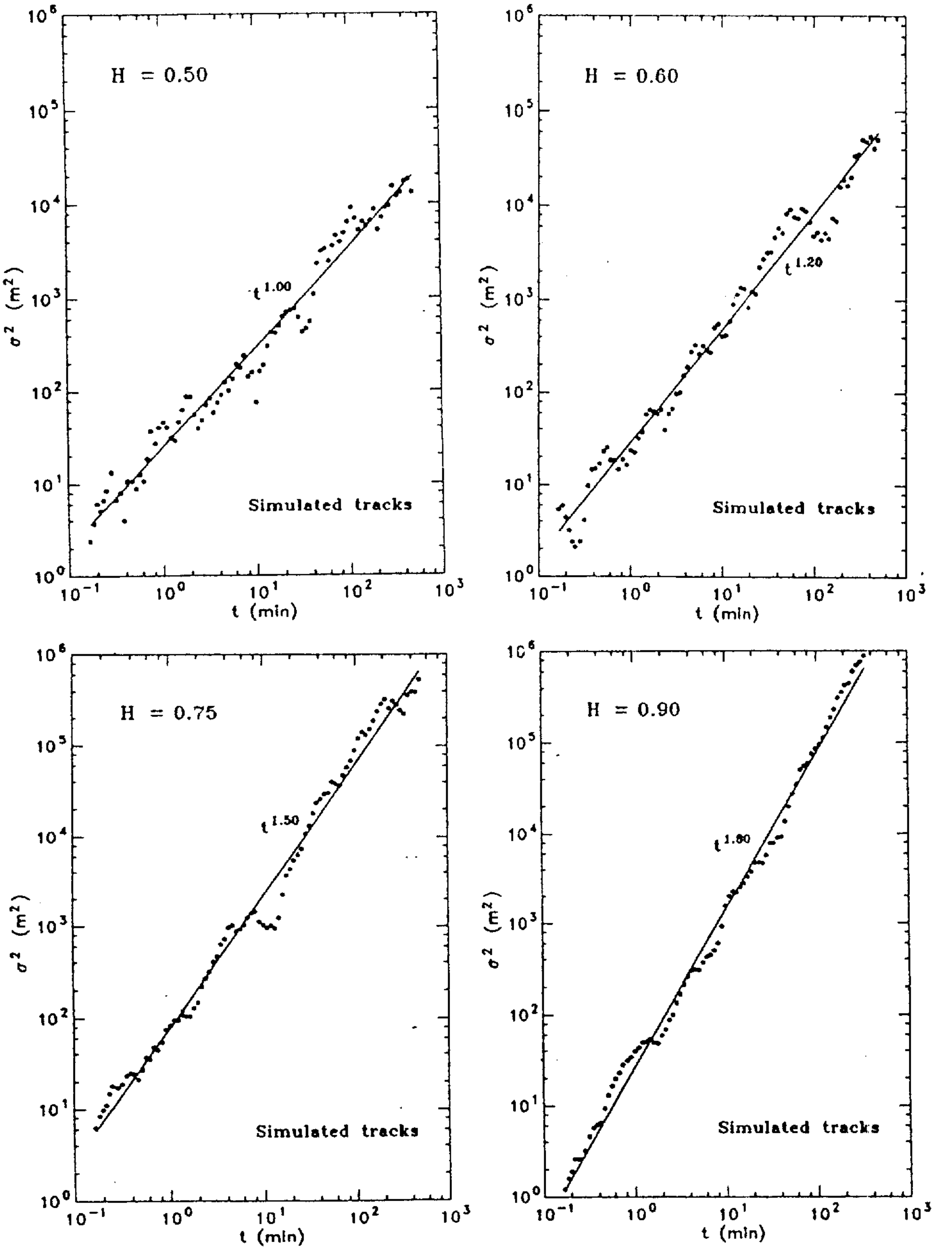

Fig. 8. Variograms of separation distances of drifters generated by the FBM model with Hurst exponents of 0.5, 0.6, 0.75 and 0.9.

be achieved by a diffusion model with a constant diffusivity. Monte Carlo simulations of the oceanic diffusion, based on random walk approach, also cannot reproduce the observed scaling exponent. In this paper we demonstrated that the FBM model can reproduce the observed scaling properties of variance. A new 'realistic' oceanic diffusion model, which incorporates the fractal nature in the variation of patch spreading, is expected to be developed in a near future.

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