The Journal of the Korean Society of Oceanography
Vol. 29, No. 3, pp. 228-238, September 1994

Interpolation and Reconstruction of the Holocene Sea-levels Using Inverse Fractal Interpolation Functions

SANG YONG CHUNG, DAE CHOUL KIM AND HI-IL YI*
Department of Applied Geology, National Fisheries Univesity of Pusan, 608-737, Korea
**Marine Geology & Geophysics Division, Korea Ocean Research and Development Institute*

프랙탈 내삽함수 역산법을 이용한 홀로세 해수면의 내삽 및 재구성

정상용 · 김대철 · 이희일*
부산수산대학교 응용지질학과
*한국해양연구소, 해양지질연구부

The change of sea-level is a good indicator of the change of climate during the Quaternary period. The sea-levels in the world have been changing very irregularly during that time. The pattern of the Quaternary sea-level change was assumed to be a stochastic fractal in this study. We measured fractal dimensions of the Holocene sea-levels of the Hudson River estuary and the Delaware coast. A box counting method gave almost the same values, i.e., D=1.358 for the Hudson sea-level changes and D=1.346 for the Delaware sea-level changes. The ability of the inverse method of fractal interpolation functions (IFIF) was examined by the interpolation of the Hudson and the Delaware sea-levels. IFIF reproduction the realistic sea-levels for the both of them. The Delaware sea-level data made less statistical errors for the interpolation of IFIF than the Hudson sea-level data. This suggests that the Delaware sea-leval data are more reliable than the Hudson sea-leval data. IFIF was also used to estimate sea-levels of the Korean coasts because they don't have enough sea-level data during the Holocene Epoch. The vertical scaling factor of IFIF for the Korean sea-level data was calculated from the fractal dimension of the Delaware sea-level data. Fractal interpolation functions (FIF) was used to reconstruct the peleosea-levels of the Korean coasts and the Atlantic Ocean coasts of the United States. The Korean Paleosea-level change generacted by FIF is different from the paleosea-level change of the eastern U.S.. The Korean paleosea-levels are much higher than the eastern U.S. paleosea-levels, comparing to each other from the present to 8,000 BP.

제4기 동안에 일어난 해수면 변화는 지역적 그리고 나아가 전 지구 기후변화에 좋은 지시자가 되고 있다. 세계 해수면은 제4기 동안 매우 불규칙하게 변해왔는데 여기에 대한 정밀한 수학적 접근은 어려운 실정이었다. 이 연구에서 처음으로 세계 해수면 변동자료에 확율 프랙탈을 적용하여 제4기 해수면 변동을 재구성 하였다. 미국 대서양 연안에 위치한 허드슨만과 델라웨어 해안의 홀로세 해수면 변화에 대한 프랙탈 차원을 상자 계산 방법으로 측정한 결과, 허드슨만 해수면 변화는 프랙탈 차원이 1.358이고 델라웨어 해수면 변화는 1.346이다. 프랙탈 내삽함수의 역산 방법 (IFIF)의 유효성이 허드슨만과 델라웨어 해수면의 내삽에 의해서 검증되었고, IFIF에 의해서 두지역에서의 합리적인 해수면들이 복원되었다. 델라웨어 해수면 자료는 허드슨 해수면 자료보다 IFIF의 내삽에 대해 통계적 오차가 적게 나타나는데, 이는 델라웨어 해수면 자료가 허드슨 자료보다 더 신빙성이 높다는 것을 의미한다. IFIF는 한국 해안의 해수면을 추정하는 데도 이용되었는데, 한국 해안의 해수면 자료는 허드슨만이나 델라웨어 지역보다 자료가 충분치 못하다. 따라서 IFIF의 적용 통계 오차가 적은 델라웨어 해수면 자료에서 구한 프랙탈 차원을 사용하여 한국 해안에 대한 IFIF의 수직 비율 인자를 계산하였다. 프랙탈 내삽

함수(FIF)를 한국 해안과 미국 대서양 연안의 고해수면의 재구성에 적용한 결과, 한국의 고해수면은 미국의 고해수면과 다른 변화를 보였다. 8,000년 전부터 현재까지 두 지역의 해수면은 비교해 보면 한국 해수면이 미국 해수면 보다 훨씬 높았다.

INTRODUCTION

It is believed that the sea-levels of the world have risen since the last Glacial Age. They are oscillating within the rise of sea levels because of the continuous climatic changes during the Quaternary Period. Hence, the global sea-level history reflects the climatic changes. By the way, we have difficulties in obtaining paleosea-level data because ^{14}C age data of peats are necessary for them. It is very hard and expensive to get the accurate ^{14}C data from the age dating. It may be a good way to estimate paleo sea-levels with statistical methods for the acquisition of more sea-level data and the better understanding of paeoclimates. The variation of sea-levels during the Quaternary Period is very irregular. The normal statistical methods such as least squares, spline, and polynomial fitting cannot produce the very irregular sea levels realistically.

Fractal geometry is a good tool for the realization of very complicated shapes, images, or phenomena. Fractal geometry was applied to the reconstruction of sea floor topography (Mareschal, 1989), dispersivity distributions (Wheatcraft and Tyler, 1988) and hydraulic conductivity distributions (Chung, 1992) in heterogeneous aquifers, and to the relation between the length and drainage area of mainstream (Robert and Roy, 1990; Rosso and Bacci, 1991). The prupose of this study is to apply a stochastic fractal to the interpolation of sparse ^{14}C data and the reconstruction of the Holocene sea-level curve. Two data sets of sea-levels, i.e., the data sets of the Hudson River estuary and the Delaware coast along the Atlantic Ocean were used for the verification of the ability of the inverse method of fractal interpolation functions (IFIF) in interpolating sea-level data. The Holocene sea-level data of the Korean coasts were reproduced by IFIF because they have only a small number of paleosea-level data. Fractal interpolation functions (FIF) were used for the reconstruction of the Holocene sea-levels of the Hudson River estrary, the Delaware coast, and the Korean coasts.

FRACTALS AND FRACTAL DIMENSIONS

Fractal geometry was initiated by Mandelbrot (1967). It comes from the Latin adjective *fractus* which means the both "fragmented" and "irregular." Euclidean geometry cannot describe complex natural objects realistically such as clouds, mountains, coastlines, snow flakes and asteroids because Euclidean geometry is based on a characteristic scale and specific formulas. On the other hand, fractal geometry is not dependent on a specific scale and it uses recursive algorithms for the generation of shapes or images. Fractal theory has progressed a great deal during the last decade. Fractal concepts and geometry have become important tools to explain complex and irregular natural phenomena. Multifractal geometry was applied for energy dissipation in turbulant flows (Sreenivasan et al., 1989), viscous fingering in porous media (Malφy et al., 1985; Chen and Wilkinson, 1985), and the measurements of cloud and rain (Lovejoy and Schertzer, 1990). Moreover, fractals play a central role in the realistic rendering and modelling of natural phenomena in computer graphics.

All fractals are characterized by their own fractal dimensions. The fractal dimension of a fractal line is $1.0 \leq D < 2.0$, a fractal surface is $2.0 \leq D < 3.0$, and a fractal volume is $3.0 \leq D < 4.0$. The fractal dimension of a self-similar fractal is easily determined by a simple formula (Mandelbrot, 1985). The determination of the fractal dimension of a self-affined fractal needs a divider method, variogram method, spectral method, or box counting method. The divider method often gives an anomalously low fractal dimension because of crossover length problem (Brown, 1987; Malinverno, 1990). The variogram method tends to overestimate the fractal dimension of the self-affined fractal of a stationary random function (Chung, 1992). The spectral method introduced by Voss (1985) needs the transformation of fractional Brownian motion (fBm) in time domain to the corresponding spectral density in frequency domain. The box counting method always gives the Euclidean dimension to self-affined fractals within crossover length (Mandelbrot, 1985). Brown's gird method (1988) is similar to the box counting method, but it solves crossover length problem. His method uses the standard deviation of the fractal amplitudes over the horizontal resolution or scale of self-affined fractals, and an equation of the standard deviation developed by Wong and Lin (1988). Brown's method produces the power law which doesn't include the crossover length:

$$N(n)=n^{D} \tag{1}$$

where

$N(n)$ is the total number of boxes covering fractals,

n is the horizontal resolution or scale of fractals,

D is the fractal dimension.

The fractal dimension of a self-affined fractal is determined from the slope of log N(n) versus log n.

FRACTAL INTERPOLATION FUNCTIONS (FIF)

Fractal interpolation functions (FIF) were developed by Barnsley (1986) to simulate one-dimensional landscapes such as the profiles of mountain ranges, tops of clouds, the stalactite-hung roofs of caves and the horizons over forests. Mareschal (1989) used FIF for the construction of bathymetry profiles.

FIF uses an iterated function system (IFS) with 1-D affine transformation

$$w_n\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} a_n & 0 \\ c_n & d_n \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}+\begin{pmatrix} e_n \\ f_n \end{pmatrix} \tag{2}$$

where a_n, c_n, d_n, e_n and f_n are real constants.

FIF needs two constraints for the determination of real constants in the 1-D affine transformation

$$w_n\begin{pmatrix} X_0 \\ F_0 \end{pmatrix}=\begin{pmatrix} X_{n-1} \\ F_{n-1} \end{pmatrix}, \quad w_n\begin{pmatrix} X_N \\ F_N \end{pmatrix}=\begin{pmatrix} X_n \\ F_n \end{pmatrix} \tag{3}$$

where

n is 1, 2, ⋯, N.,

(X_0, F_0) is the initial value of interpolation data,

(X_N, F_N) is the last value of interpolation data.

The transformation w_n is shear transformation and d_n is a free parameter. When L is assumed to be a line segment parallel to the y-axis, $w_n(L)$ is also a line segment parallel to the y-axis. The ratio of the length of $w_n(L)$ to the length of L is $|d_n|$. The d_n is called a vertical scaling factor in the transformation w_n. The value of d_n is $0<|d_n|<1$ because the IFS has a finite set of contraction mappings. A piecewise-linear interpolation function is generated for $d_n=0$.

For real constants are determined from Eqs. (2) and (3) with known interpolation points and d_n:

$$a_n=(X_n-X_{n-1})/(X_N-X_0), \tag{4}$$

$$e_n=(X_NX_{n-1}-X_0X_n)/(X_N-X_0), \tag{5}$$

$$c_n=(F_n-F_{n-1})/(X_N-X_0)-d_n(F_N-F_0)/(X_N-X_0), \tag{6}$$

$$f_n=(X_NF_{n-1}-X_0F_n)/(X_N-X_0)-d_n(X_NF_0-X_0F_N)/(X_N-X_0). \tag{7}$$

New interpolation points are determined from the above four real constants and Eq. (2), using a random iteration algorithm.

Fractal dimension of fractal interpolation functions

Barnsley (1988) explained the fractal dimension of FIF as follows:

If $\sum_{n=1}^{N}|d_n|>1$ from Eq. (2), and the interpolation points don't all lie on a single straight line, the fractal dimension of FIF is the unique real solution D of

$$a_n^{D-1}\sum_{n=1}^{N}|d_N|=1 \tag{8}$$

where N is the number of intervals which the vertical scaling factor occupies. Otherwise, the fractal dimension of FIF is 1.0.

If interpolation points are equally spaced between X_0 and X_N, the X_i at any point i is

$$X_i = X_0 + \frac{i}{N}(X_N - X_0) \quad \text{for } i=0, 1, 2, \cdots, N. \quad (9)$$

The real constant a_n is $\frac{1}{N}$ from Eq. (4) and Eq (9). Therefore, the fractal dimension of FIF is 1.0.

$$D = 1 + \frac{\log(\sum_{i=1}^{N} |d_n|)}{\log(N)} \quad (10)$$

The fractal dimension of FIF is less than 2.0 because $\sum_{n=1}^{N} |d_n| < N$. For equally spaced interpolation points, the vertical scaling factor is determined from Eq. (10) as

$$|d_n| = N^{D-2} \quad (11)$$

INVERSE METHOD OF FRACTAL INTERPOLATION FUNCTIONS (IFIF)

The inverse method (IFIF) of fractal interpolation functions gives estimates at desired interpolation points from the fractal distribution generated by FIF. IFIF was developed to obtain estimates of the hydraulic conductivities in a heterogeneous sand aquifer (Chung, 1992). It is a new interpolation technique using fractal geometry. The algorithms of IFIF are as follows:

1. Determine the fractal dimension of a fractal distribution determined by experimental data.
2. Determine its vertical scaling factor from Eq. (11). If the difference of values between two consecutive data is greater than or equal to 0, the sign of d_n is positive. If not, it is negative.
3. Generate a fractal distribution with sample data and vertical scaling factors, using a random iteration algorithm.
4. Determine interpolation values at desired interpolation points. All interpolated points don't match exactly the desired interpolation points. A tolerance limit between an actual interpolated point and a desired interpolation point is required to determine an interpolation value at a desired point. The tolerance limit should be very small so that the selected interpolated point can be accepted as a desired interpolation point. If more than one interpolated point are produced in a tolerance limit, an estimate should be selected at the closest point to a desired interpolation point. If the number of iterations for the random iteration algorithm of FIF is increased, a tolerance limit can be decreased. However, computer execution time is increased.

APPLICATION OF IFIF TO THE HOLOCENE SEA-LEVELS

Sea-level Data

Newman et al., (1987) studied the Holocene neotectonics and the Ramapo fault zone using the Quaternary sea-level anomaly at the lower Hudson River estuary (Figure 1). They used the Quaternary sea-level data originated from the ^{14}C dated basal peats, organic silts, and wood samples. The first experimental data set uses only the sea-level data of the radiocarbon-dated basal peats among the Hudson River data and excludes the anomalous data originated from downfaulted blocks. The number of experimental data is 94. Figure 2 shows the variation of the experimental data for the Hudson River estuary.

The second experimental data set is 77 sea-level data originated from the ^{14}C dated peats of the Delaware coasts. The Delaware ^{14}C data were collected by Belknap (1975), Kraft (1976), and Fletcher et al. (in press). Figure 3 is the location map of the Delaware Bay estuary and part of the Atlantic Ocean coasts (Yi, 1993). The variation of the experimental data for the Delaware Bay estuary and part of Atlantic Ocean coast is shown in Figure 4.

The third experimental data set is Park (1983)'s sea-level data originated from the radiocarbon-dated organic sediments of the Korean coasts. The data are only selected from ^{14}C dating of basal peats because of two reasons. One is ^{14}C dating from basal peats have less error range than other organic materials such as shells. The other reason

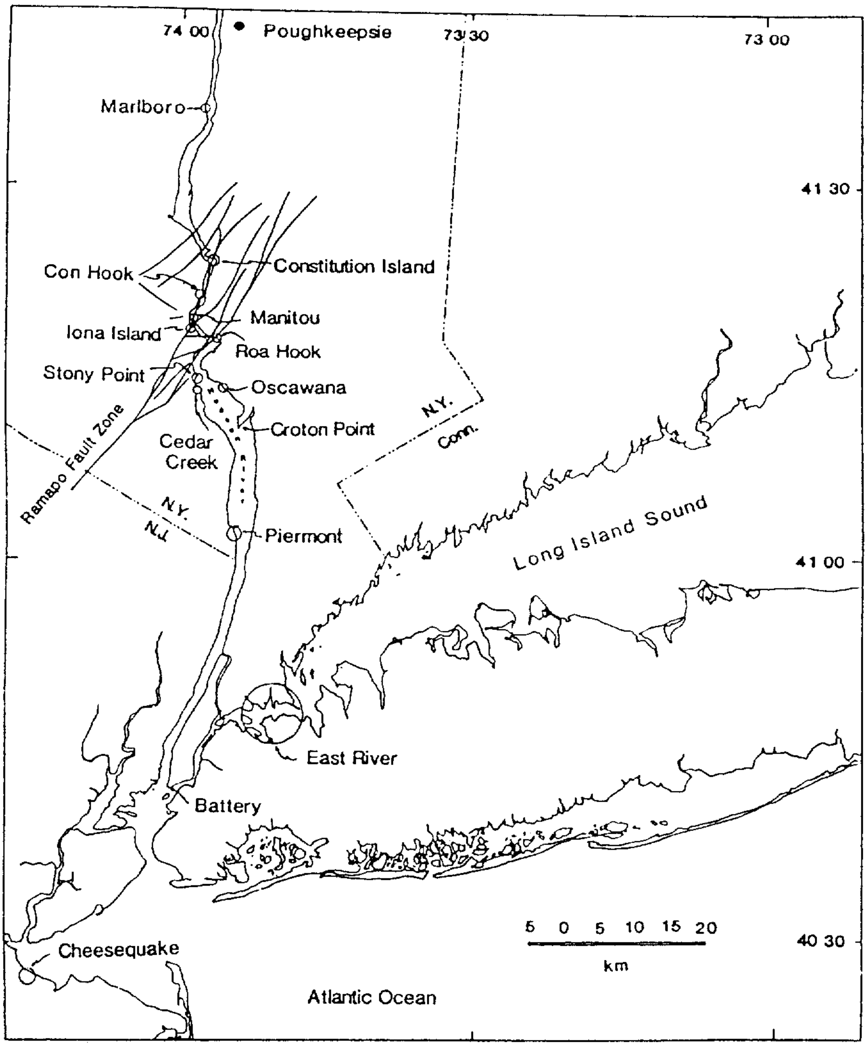

Fig. 1. Hudson River estuary and surrounding region. Open circles indicate tidal marsh sampling stations. Solid lines follow some of the fault traces associated with the Ramapo Fault Zone (after Newman et al., 1987).

is the sea-level data from the two other locations above (the Hudson River estuary and the Delaware coast) were also chosen at only peat organic deposits. Figure 5 is sample sites at the Korean coasts. Figure 6 is the graph of the Korean Quaternary sea-level versus the age before present.

Fractal Dimensions of Sea-Level Data

Brown's grid method of Eq. (1) was used for the determination of fractal dimensions of the experimental data because the distributions of sea-levels were assumed to be stochastic fractals. The fractal dimension of the sea-level data of the Hudson River estuary is 1.358, and that of the Delaware coast is 1.346. The absolute values of the Hudson paleosea-levels are different from those of the Delaware paleosea-levels. However, their fractal dimensions are almost the same. It means the fractal model is a good assumption for the distributions of the Quaternary sea-level data.

Experiments and Discussions

The number of experimental data was reduced

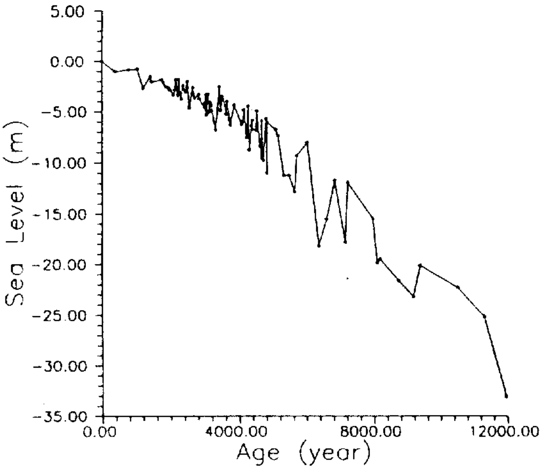

Fig. 2. Paleosea-level data of the Hudson River estuary since 12,000 BP.

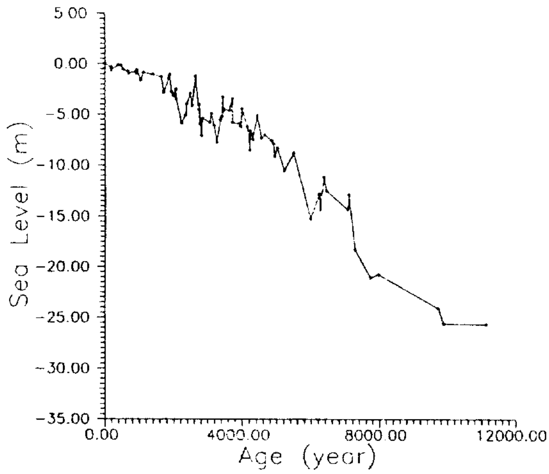

Fig. 4. Paleosea-level data of the Delaware Bay estuary since about 12,000 BP.

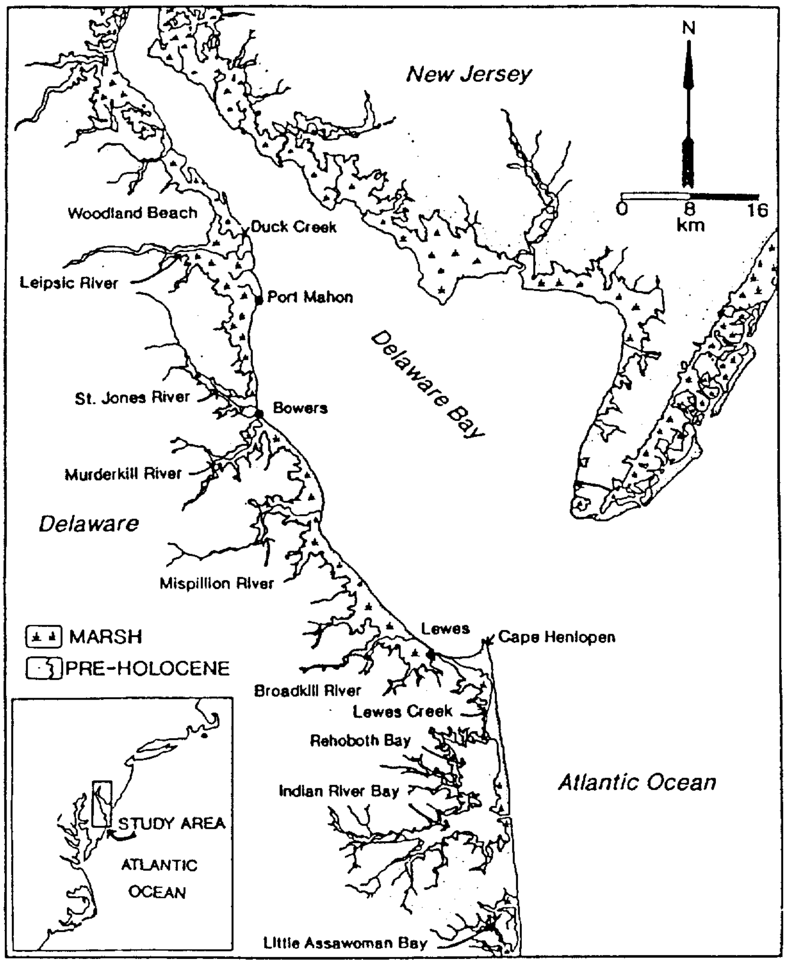

Fig. 3. Delaware Bay estuary and part of Atlantic Ocean coast (after Yi, 1993).

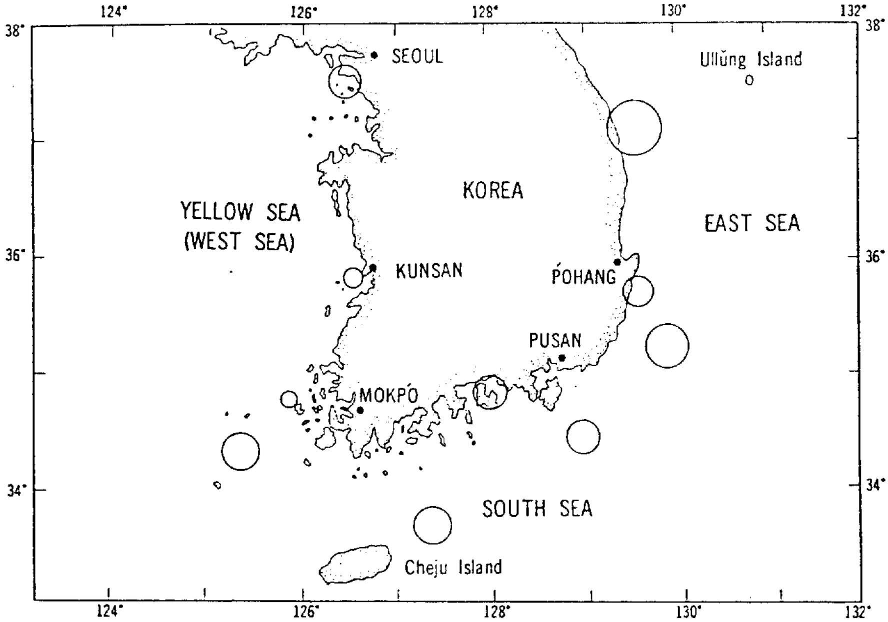

Fig. 5. Korean coastal and offshore sampling sites sea-level study (after Par, 1983).

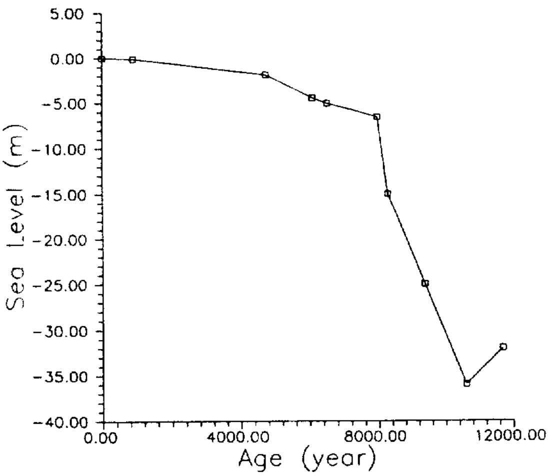

Fig. 6. Paleosea-level data of the Korean coast since about 12,000 BP.

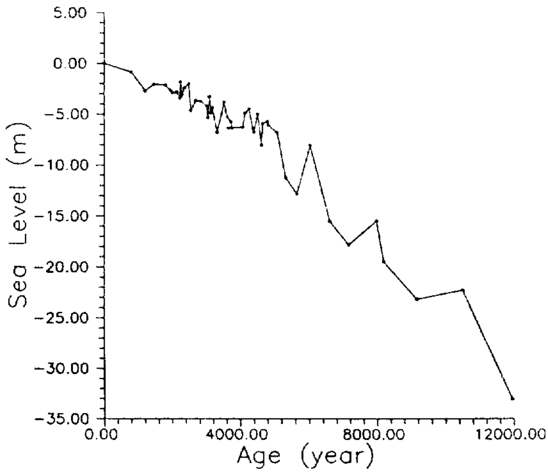

Fig. 7. Distribution of the reduced Hudson sample data for the application of IFIF.

to a half of the whole data to verify the interpolation ability of IFIF, picking up sea-level data at every other point. The number of sample data is 48 for the Hudson River estuary and 39 for the Delaware coast. Figures 7 and 8 are the distributions of the reduced sample data for the Hudson River estuary and the Delaware coast, respectively. IFIF was used to reproduce the missing data between sample data. The spacing of sample data is assumed to be equal. The vertical scaling factors of two sample data sets were calculated by Eq. (11). The vertical scaling factor of the Hudson sample data is 0.084, and that of the Delaware sample data is 0.093. Some statistical errors were used to evaluate the accuracy of the data reproduced by IFIF:

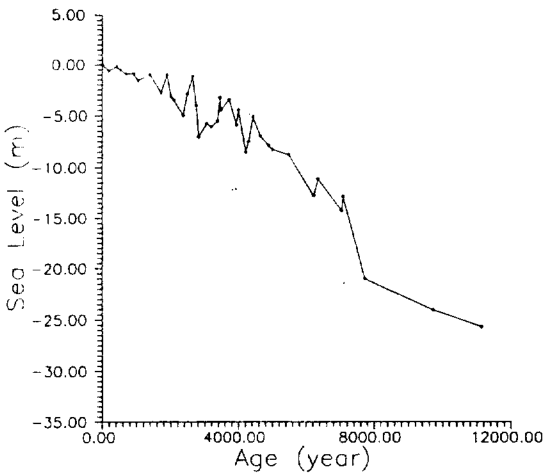

Fig. 8. Distribution of the reduced Delaware sample data for the application of IFIF.

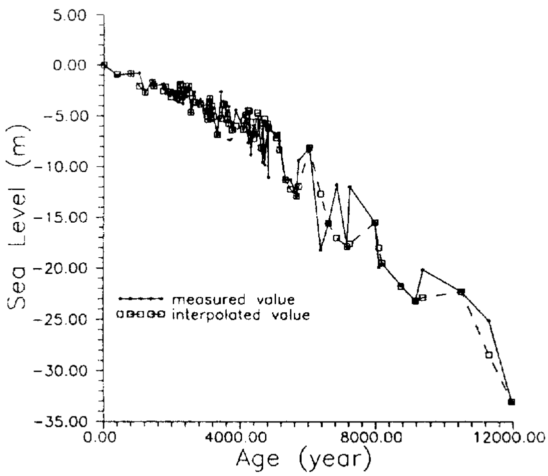

Fig. 9. Interpolation result of the Hudson sample data by IFIF.

Table. 1. Statistical errors resulting from the sea-level data interpolated by IFF

Sample	ME	MSE	SRMSE	VE	SD
Hudson River	−0.03	2.46	1.57	2.48	1.58
Delaware coast	0.25	0.90	0.95	0.85	0.92

Error $(E)=Z(x)_i-Z^*(x)_i$

Mean Error $(ME)=\frac{1}{N}\sum_{i=1}^{N}[Z(x)-Z^*(x)]_i$ (12)

Mean Square Error (MSE)

$$=\frac{1}{N}\sum_{i=1}^{N}[Z(x)-Z^*(x)]_i^2 \quad (13)$$

Square Root Mean Square Error (SRMSE)

$$=\sqrt{MSE} \quad (14)$$

Variance of Error $(VE)=\frac{1}{N-1}\sum_{i=1}^{N}(E_i-ME)^2$ (15)

Standard Deviation $(SD)=\sqrt{VE}$ (16)

where

$Z(x)$ is true sample data

$Z^*(x)$ is interpolated data by IFIF

N is the number of data

Table 1 shows the result of the statistical verification of the interpolation data obtained from the sample data of the Hudson River estuary and the Delaware coast, respectively. Statistical errors of two samples are not large. Mean error is not useful for the statistical cross-validation of two samples. The SRMSE and SD of the Delaware sample data are much less than those of the Hudson sample data, suggesting that the sea-level data of the Delaware coast are more reliable than those of the Hudson River estuary.

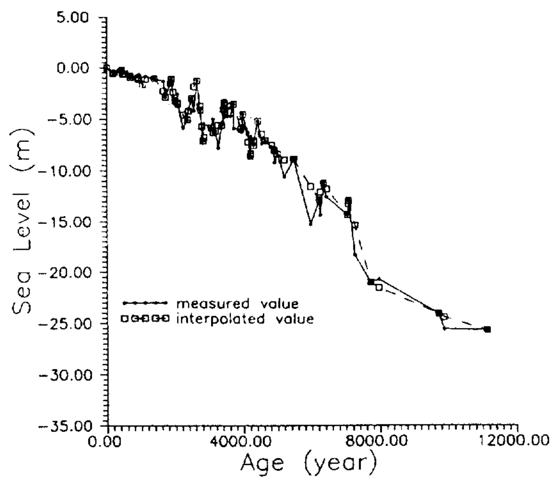

Fig. 10. Interpolation result of the Delaware sample data by IFIF.

Figure 9 is the interpolation result of the Hudson sample data by IFIF. The interpolation values are in good accordance to the measured values from the present to 6,000 BP. However, the inter-

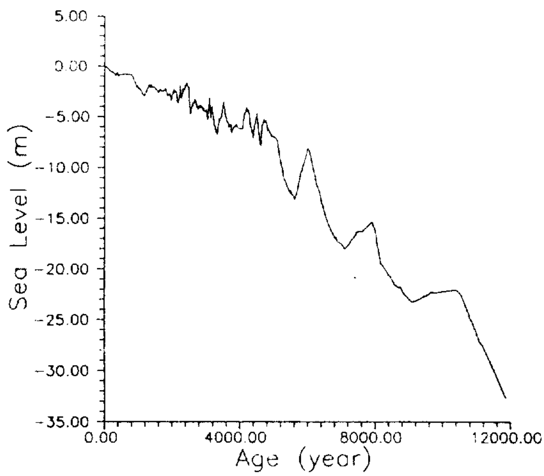

Fig. 11. Estimated sea-level curve of the Hudson River estuary using FIF with 2,000 iterations.

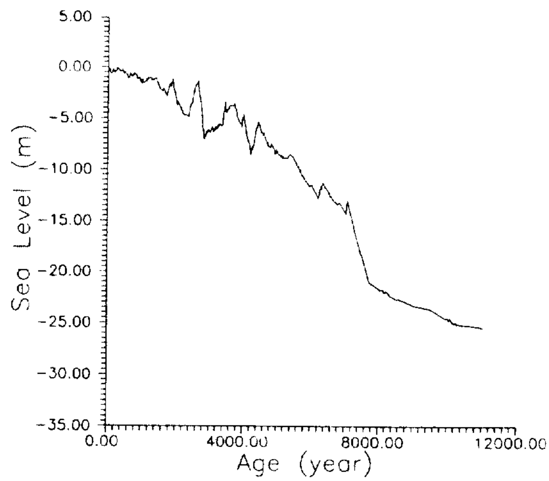

Fig. 12. Estimated sea-level curve of the Delaware Bay estuary using FIF with 2,000 iterations.

polation values older than 6,000 BP deviate largely from the measured values. There are two reasons for the deviation. One is that the number of sample data older than 6,000 BP is much less than that of from the present to 6,000 BP. The other is the large fluctuation of sample data older than 6,000 BP. Figure 10 is the interpolation result of the Delaware sample data by IFIF. The interpolation values are in good accordance to the measured values from the present to 5,000 BP. However, the interpolation values older than 5,000 BP are more deviated from the measured values than those from the present to 5,000 BP. This deviation has the same reason as the deviation of the Hudson sample data. The more sample data we make, the more accurate interpolation values can be produced. When the Delaware interpolation data older than 5,000 BP are compared to the Hudson interpolation data older than 6,000 BP, they have much less deviations from the measured data. It can be said that the Delaware sea-level data after 5,000 BP are more reliable than the Hudson sea-level data after 6,000 BP. Therefore, the larger statistical errors of the Hudson sea-level data than the Delaware sea-level data are resulted from the Hudson sea-level data older than 6,000 BP.

Figures 11 and 12 show the estimated sea-level curves of the Hudson River estuary and the Delaware coast respectively, using FIF with 2,000 iterations. The estimated sea-level curve of the Hudson River estuary younger than 5,000 BP is similar to that of the Delaware coast. However, the estimated sea-level curve of the Hudson River estuary older than 5,000 BP has the larger amplitudes of sea-level change than that of the Delaware coast older than 5,000 BP. This is due to the larger fluctuations of the Hudson sample data, than the Delaware sample data after 5,000 BP.

INTERPOLATION OF THE HOLOCENE SEA-LEVELS OF KOREAN COASTS

The ^{14}C age dating of peats and shells along the Korean coasts was collected by Park (1983). As mentioned above, the reliable ^{14}C data obtained from the basal peat deposits were used for the sample data of this experiment. Figure 6 shows Korean paleosea-level already reached the present coast-line about 8,000 years ago. The sea-levels at that time was 6.6 m below the present mean sea-level. On the other hand, the sea-levels of the Hudson River estuary and the Delaware coast arrived at the present coastlines about 4,000 years ago. Hence, the variation of Korean paleosea-level is different from that of paleosea-levels of the Atlantic Ocean coasts of the United States. However, the patterns of their variations are assumed to be the same in this study IFIF was used for the inte-

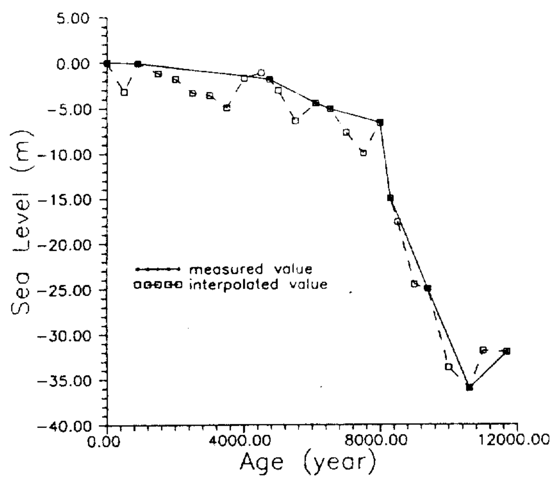

Fig. 13. Interpolation result of the Korean sample data by IFIF.

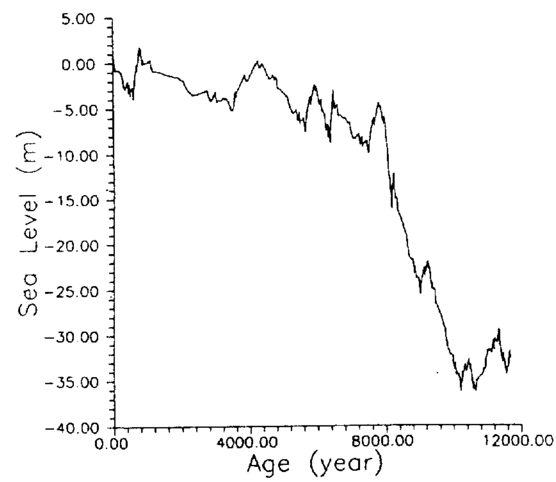

Fig. 14. Estimated sea-level curve of the Korean coasts using FIF with 2,000 iterations.

rpolation of Korean paleosea-levels at the intervals of about 500 years. The vertical scaling factor for the Korean coasts was determined from the fractal dimension (D=1.346) of Delaware sea-level data because the Delaware data were more reliable than the Hudson data. This experiment assumed that the sample data had the equal spacing between sea-level data. The calculated vertical scaling factor was 0.238. Figure 13 is the interpolation result of the Korean sample data using IFIF. The interpolation was made at the intervals of about 500 years because the ^{14}C age dating usually had the error of several hundreds of years. The interpolated data made small fluctuations of paleosea-levels. Figure 14 is an estimated sea-level curve around the Korean coasts, using FIF with 2,000 iterations. It has some abrupt changes and more sharpness than two American estimated sea-level curves. This is due to the small number of the Korean sample data.

CONCLUSIONS

The application of fractal geometry to the Quaternary sea-levels gives the following conclusions:

1. The fractal dimension of the Hudson sea-level data is almost the same as that of the Delaware sea-level data, even though the absolute values of their sea-levels are different.

2. IFIF produced reasonable interpolation data from the Hudson and the Delaware sample data with small statistical errors.

3. The Delaware sea-level data are more reliable than the Hudson sea-level data because the statistical errors of the Delaware interpolation data are much less than those of the Hudson interpolation data.

4. IFIF makes large errors for the Dealware interpolation data older than 5,000 BP and the Hudson interpolation data older than 6,000 BP because the sample data at these age are not enough. However, the Delaware interpolation data older than 5,000 BP have much less deviations from the measured data than the Hudson interpolation data older than 6,000 BP. It can be said that the Delaware sea-level data after 5,000 BP are more reliable than the Hudson sea-level data after 6,000 BP.

5. The change of Korean paleosea-levels is different from that of the eastern U.S. paleosea-levels. The Korean paleosea-levels are much higher than the eastern U.S. paleosea-levels, when two sea-level curves are compared to each other from the present to 8,000 BP.

6. The Korean sea-level curve interpolated by IFIF produced reasonable fluctuations on the basis of the sea-level curves of the Hudson River estuary and the Delaware coast of U.S..

7. FIF produced Korean sea-level curve with

some abrupt changes and more sharpness than two American sea-level curves due to the limited number of sample data.

ACKNOWLEDGEMENT

This research was supported partly by the '93 Circum-Pacific International Symposium on Earth Environment organized by National Fisheries University of Pusan. We thank both reviewers, Dr. Y.A. Park at Seoul National University and Dr. S.J. Han at KORDI (Korea Ocean Research and Development Institute) for giving constructive comments on this paper.

REFERENCES

Barnsley, M. F., 1986, Fractal Functions and Interpolation, Constructive Approximation, **2**: 303-329.

Barnsley, M. F., 1988, Fractals Everywhere, Academic Press, Inc., 394p.

Belknap, D. F., 1975, Dating of Late Pleistocene and Holocene Relative Sea Levels in Coastal Delaware: Unpublished M.S. Thesis, Department of Geology, University of Delaware, Newark, Delaware, 95p.

Brown, S. R., 1987, A Note on the Description of Surface Roughness using Fractal Dimension, Geophysical Research Letters, **14**: 1095-1098.

Brown, S. R., 1988, Derivation of Box Method - Self Affine Fractal, personal communication with Stephen W. Wheatcraft, 7p.

Chen, J. D. and Wilkinson, 1985, Pore-Scale Viscous Fingering in Porous Media, Phys. Rev. Lett., **55**: 18 92-1895.

Chung, S. Y., 1992, Development of Inverse Fractal Interpolation Functions for Modeling Hydraulic Conductivity Distributions, University of Nevada-Reno, Ph. D. Dissertation, 207p.

Fletcher, C. H., J. E., Van Pelt and G. Bruch, in press, Tidal Wetlands Record of Holocene Sea-level Movements and Climate History: Paleogeography, Paleoclimatology, Paleoecolgy.

Kraft, J. C., 1976, Radiocarbon Dates in the Delaware Coastal Zone (Eastern Atlantic Coast of North America): University of Delaware Sea Grant Program Project R/G-12, **2**: 293-314.

Lovejoy, S. and D. Schertzer, 1990, Multifractals, Universality Classes and Sattellite and Rain Fields, Jour. of Geophysical Research, **95**: 2021-2034.

Malinverno, A., 1990, A Simple Method to Estimate the Fractal Dimension of A self-Affine Series, Geophysical Research Letters, **17**: 1953-1956.

Maløy, K. J., J. Feder and T. Jossang, 1985, Viscous Fingering Fractals in Porous Media, Phys. Rev. Lett., **55**: 2688-2691.

Mandelbrot, B. B., 1967, How Long is the Coast of Britain ? Statistical Self-Simility and Fractal Dimension, Science, **156**: 636-638.

Mandelbrot, B. B., 1985, Self-Affine Fractals and Fractal Diemension, Physica Scripta, v. 32.

Mareschal, J., 1989, Fractal Reconstruction of Sea-Floor Topography, Pure Applied Geophysics, 131(1/2), Fractals in Geophysics edited by C. H. Scholtz and B. B. Mandelbrot, Birkhauser Verlag, 197-210.

Newman, W. S., L. J. Cinquemani, J. A. Sperling, L. F. Marcus, and R. R. Pardi, 1987, Holocene Neotectonics and the Ramapo Fault Zone Sea-level Anomaly: A Study of Varying Marine Transgression Rates in the Lower Hudson Estuary, New York and New Jersey, in Sea-Level Fluctuation and Coastal Evolution, SEPM Spec. Publ. No. 41, Nummedal, D., O. H. Pilkey and J. D. Howard eds. 97-111.

Park, Y. A. 1983, Late Quaternary sedimentation on the continental shelf off the southeast coasct of Korea, RIBS-ED-82-507, p. 163-186.

Robert, A. and A. G. Roy, 1990, On the fractal Interpretation of the Mainstream Length-Drainage Area Relationship, Water Resources Research, **26**: 839-842.

Rosso, R. and B. Bacci, 1991, Fractal Relation of Mainstream Length to Catchment Area in River Networks, Water Resources Research, **27**: 381-387.

Sreenivasan, K. R., R. R. Prasad, C. Meneveau, and R. Ramshankar, 1989, The Fractal Geometry of Interfaces and the Multifractal Distribution of Dissipation in Fully Turbulent Flows, reprinted from PAGEOPH, 131(1/2), Fractals in Geophysics edited by C. H. Scholz, and B. B. Mandelbrot, 313p.

Voss, R. F., 1985, Random Fractal Forgeries, in Fundamental Algorithms for Computer Graphics, edited by R. A. Earnshaw, 805-835, Springer-Verlag, New York.

Wheatcraft, S. W. and S. W. Tyler, 1988, An Explanation of Scale-Sependent Dispersivity in Heterogeneous Auqifers using Concepts of Fractal Geometry, Water Resources Research, **24**: 566-578.

Wong, P. Z. and J. S. Lin, 1988, Studying Fractal Geometry on Submicron Length Scale by Small-Angle Scattering, Mathematical Geology, **20**: 655-665/

Yi, H. L., 1993, Stratigraphy, Microfacies Analysis, and Paleoenvironment Evoloution of the Wetlands of Delaware Bay Estuarine Coast and Atlantic Ocean Coast of Delaward, Ph.D. Dissertation, University of Delaware, Delaware, U.S.A., 561p.

Accepted July 6, 1994