論文 94-19-6-12

Optimal Grayscale Morphological Filters Under the LMS Criterion

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LMS 알고리즘을 이용한 형태학 필터의 최적화 방안에 관한 연구

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ABSTRACT

This paper presents a method for determining optimal grayscale function processing(FP) morphological filters under the least square (LMS) error criterion. The optimal erosion and dilation filters with a grayscale structuring element(GSE) are determined by minimizing the mean square error (MSE) between the desired signal and the filter output. It is shown that convergence of the erosion and dilation filters can be achieved by a proper choice of the step size parameter of the LMS algorithm.

In an attempt to determine optimal closing and opening filters, a matrix representation of both opening and closing with a *basis matrix* is proposed. With this representation, opening and closing are accomplished by a local matrix operation rather than cascade operations. The LMS and back-propagation algorithms are utilized for obtaining the optimal basis matrix for closing and opening. Some results of optimal morphological filters applied to 2-D images are presented.

要 約

본 논문에서는 최소자승오차법(LMSE, least mean square error)을 이용하여 function processing (FP) 형태학 필터를 최적화하는 알고리즘을 제안한다. Erosion이나 dilation 연산들은 원하는 신호와 실제 필터의 출력 신호 사이의 평균자승오차(MSE, mean square error)를 최소화하는 농담(濃淡) 구조요소(GSE, grayscale structuring element)를 결정함으로써 최적화 된다. 본 논문에서는 LMS 트레이닝 알고리즘을 형태학 필터의 최적화에 적용하기 위하여 스텝 사이즈 매개변수 η가 만족해야 하는 조건을 보이고, 이를 이용하여 erosion이나 dilation 형태학 필터들의 최적 GSE를 결정할 수 있음을 보였다.

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接受日字 : 1994年 1月 15日

또한, 본 논문에서는 행렬을 사용하여 복합 형상학 연산들을 소역 행열 연산자로 새롭게 정의하고, 이 소역 연산들에 LMS 알고리즘과 back-propagation 알고리즘을 적용하여 복합 형태학 필터들을 최적화할 수 있음을 보였다. 실험 부분에서는 제안된 최적 형상학 필터들을 2-D 영상에 적용한 결과를 보였다.

I. INTRODUCTION

Mathematical morphology involves the study of the different ways in which a structuring element interacts with a given set, modifies its shape, and extracts the resultant set [1]-[3]. The basic operations are erosion and dilation. Based on these operations, closing and opening are defined. The morphological operations have been successfully used in many applications including object recognition, image enhancement, texture analysis, and industrial inspection [2], [3].

In mathematical morphology, several techniques have been used for finding optimal grayscale morphological filters. Some techniques use neural networks and fuzzy systems in which synaptic weights are represented by a structuring element and trained by a variety of neural learning algorithms [4]-[7]. Recently, Dougherty [7] introduced the derivation of the fundamental set for morphological filters, this set being a minimal family of structuring elements from which it is always possible to select the erosions and dilations comprising an optimal filter. The advantage of this scheme is that *a priori* information concerning the multivariate distribution of the filter input signal is not required. However, for morphological filters of size greater than three pixels, the fundamental set becomes a big search space without clear means to find an optimal morphological filter.

The LMS algorithm has been widely used in linear and nonlinear adaptive filtering due to its capability to handle an unknown joint distribution function. Recently, the microstatistic [8] and neural filters [9] have successfully exploited the LMS algorithm. However, the LMS algorithm has never been applied to grayscale morphology. This paper presents a method for determining optimal grayscale FP morphological filters with any size of GSE under the LMS error criterion [10], [11]. The optimal erosion and dilation filters with a grayscale structuring element are determined by minimizing the MSE between the desired signal and filter output. Convergence properties of this scheme are studied by utilizing a method in [12]. It is shown that convergence of the erosion and dilation filters can be reached if the step size parameter η of the LMS algorithm is restricted to $0<\eta<1$.

The second major part of this paper is concerned with the optimality of compound filters suchas opening and closing. In an attempt to determine the optimal compound filters, we propose a matrix representation of these filters using a basis matrix by extending the *basis function* theorem [13], [14]. With this proposed representation, the closing and opening operations are accomplished by a local matrix operation rather than cascade operations. Furthermore, the analysis of the basis matrix shows that the basis matrix is skew symmetric, permitting to derive a simpler matrix representation for compound morphological operators [15]. The LMS and back-propagation algorithms are utilized for obtaining the optimal basis matrix. Each entry of the basis matrix is found through a back propagation algorithm. It is shown that convergence of these compound operations is reached in a few iterations. Some results of optimal morphological filters applied to 2-D images are presented.

Section II, some basic morphological definitions are presented and the optimal dilation and erosion filters are determined by using the LMS algorithm. Convergence properties are also investigated. In Section III, the optimal opening and closing filters are obtained. In section IV, experimental results are presented. Conclusions are given in section V.

II. OPTIMAL EROSION AND DILATION FILTERS

Before stating the optimal filtering problem over the class of the grayscale morphological filters, we review the grayscale erosion and dilation operations and introduce some common terminolgy. Let the GSE with size N be denoted by $\mathbf{k}=\{k(0), k(1),\cdots,k(N-1)\}$ and a set of samples covered by $\mathbf{k}$ at time n be denoted by $\mathbf{f}(n)$. The output $g(n)$ of a grayscale morphological operation is given by

$$g(n)=\theta[\mathbf{f}(n)], \tag{1}$$

where θ is a Min/Max operator representing either a single operation (Min or Max) or a combined operation of Min and Max (Min of Maxima or Max of Minima). For instance, the output of the grayscale dilation is given by

$$g_d(n)=\mathbf{f}(n)\oplus\mathbf{k}=\max[f(n)+k(0), f(n-1)+k(1),\cdots,f(n-N+1)+k(N-1)], \tag{2a}$$

and the output of the grayscale erosion is given by

$$g_e(n)=\mathbf{f}(n)\ominus\mathbf{k}=\min[f(n)-k(0), f(n+1)-k(1),\cdots,f(n+N-1)-k(N-1)], \tag{2b}$$

In mathematical morphology, erosion and dilation are the most fundamental and important operations. This is so because all the compound morphological operations can be implemented as a union/an intersection of erosions and dilations. Next we present a method based on the LMS algorithm for determining the optimal GSE for erosion and dilation.

1. LMS Algorithm for Optimizing Erosion and Dilation

The MSE is given by

$$MSE=E\{d(n)-\theta[\mathbf{f}(n)]\}^2=E[\xi^2(n)], \tag{3}$$

where $E\{\cdot\}$ is the expectation operator, $d(n)$ stands for the desired signal (target value) at time n, and $\xi(n)$ is the difference between the desired signal and the output value. The optimal filtering problem under the LMS error criterion usually requires the knowledge of the input correlation matrix. On the other hand, the LMS algorithm [10], [11] which has been applied in adaptive signal processing, does not not require the input correlation matrix. With the LMS algorithm, optimal morphological filter can be obtained by finding the optimal GSE minimizing the MSE.

The LMS algorithm is based on a learning rule as follows : The updated value of GSE $\mathbf{k}$ at the $(p+1)th$ iteration is given by

$$\mathbf{k}_{p+1}=\mathbf{k}_p-\eta\Delta\mathbf{k}_p. \tag{4}$$

Here the increment $\Delta\mathbf{k}_p$ is obtained by estimating the gradient of the MSE between the desired and actual output signal. The parameter η in Eq. (4) is related to the covergence of the steepest-decent algorithm [10], [11]. At each iteration, we have a gradient estimate given by

$$\Delta\mathbf{k}_p=\begin{pmatrix}\dfrac{\partial E[\xi^2(n)]}{\partial k(0)}\\ \dfrac{\partial E[\xi^2(n)]}{\partial k(1)}\\ \vdots \\ \dfrac{\partial E[\xi^2(n)]}{\partial k(N-1)}\end{pmatrix}, \tag{5}$$

where $E[\cdot]$ is the expected value and the $(m+1)$ th element of this vector is

$$\frac{\partial E[\xi^2(n)]}{\partial k(m)}=E\left[\frac{\partial\xi^2(n)}{\partial k(m)}\right]=2\cdot E\left[\xi(n)\frac{\partial\xi(n)}{\partial k(m)}\right]. \tag{6}$$

With this definition, the derivative $\dfrac{\partial\xi(n)}{\partial k(m)}$ is given by

$$\frac{\partial\xi(n)}{\partial k(m)}=-\frac{\partial\theta[\mathbf{f}(n)]}{\partial k(m)}, \tag{7}$$

For dilation, this derivative can be obtained using the following formula

$$\frac{\partial\theta[\mathbf{f}(n)]}{\partial k(m)} = \begin{cases} 1, & \text{if } g(n)=f(n-m)+k(m), \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

For erosion, the derivative is given by

$$\frac{\partial\theta[\mathbf{f}(n)]}{\partial k(m)} = \begin{cases} -1, & \text{if } g(n)=f(n+m)-k(m), \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Next, we make an attempt to smooth the estimate in Eq.(6) by using an averaged gradient of the MSE. The *mth* element of the gradient vector can be estimated as

$$E\left[\xi(n)\frac{\partial\xi(n)}{\partial k(m)}\right] = \frac{\sum_{j=1}^{L}\frac{\partial\xi(j)}{\partial k(m)}(d(j)-\theta[\mathbf{f}(j)])}{\left|\sum_{j=1}^{L}\frac{\partial\xi(j)}{\partial k(m)}\right|}, \quad (10)$$

where L is the total number of samples in the input signal, and the denominator represents the total number of times that the *mth* element in the input vector becomes the output.

2. Convergence Properties

In applying the LMS algorithm to adaptive linear filtering, it is found that the convergence depends on the ratio $\frac{\lambda_{max}}{\lambda_{min}}$ where λ_{max} and λ_{min}, respectively, are the maximum and minimum eigenvalues of the input correlation matrix **R** [12]. If this ratio is large, *i.e.*, the eigenvaluse of **R** are dispersed, the algorithm converges slowly. In practice, the eigenvalues of **R** are difficult to evaluate. However, in order to show the convergence of the adaptation algorithm for dilation and erosion, we utilize an approach similar to the one in [11], [12] where the knowledge of the matrix **R** is not needed.

Proposition 1: Convergence for dilation and erosion can be achieved if $0<\eta<1$.

Proof: For dilation, the error at iteration $p+1$ can be written as

$$\xi_{p+1}(n) = d(n) - \max_{i=0}^{N-1}\,[f(n-i)+k_{p+1}(i)], \quad (11)$$

where $k_{p+1}(i)$ is the $(i+1)th$ structuring element at iteration p+1. Furthermore,

$$k_{p+1}(i) = k_p(i) - 2\eta\xi_p(n)\frac{\partial\xi_p(n)}{\partial k_p(i)}. \quad (12)$$

Substituting Eq. (12) into Eq. (11) gives

$$\xi_{p+1}(n) = d(n) - \max_{i=0}^{N-1}\,[f(n-i)+k_p(i) - 2\eta\xi_p(n)\frac{\partial\xi_p(n)}{\partial k_p(i)}].$$

Suppose that the $(j+1)th$ element is the maximum in the above max operation. Then

$$\xi_{p+1}(n) = d(n) - [f(n-j)+k_p(j)-2\eta\xi_p(n)\frac{\partial\xi_p(n)}{\partial k_p(j)}].$$

Since from Eq. (7) and Eq. (8),

$$\frac{\partial\xi_p(n)}{\partial k_p(j)} = \begin{cases} -1, & \text{if } f(n-j)+k_p(j)=\theta[\mathbf{f}(n)], \\ 0, & \text{otherwise.} \end{cases}$$

Thus,

$$\xi_{p+1}(n) = d(n) - [f(n-j)+k_p(j)+2\eta\xi_p(n)].$$

But

$$\xi_p(n) = d(n) - [f(n-j)+k_p(j)].$$

Thus,

$$\xi_{p+1}(n) = (1-2\eta)\xi_p(n).$$

At iteration $p+2$,

$$\xi_{p+2}(n) = (1-2\eta)\xi_{p+1}(n) = (1-2\eta)^2\xi_p(n).$$

Thus, at iteration $p+q$,

$$\xi_{p+q}(n) = (1-2\eta)^q\xi_p(n).$$

Taking the expected values from both sides gives

$$E[\xi_{p+q}(n)] = (1-2\eta)^q E[\xi_p(n)].$$

This error will converge to zero at $q=\infty$ only if

$|1-2\eta|<1.$

In other words, for convergence,

$0<\eta<1.$

Similarly, the convergence for erosion can be proved.

It is necessary to examine the effect of the parameter η as η varies within the above condition. In Fig.1, each convergence curves of dilation are visualized with respect to the number of iterations

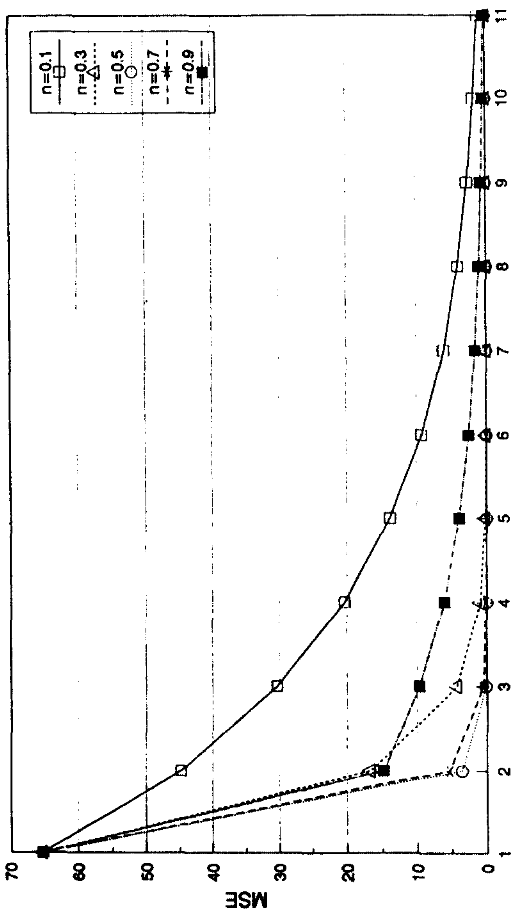


Fig 1. The variations of the MSE convergence curve when the parameter η moves in $0<\eta<1$.

for $\eta=0.1, 0.3, \cdots, 0.9$. In the Fig.1, it is found that the optimization process with $\eta=0.5$ can be terminated(MSE$=0$) at the smallest number of iterations.

Ⅲ. OPTIMAL OPENING AND CLOSING FILTERS

In this section, results for erosion and dilation are extended to the optimization of opening and closing. To find the optimal GSE for opening and closing, we first introduce a matrix representation of both opening and closing with a basis matrix by extending the *basis function* theorem [2], [3], [13]. With this representation, opening and closing are accomplished by a single local matrix operation rather than cascade operations that introduce delays and require additional storage.

1. Basis Representation of Opening and Closing

The next proposition formalizes the basis representation of grayscale opening and closing.

Proposition 2: The grayscale opening of $\mathbf{f}(n)$ by an structuring element $\mathbf{k}$ is given by

$$g_o(n)=\max_{z}\{\min_{z'}[f(n-z+z')+b(z,z')]\},$$

and grayscale closing is given by

$$g_c(n)=\min_{z}\{\max_{z'}[f(n+z-z')-b(z,z')]\},$$

where $b(z, z')$ is called the basis element of grayscale opening (closing) and it is defined as $b(z,z')=k(z)-k(z')$.

Proof: From the definition of grayscale opening,

$$\begin{aligned} g_o(n) &= [(\mathbf{f}\ominus\mathbf{k})\oplus\mathbf{k}](n) \\ &= (g_e\oplus\mathbf{k})(n) \\ &= \max_{z}\{g_e(n-z)+k(z)\} \\ &= \max_{z}\{\min_{z'}[f(n-z+z')-k(z')]+k(z)\}. \end{aligned}$$

Since $\min[a, b]+c=\min[a+c, b+c]$, g_o can be represented as

$$g_o(n)=\max_z\{\min_{z'}[f(n-z+z')+b(z,z')]\}.$$

$g_c(n)$ can be derived similarly.

Using this proposition, opening and closing can be realized by local operations based on sums and differences of f(n) and **k**. For example, consider a GSE $\mathbf{k}=\{k(0), k(1), k(2)\}$. The outputs of opening and closing, respectively, are given by

$$g_o(n)=\max\{\min[f(n), f(n+1)+b(0,1), f(n+2)+b(0,2)],$$
$$\min[f(n-1)+b(1,0), f(n), f(n+1)+b(1,2)],$$
$$\min[f(n-2)+b(2,0), f(n-1)+b(2,1), f(n)]\},$$

and closing

$$g_c(n)=\min\{\max[f(n), f(n-1)-b(0,1), f(n-2)-b(0,2)],$$
$$\max[f(n+1)-b(1,0), f(n), f(n-1)-b(1,2)],$$
$$\max[f(n+2)-b(2,0), f(n+1)-b(2,1), f(n)]\}.$$

The grayscale opening and closing of $\mathbf{f}(n)$ by **k** of size N can be expressed in a compact form using matrix notation : Let the $N\times N$ input matrix be denoted by

$$F(n)=\begin{pmatrix} f(n) & f(n+1) & \cdots & f(n+N-1) \\ f(n-1) & f(n) & \cdots & f(n+N-2) \\ \vdots & \vdots & \ddots & \vdots \\ f(n-N+1) & f(n-N+2) & \cdots & f(n) \end{pmatrix}. \tag{13}$$

Let the $N\times N$ basis matrix B, whose elements consist of $\{b(i,j)\}$, be given by

$$B=\begin{pmatrix} b(0,0) & b(0,1) & \cdots & b(0,N-1) \\ b(1,0) & b(1,1) & \cdots & b(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ b(N-1,0) & b(N-1,1) & \cdots & b(N-1,N-1) \end{pmatrix}. \tag{14}$$

It is interesting to observe some proporties of the basis matrix. First, each row represents a basis function defined by [2], [3], [13]. Second, since $b(i,j)=k(i)-k(j)$, $b(i,i)=0$, and $b(i,j)=-b(j,i)$. Thus, the basis matrix can be written as :

$$B=\begin{pmatrix} 0 & b(0,1) & \cdots & b(0,N-1) \\ -b(0,1) & 0 & \cdots & b(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ -b(0,N-1) & -b(1,N-1) & \cdots & 0 \end{pmatrix}. \tag{15}$$

Note that the basis matrix is skew-symmetric, that is, $B^T=-B$, where B^t is the transpose of B. This matrix also implies that the total number of unknown weights can be reduced from N^2 down to $(N^2-N)/2$.

The output of opening (closing) at time n is obtained according to the following steps : Add matrices $F(n)$ in Eq. (13) and B in Eq. (15), and at each row (column) of the resultant matrix, find the minimum (maximum) of the row(column) elements. The maximum (minimum) of the minima (maxima) represents the output of opening(closing) at n.

This implementation using local operators for opening and closing can lead to two layer neural network representations as shown in Fig.2. In these representations, each basis element in Eq. (15) is adopted as a synaptic weight to be added to each node input, and the output of each node is determined by the minimum or maximum operation rather than the summation operation used in general neural networks. The minimum and maximum operators are symbolized as • and ∘ in Fig.2.

In Eq.(15), it is interesting to note that among the only $(N^2-N)/2$ unknown entries in the basis matrix, the $N-1$ entries are linearly independent with the remaining ones represented as linear combination of the others. The next proposition formalizes this observation.

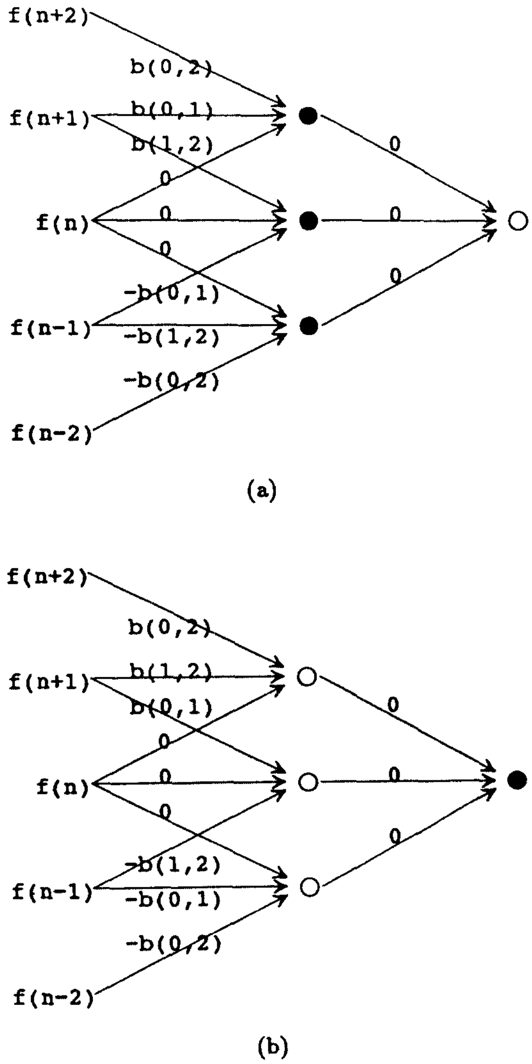


Fig 2. The two-layer neural network representation of the morphological operations (size of GSE $N=3$) : (a) opening, (b) closing.

Proposition 3: The basis matrix contains only $N-1$ linearly independent entries.

Proof : Without loss of generality, it can be proved for $N=3$. Since $b(i,j)=k(i)-k(j)$, $b(0,2)=b(0,1)+b(1,2)$ and $b(0,2)=-b(2,0)$. For $N=3$, there are only 2 unknown basis elements, $b(0,1)$ and $b(1,2)$, since b(0, 2) is represented as linear combination of the former two. It can be readily proved by induction for any GSE with $N>3$.

Proposition 3 raises an interesting question. For 2 basis elements obtained from a 3-point GSE, how do we obtain the original structuring element? To obtain this, we may put an extra constraint in the original values of the GSE. For example, $k(0)+k(1)+k(2)=1$. However, it is felt that this problem is not significant because the knowledge of the basis matrix is sufficient to perform opening or closing.

2. LMS Algorithm for Optimizing Opening and Closing

In the previous subsection, it is shown that opening and closing can have a structure similar to a two-layer neural network in which synaptic weights are represented by basis elements. To train the morphological neural network, we utilize a learning rule based on the LMS algorithm. Each entry of the optimal basis matrix is found through the back propagation algorithm.

The updated basis matrix B at iteration $p+1$ is given by

$$B_{p+1}=B_p-\eta\Delta B_p. \tag{16}$$

The increment $\Delta \mathbf{B}_p$ is obtained by estimating the gradient of the MSE. At each iteration, we have a gradient estimate given by

$$\Delta B_p=\begin{pmatrix} 0 & \Delta b(0,1) & \cdots & \Delta b(0,N-1) \\ \Delta b(1,0) & 0 & \cdots & \Delta b(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ \Delta b(N-1,0) & \Delta b(N-1,1) & \cdots & 0 \end{pmatrix}, \tag{17}$$

where

$$\Delta b(i,j)=\frac{\partial E[\xi^2(n)]}{\partial b(i,j)}=2\cdot E\left[\xi(n)\frac{\partial \xi(n)}{\partial b(i,j)}\right] \tag{18}$$

From Eq. (3), the derivative $\frac{\partial \xi(n)}{\partial b(i,j)}$ is given by

$$\frac{\partial \xi(n)}{\partial b(i,j)} = -\frac{\partial \theta[\mathbf{f}(n)]}{\partial b(i,j)} \tag{19}$$

Using Eq.(13) and Eq.(15), the opening operation is expressed as the row-wise basis expansion as follows

$$\theta[\mathbf{f}(n)] = \max(\alpha_0, \alpha_1, \cdots, \alpha_{N-1}), \tag{20}$$

where

$$\alpha_0 = \min[f(n), f(n+1)+b(0,1), \cdots, f(n+N) + b(0, N-1)],$$
$$\alpha_1 = \min[f(n-1)-b(0,1), f(n), \cdots, f(n+N-2) + b(1, N-1)],$$
$$\vdots$$
$$\alpha_{N-1} = \min[f(n-N+1)-b(0,N-1), f(n-N+2) - b(1, N-1), \cdots, f(n)].$$

Therefore, $\frac{\partial \theta[\mathbf{f}(n)]}{\partial b(i,j)}$ can be obtained by applying the chain rule similar to the back propagation algorithm

$$\frac{\partial \theta[\mathbf{f}(n)]}{\partial b(i,j)} = \frac{\partial \theta[\mathbf{f}(n)]}{\partial \alpha_u} \frac{\partial \alpha_u}{\partial b(i,j)}, \tag{21}$$

where

$$\frac{\partial \theta[\mathbf{f}(n)]}{\partial \alpha_u} = \begin{cases} 1, & \text{if } \alpha_u = \theta[\mathbf{f}(n)], \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

and for any m, $n-N+1 \le m \le n+N-1$,

$$\frac{\partial \alpha_u}{\partial b(i,j)} = \begin{cases} 1, & \text{if } \alpha_u = f(m)+b(i,j), \\ -1, & \text{if } \alpha_u = f(m)-b(i,j), \\ 0, & \text{otherwise,} \end{cases} \tag{23}$$

Next, we make an attempt to smooth the estimate in Eq.(18) by using an averaged gradient of the MSE. The gradient vector can be estimated as

$$E\left[\xi(n)\frac{\partial \xi(n)}{\partial b(i,j)}\right] = \frac{\sum_{l=1}^{L} \frac{\partial \xi(l)}{\partial b(i,j)} (d(l) - \theta[\mathbf{f}(l)])}{\left|\sum_{l=1}^{L} \frac{\partial \xi(l)}{\partial b(i,j)}\right|}, \tag{24}$$

where the denominator of Eq.(24) represents the total number of times that the $(i, j)th$ entry of the basis matrix contributes to the output.

Ⅳ. EXPERIMENTAL RESULTS

In this section, the optimization of the grayscale morphological filters is experimentally demonstrated. Target images were generated by processing an original image by each morphological filters having an arbitrary GSE. The original image has been used as an input image to each morphological operators. Each filters have been trained by the LMS algorithm described in previous sections. In this experiment, all the elements of the GSE was initially set to zero. The MSE for the training process and the values of the GSE obtained in sucessive training steps were calculated.

1. Erosion and Dilation

The original image is shown in Fig. 3. A GSE was selected as $\mathbf{k} = [15, 8, 5]$, and erosion and dilation with this GSE were applied to the original image to obtain target images. The target images for erosion and dilation are shown in Fig.4(a) and Fig.4(b). The training was performed while the step size η was set to 0.5. For erosion and dilation, the final value for the the MSE is equal to zero with convergence reached in 5 iterations in both cases. The final value for the GSE is $\mathbf{k} = [15, 8, 5]$ which is equal to the original GSE. The values of the GSE obtained in successive training steps for erosion and dilation are visualized in Fig. 6(a) and Fig. 6(b). The MSE convergence curves are shown in Fig. 7(a) and Fig. 7(b).

2. Opening and Closing

For GSE $\mathbf{k} = [15, 8, 5]$, the basis matrix for opening and closing is obtained by the method described in section III, and is given by

$$\mathbf{B} = \begin{pmatrix} 0 & 7 & 10 \\ -7 & 0 & 3 \\ -10 & -3 & 0 \end{pmatrix}$$

Fig 3. The original image (128 × 128).

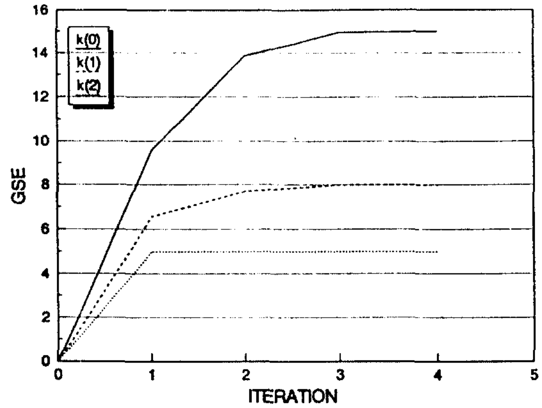
(a) (b)

Fig 4. The target image for erosion and dilation : (a) erosion, (b) dilation. (**k** = [15, 8, 5] for both cases)

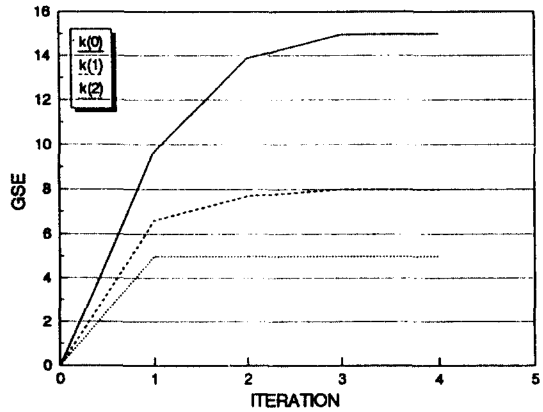
(a) (b)

Fig 5. The target image for opening and closing : (a) opening, (b) closing. (**k** = [15, 8, 5] for both cases)

Opening and closing with the above basis matrix were applied to the original image to obtain target images. The target images for opening and closing are shown in Fig. 5(a) and Fig. 5(b). The updating procedure was applied using the original and target images for the optimization of opening and closing. In this experiment, the step size η was also set to 0.5. The final value for the MSE



(a)

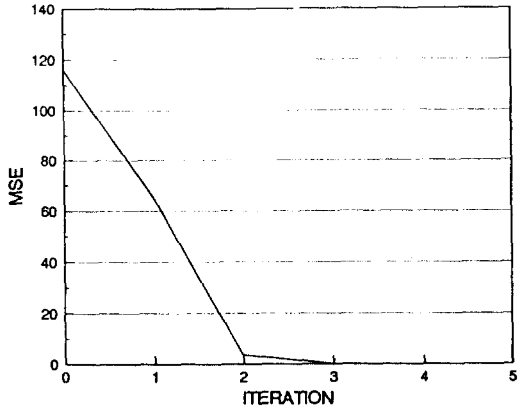


(b)

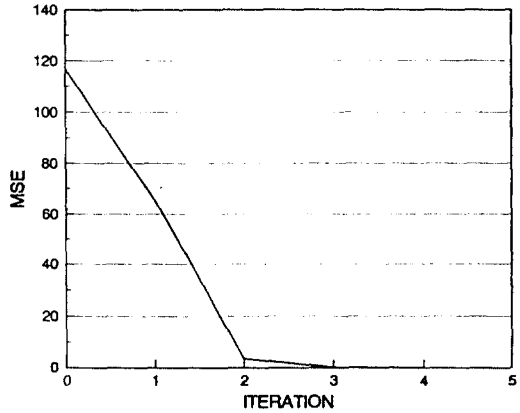
Fig 6. The convergence curve of the GSE : (a) erosion, (b) dilation.

is equal to zero with convergence reached in 9 iterations for opening and in 10 iterations for closing. For both cases, the original basis matrices were obtained after the final iteration.

Since the 3 × 3 basis matrix consists of only two linealy independent elements $b(0, 1)$ and $b(1, 2)$ from Proposition 3, the convergence of these elements was examined. The results for opening and closing are shown in Fig. 8(a) and Fig. 8(b). The MSE convergence curves are presented in Fig. 9(a) and Fig. 9(b).

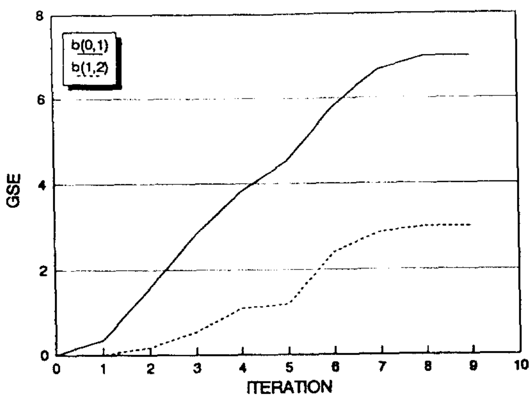


(a)

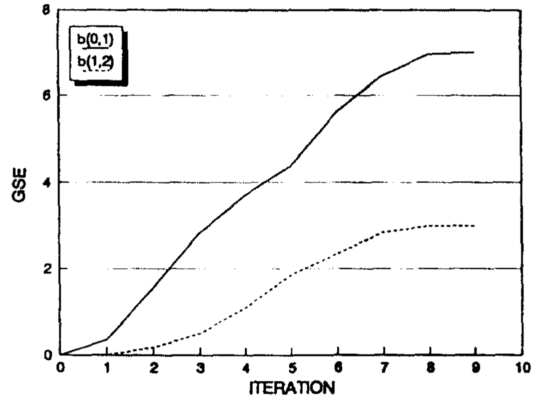


(b)

Fig 7. The MSE convergence curve : (a) erosion, (b) dilation.

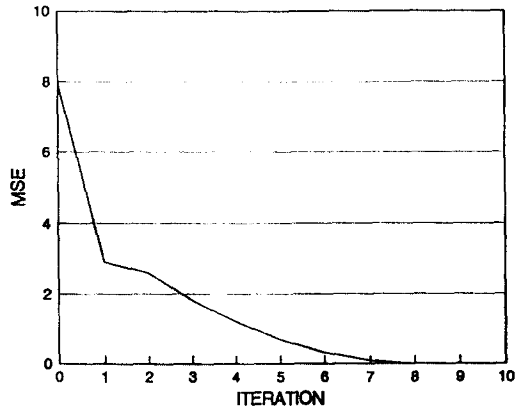


(a)

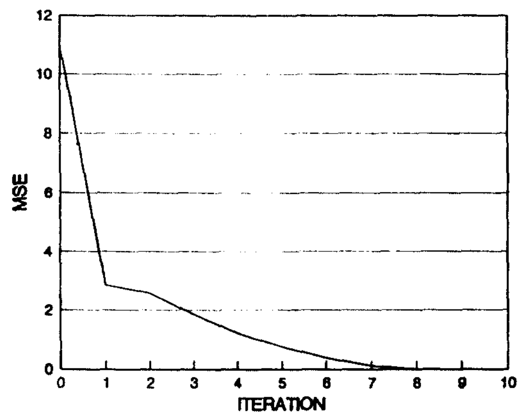


(b)

Fig 8. The convergence curve of linearly independent elements in the basis matrix : (a) opening, (b) closing.



(a)



(b)

Fig 9. The MSE convergence curve : (a) opening, (b) closing.

V. CONCLUSIONS

We presented a method for determining optimal grayscale function processing morphological filters under the LMS error criterion. We showed that convergence of the erosion and dilation filters can be reached if the step size parameter η of the LMS algorithm is restricted to $0 < \eta < 1$.

For the opening and closing filters, we proposed the matrix opening and closing representations using the basis matrix and characterized properties of the basis matrix. The LMS and back-propagation algorithms were utilized for obtaining the optimal basis matrix for opening and closing. Experimental results indicated that optimal morphological filters under the LMS error criterion were obtained in a few iterations.

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본 논문은 과학재단 핵심전문 연구비에 의해 연구되었음

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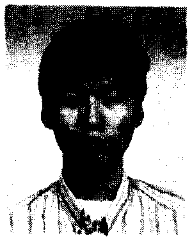

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