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# Estimation for the Exponential ARMA Model[1)]

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## Abstract

The Yule-Walker estimator and the approximate conditional least squares estimator of the parameter of the EARMA(1,1) model are obtained. These two estimators are compared by simulation study. It is shown that the approximate conditional least squares estimator is better in the sense of the mean square error than the Yule-Walker estimator.

## 1. Introduction

Recently there has been a considerable attention in non-Gaussian time series model since Gaussian distribution is not appropriate to model a positive and highly skewed data, e.g., the wind speed, the service time in a queue, or the daily flows of a river.

One class of the models is the class of the exponential time series models characterized by the fact that the marginal distribution of the observations follows an exponential distribution. This class of models was first introduced by Lawrance and Lewis(1977) in the form of the exponential moving average model EMA(1) and the exponential autoregressive model EAR(1). These two models were mixed into the exponential autoregressive moving average model EARMA(1,1) by Jacobs and Lewis(1977).

The class of models was extended to the EAR(p) model and the EMA(q) model which were mixed into the EARMA(p,q) model by Lawrance and Lewis(1980).

Properties of these models have been studied extensively, so that there has been good understanding of the underlying probabilistic mechanism of the models. However, very little works have been done on estimation of the model parameters. This is because the likelihood function of the models has discontinuities which cause difficulties to obtain the the maximum likelihood estimators.

It is only known that Lawrence and Lewis(1980a) do find the Yule-Walker estimators for the parameters of the EAR(p) model and Billard and Mohamed(1991) obtain the conditional least squares estimators for the EAR(p) model.

In this paper, we attempt to estimate the parameters of the EARMA(1,1) model by the

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Yule-Walker estimation method and the conditional least square estimation method. If the moving average parameters are included in the class of the exponential models, there are some difficulties to obtain an unique estimator of the parameters. However, we seek the estimators which satisfy the invertible condition.

Furthermore, we compare the conditional least squares estimators with the Yule-Walker estimators by simulation study.

## 2. The properties of the EARMA(1,1) model

Let $\{X_t\}$ be a stationary sequence of random variables whose marginal distribution is exponential with parameter $\mu$. Then the EAR(p) model is defined by

$$X_t = \left\{ \begin{array}{ll} a_1 X_{t-1} & \text{w.p. } a_1 \\ a_2 X_{t-2} & \text{w.p. } a_2 \\ \vdots & \vdots \\ a_p X_{t-p} & \text{w.p. } a_p \end{array} \right\} + \varepsilon_t, \tag{2.1}$$

where

$$\begin{aligned} a_1 &= 1-\alpha_2 \\ a_i &= \prod_{j=2}^{i} \alpha_j (1-\alpha_{i+1}), \quad i=2,3,\cdots,p-1 \\ a_p &= \prod_{j=2}^{p} \alpha_j \end{aligned} \tag{2.2}$$

for $0<\alpha_j<1$, $j=1,2,\cdots,p$, and where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed (iid) random variables.

Lawrance and Lewis(1980) showed that if the $X_t$ has a marginal exponential distribution, then the $\varepsilon_t$ is a mixture of a discrete component and exponential distributions. For example, if p=1,

$$\varepsilon_t = \left\{ \begin{array}{ll} 0 & \text{w.p. } \alpha \\ E_t & \text{w.p. } (1-\alpha), \end{array} \right. \tag{2.3}$$

where $E_t$ is an exponentially distributed random variable with parameter $\mu$.

The EMA(q) model is defined by

$$X_t = \left\{ \begin{array}{ll} \beta_q E_t & \text{w.p. } b_{q+1} \\ \beta_q E_{t-1} + \beta_{q-1} E_{t-1} & \text{w.p. } b_q \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \beta_q E_t + \beta_{q-1} E_{t-1} + \cdots + \beta_1 E_{t-q+1} & \text{w.p. } b_2 \\ \beta_q E_t + \beta_{q-1} E_{t-1} + \cdots + \beta_1 E_{t-q+1} + E_{t-q} & \text{w.p. } b_1 \end{array} \right. \tag{2.4}$$

for $0 \le \beta_j \le 1$, $j=1,2,\cdots,q$, where

$$b_i = \begin{cases} \beta_q, & i=q+1 \\ (1-\beta_q)\cdots(1-\beta_i)\beta_{i-1}, & 2\le i\le q \\ \prod_{k=0}^{q-1}(1-\beta_{q-k}), & i=1. \end{cases} \tag{2.5}$$

Note that the $\beta_i$'s can be obtained uniquely from $b_i$'s.

It is known that the autocorrelation structures of the EAR(p) model and the EMA(q) model are completely analogous to those of the Gaussian AR(p) model and the MA(q) model, respectively.

Now, to construct the EARMA(p,q) model, we replace the $E_{t-q}$ variable in the EMA(q) model by $A_{t-q}^{(p)}$, the EAR(p) variable.

Then the defining equation for the EARMA(p,q) model is given by

$$X_t = \begin{cases} \beta_q E_t & \text{w.p. } b_{q+1} \\ \beta_q E_{t-1} + \beta_{q-1}E_{t-1} & \text{w.p. } b_q \\ \vdots & \vdots \\ \beta_q E_t + \beta_{q-1}E_{t-1} + \cdots + \beta_1 E_{t-q+1} & \text{w.p. } b_2 \\ \beta_q E_t + \beta_{q-1}E_{t-1} + \cdots + \beta_1 E_{t-q+1} + A_{t-q}^{(p)} & \text{w.p. } b_1, \end{cases} \tag{2.6}$$

where $b_i$'s are defined at (2.5).

Writing $X_t^{(p,q)}$ as a variable in the EARMA(p,q) model based on the moving average parameters $\beta_q, \beta_{q-1}, \cdots, \beta_1$, the EARMA(p,q) model can be defined recursively as

$$X_t^{(p,q)} = \begin{cases} \beta_q E_t & \text{w.p. } \beta_q \\ \beta_q E_t + X_{t-1}^{(p,q-1)} & \text{w.p. } (1-\beta_q). \end{cases} \tag{2.7}$$

Let $\{X_t\}$ be a stationary sequence of the EARMA(1,1) model such that for $t=1,2,\cdots$,

$$X_t = \begin{cases} \beta E_t & \text{w.p. } \beta \\ \beta E_t + A_{t-1} & \text{w.p. } (1-\beta), \end{cases} \tag{2.8}$$

and for $t=0,1,\cdots$,

$$A_t = \begin{cases} \alpha A_{t-1} & \text{w.p. } \alpha \\ \alpha A_{t-1} + E_t & \text{w.p. } (1-\alpha), \end{cases} \tag{2.9}$$

where $0\le\beta\le1$, $0\le\alpha\le1$, and $E_t$ is an exponentially iid random variable with parameter $\mu$. Then, substituting $A_{t-1}$ in (2.8) by $A_{t-1}$ in (2.9) gives

$$X_t = \begin{cases} \beta E_t & \text{w.p. } \beta \\ \beta E_t + \alpha A_{t-2} & \text{w.p. } \alpha(1-\beta) \\ \beta E_t + \alpha A_{t-2} + E_{t-1} & \text{w.p. } (1-\alpha)(1-\beta). \end{cases} \tag{2.10}$$

Substituting $A_{t-2}$ in (2.10) by $A_{t-2}$ in (2.9) gives

$$X_t = \begin{cases} \beta E_t & \text{w.p. } \beta \\ \beta E_t + \alpha^2 A_{t-3} & \text{w.p. } \alpha^2(1-\beta) \\ \beta E_t + \alpha^2 A_{t-3} + \alpha E_{t-2} & \text{w.p. } \alpha(1-\alpha)(1-\beta) \\ \beta E_t + \alpha^2 A_{t-3} + E_{t-1} & \text{w.p. } \alpha(1-\alpha)(1-\beta) \\ \beta E_t + \alpha^2 A_{t-3} + \alpha E_{t-2} + E_{t-1} & \text{w.p. } (1-\alpha)^2(1-\beta). \end{cases}$$

Continuing this process, the EARMA(1,1) model reduces to

$$X_t = \begin{cases} \beta E_t & \text{w.p. } \beta \\ \beta E_t + \alpha_{t-1} A_0 + \sum_{k=0}^{j} \begin{bmatrix} t-1 \\ k \end{bmatrix}, \quad j=0,1,2,\cdots,t-1 & \text{w.p. } \alpha^{t-1-j}(1-\alpha)^j(1-\beta), \end{cases} \tag{2.11}$$

where $\sum_j^k \begin{bmatrix} t-1 \\ k \end{bmatrix}$ denotes the sum of the combinations of choosing k terms from $E_{t-1}, \alpha E_{t-2}, \alpha^2 E_{t-3}, \cdots, \alpha^{t-2} E_1$, and $A_0$ is an exponentially distributed random variable with parameter $\mu$.

The equation (2.11) makes the analysis and the forecasts of the EARMA(1,1) model easier since $X_t$ is the linear combination of the exponentially iid random variable $E_j$, $j=0,1,2,\cdots,t$, where $A_0=E_0$.

To obtain the Yule-Walker estimators of the parameters of the EARMA(1,1) model, the autocorrelations have to be obtained. The autocorrelations of the stationary EARMA(1,1) model can be obtained by the usual device of multiplying the defining equation $X_t$ by $X_{t-k}$, $t=1,2,\cdots$ and taking expectation.

**Theorem 1.** Jacobs and Lewis(1977)

Let $\{X_t\}$ be the stationary sequence of satisfying the model(2.8) and (2.9). Then, the autocorrelation $\rho_k$ between $X_t$ and $X_{t-k}$ is given by

$$\rho_k = \begin{cases} \beta(1-\beta) + \alpha(1-\beta)(1-2\beta), & k=1 \\ \alpha \rho_{k-1}, & k \geq 2. \end{cases} \tag{2.12}$$

From (2.12), the Yule-Walker estimators $\tilde{\alpha}$ and $\tilde{\beta}$ are obtained as

$$\tilde{\alpha} = \frac{\hat{\rho}_2}{\hat{\rho}_1} \tag{2.13}$$

and

$$\tilde{\beta} = \frac{(3\hat{\rho}_2 - \hat{\rho}_1) \pm \sqrt{(\hat{\rho}_1 - \hat{\rho}_2)^2 + 4\hat{\rho}_1^{\,2}(2\hat{\rho}_2 - \hat{\rho}_1)}}{2(2\hat{\rho}_2 - \hat{\rho}_1)} \tag{2.14}$$

respectively, where $\hat{\rho}_k$, $k = 1, 2$, is the sample autocorrelation, i.e.,

$$\hat{\rho}_k = \frac{\sum_{t=k+1}^{n} (X_t - \overline{X})(X_{t-k} - \overline{X})}{\sum_{t=1}^{n} (X_t - \overline{X})^2} \qquad k=1,2,\cdots, \tag{2.15}$$

and

$$\overline{X} = \frac{1}{n}\sum_{t=1}^{n} X_t. \tag{2.16}$$

It is immediately seen in the equation (2.14) that there are two estimators of $\beta$. In this case, some difficulties arise in model building and forecasting since the unique model does not exist. Therefore, a condition is required to select an unique estimator. This condition can be obtained by the invertibility.

**Definition.** Granger and Andersen(1978)

Let $\{X_t\}$ be a sequence generated by a difference equation of the form

$$X_t = f(X_{t-i}, \varepsilon_{t-j}) + \varepsilon_t, \quad i=1,2,\cdots,p,\ j=1,2,\cdots,q \tag{2.17}$$

where f is a known function and q starting up values of the $\varepsilon_t$ are also given.

Let $\hat{\varepsilon}_t$ be generated by

$$\hat{\varepsilon}_t = X_t - f(X_{t-j}, \varepsilon_{t-j}) \tag{2.18}$$

where $\hat{\varepsilon}_j = \overline{\varepsilon_j}$ for $j \le 0$.

Now, we define the variable

$$e_t = \varepsilon_t - \hat{\varepsilon}_t.$$

Then the model (2.17) is said to be invertible if

$$E(e_t) \to 0 \quad \text{as} \quad t \to \infty, \tag{2.19}$$

and

$$E(e_t^2) \to \text{constant} < \infty \quad \text{as} \quad t \to \infty . \tag{2.20}$$

**Theorem 2.** Let $\{X_t\}$ be the stationary sequence of satisfying the model (2.8) and (2.9). Then $\{X_t\}$ is invertible if

$$\frac{(1-\alpha)(1-\beta)}{\beta} < 1. \tag{2.21}$$

**Proof** From (2.8) and (2.9), we have

$$E(E_t) = \frac{1}{\beta} E(X_t) - \frac{1-\beta}{\beta}\{\alpha E(A_{t-2}) + (1-\alpha) E(E_{t-1})\}. \tag{2.22}$$

Therefore,

$$\begin{aligned} E(e_t) = E(E_t - \hat{E}_t) &= -\frac{(1-\beta)(1-\alpha)}{\beta} E(E_{t-1} - \hat{E}_{t-1}) \\ &= -\frac{(1-\beta)(1-\alpha)}{\beta} E(e_{t-1}). \end{aligned} \tag{2.23}$$

Hence if $(1-\beta)(1-\alpha)/\beta < 1$, then $E(e_t) \to 0$ as $t \to \infty$. Similarly, we have

$$E(e_t^2) = \frac{(1-\alpha)^2(1-\beta)^2}{\beta^2} E(e_{t-1}^2). \tag{2.24}$$

Hence if $(1-\beta)(1-\alpha)/\beta < 1$, then $E(e_t^2) \to 0$ as $t \to \infty$. ■

Note that the invertible condition (2.21) determines the unique estimator $\hat{\beta}$ of $\beta$ in (2.14).

## 3. Conditional least squares estimators

In this section, we derive the conditional least squares(CLS) estimators for the parameters of the EARMA(1,1) model by using the technique of Klimko and Nelson(1978).

Taking expectation given $X_{t-1}, X_{t-2}, \cdots, X_1$ in (2.8), we have

$$E(X_t | X_{t-1}, X_{t-2}, \cdots, X_1) = \beta\mu + (1-\beta) E(A_{t-1}, | X_{t-1}, X_{t-2}, \cdots, X_1). \tag{3.1}$$

Taking expectation given $X_{t-1}, X_{t-2}, \cdots, X_1$ in (2.9), we have

$$E(A_{t-1} | X_{t-1}, X_{t-2}, \cdots, X_1) = \alpha E(A_{t-2}, | X_{t-1}, X_{t-2}, \cdots, X_1) + (1-\alpha) E(E_{t-1} | X_{t-1}, X_{t-2}, \cdots, X_1). \tag{3.2}$$

Rearranging (2.8) about $E_t$ and taking expectation given $X_{t-1}, X_{t-2}, \cdots, X_1$, we have

$$E(E_{t-1} | X_{t-1}, X_{t-2}, \cdots, X_1) = \frac{1}{\beta} \{X_{t-1} - (1-\beta) E(A_{t-2} | X_{t-1}, X_{t-2}, \cdots, X_1)\}. \tag{3.3}$$

Hence, the conditional mean of $X_t$ given $X_{t-1}, X_{t-2}, \cdots, X_1$ is

$$E(X_t \mid X_{t-1}, X_{t-2}, \cdots, X_1) = \beta\mu + (1-\beta)\left(\frac{\alpha+\beta-1}{\beta}\right)^{t-1} E(A_0 \mid X_{t-1}, X_{t-2}, \cdots, X_1) + \frac{(1-\alpha)(1-\beta)}{\beta} \sum_{i=1}^{t-1} \left(\frac{\alpha+\beta-1}{\beta}\right)^{i-1} X_{t-i}, \tag{3.4}$$

where we assume $E(A_0 | X_{t-1}, X_{t-2}, \cdots, X_1) = \mu$.

Denote $E(X_t | X_{t-1}, X_{t-2}, \cdots, X_1)$ by $g_{t-1}(\alpha, \beta, \mu)$. Then the CLS estimators of $\alpha, \beta$, and $\mu$ are those values which minimize

$$Q_n(\alpha, \beta, \mu) = \sum_{t=2}^{n} \{X_t - g_{t-1}(\alpha, \beta, \mu)\}^2 \tag{3.5}$$

for a given set of observations $X_t$, $t=1,2,\cdots,n$.

Differentiating $Q_n(\alpha, \beta, \mu)$ with respect to $\mu$, $\alpha$, $\beta$, and setting the derivatives to zero, we can obtain the CLS estimators $\hat{\mu}, \hat{\alpha}, \hat{\beta}$.

However, the CLS estimators are of the complicate form and are unable to be used for the practical application.

For the practical purpose, the approximate CLS estimators can be obtained from the following approximation of (3.4).

$$E(X_t | X_{t-1}, X_{t-2}, \cdots, X_1) \approx \beta\mu + (1-\beta) \sum_{i=1}^{\infty} w_i X_{t-i}, \tag{3.6}$$

where $w_i = (1-\alpha/\beta)(1-(1-\alpha)/\beta)^{i-1}$ is an exponentially decreasing weight, since $\sum_{i=1}^{\infty} w_i = 1$.

Then the approximate CLS estimator $\hat{\mu}$ of $\mu$ can be solved by substituting (3.6) to (3.5), which is approximately equal to the sample mean of $X_t$, $t=1,2,\cdots,n$, for large n, i.e.,

$$\hat{\mu} \approx \overline{X}_t. \tag{3.7}$$

The apparent advantage of the CLS estimation method provides the estimator of $\mu$, while the Yule-Walker estimation method does not.

For simplification to obtain the approximate CLS estimator $\hat{\beta}$, let wi be $A(1-A)^{i-1}$ where A is a constant in (0,1) which is satisfying

$$\frac{(1-\tilde{\alpha})(1-\beta)}{\beta} < 1$$

where $\beta = (1-\tilde{\alpha})/A$ and $\tilde{\alpha} = \hat{\rho}_2/\hat{\rho}_1$.

Then the approximate CLS estimator $\hat{\beta}$ with an initial value of wi(0) is obtained as

$$\hat{\beta}^{(1)} \approx \frac{\sum_t\sum_i w_i^{(0)}X_tX_{t-i} - \sum_t\sum_i\sum_j w_i^{(0)}w_j^{(0)}X_{t-i}X_{t-j}}{n\overline{X}_t^2 - \sum_t\sum_i\sum_j w_i^{(0)}w_j^{(0)}X_{t-i}X_{t-j}} \tag{3.8}$$

Since the initial value of wi(0) is chosen in (0,1), the better approximate CLS estimator can be obtained recursively as

$$\hat{\beta}^{(k)} \approx \frac{\sum_t\sum_i w_i^{(k-1)}X_tX_{t-i} - \sum_t\sum_i\sum_j w_i^{(k-1)}w_j^{(k-1)}X_{t-i}X_{t-j}}{n\overline{X}_t^2 - \sum_t\sum_i\sum_j w_i^{(k-1)}w_j^{(k-1)}X_{t-i}X_{t-j}} \tag{3.8}$$

where $w_i^{(k-1)}$ is calculated from $w_i^{(k-1)} = 1-(\tilde{\alpha}/\beta^{(k-1)})(1-(1-\tilde{\alpha})/\beta^{(k-1)})^{i-1}$ and k is the number iterations satisfying the predetermined difference limit between $\hat{\beta}^{(k)}$ and $\hat{\beta}^{(k-1)}$, for example, $|\hat{\beta}^{(k)} - \hat{\beta}^{(k-1)}| < 0.01$.

It is noted that the approximate CLS estimator of $\alpha$ can not be obtained since $w_i$'s are assumed to be constant and hence (3.7) is not function of $\alpha$.

## 4. Simulation study

To compare the approximate CLS estimator $(\tilde{\alpha}, \hat{\beta})$ with the Yule-Walker estimator $(\tilde{\alpha}, \tilde{\beta})$ of the parameters of the EARMA(1,1) model, Monte Carlo simulations are performed. An algorithm developed by Lawrance and Lewis(1980b) for simulating dependent sequence of random variables from the EARMA(1,1) model is used to generate 50 sequences each of size n=20, 50, 100, 200 for each values of $(\alpha,\beta)$ = (0.2, 0.5), (0.5, 0.6), (0.8, 0.3) and $\mu$=10.

Table I shows the approximate biases and the mean square errors(MSE) of the approximate CLS estimator $(\tilde{\alpha},\hat{\beta})$ and the Yule-Walker estimator $(\tilde{\alpha},\tilde{\beta})$.

Table I. Approximate biases and mean square errors

| | $(\alpha,\beta)$ | n | 20 | 50 | 100 | 200 |
|---|---|---|---|---|---|---|
| $(\tilde{\alpha},\hat{\beta})$ | (0.2, 0.5) | bias | (0.13, 0.16) | (0.11, 0.13) | (0.08, 0.10) | (0.05, 0.07) |
| | | MSE | (0.11, 0.11) | (0.10, 0.10) | (0.08, 0.07) | (0.06, 0.07) |
| | (0.5, 0.6) | bias | (0.09, 0.14) | (0.08, 0.11) | (0.06, 0.08) | (0.03, 0.06) |
| | | MSE | (0.08, 0.12) | (0.08, 0.09) | (0.05, 0.07) | (0.04, 0.07) |
| | (0.8, 0.3) | bias | (0.10, 0.18) | (0.07, 0.13) | (0.04, 0.10) | (0.01, 0.09) |
| | | MSE | (0.09, 0.13) | (0.07, 0.10) | (0.07, 0.09) | (0.06, 0.08) |
| $(\tilde{\alpha},\tilde{\beta})$ | (0.2, 0.5) | bias | (0.13, 0.16) | (0.11, 0.15) | (0.08, 0.12) | (0.05, 0.09) |
| | | MSE | (0.11, 0.10) | (0.10, 0.11) | (0.08, 0.09) | (0.06, 0.08) |
| | (0.5, 0.6) | bias | (0.09, 0.14) | (0.08, 0.13) | (0.06, 0.11) | (0.03, 0.10) |
| | | MSE | (0.08, 0.11) | (0.08, 0.10) | (0.05, 0.09) | (0.04, 0.08) |
| | (0.8, 0.3) | bias | (0.10, 0.17) | (0.07, 0.14) | (0.04, 0.13) | (0.01, 0.11) |
| | | MSE | (0.09, 0.14) | (0.07, 0.12) | (0.07, 0.12) | (0.06, 0.10) |

It is noted that the bias of $\tilde{\alpha}$ decreases as n increases for any values of $\alpha$. This fact implies that $\tilde{\alpha}$ is asymptotically unbiased.

The bias of $\hat{\beta}$ is smaller than those of $\tilde{\beta}$ for $n \geq 50$. This fact is also true for the mean squared error. Hence, it is conjectured that the approximate CLS estimator $\hat{\beta}$ is better than the Yule-Walker estimator $\tilde{\beta}$. It is also seen that as n increases, the bias and MSE of both of $\hat{\beta}$ and $\tilde{\beta}$ decrease on the whole.

In conclusion, the simulation results suggest that the Yule-Walker estimator $\tilde{\alpha}$ is asymptotically unbiased and the approximate CLS estimator $\hat{\beta}$ is better in the sense of mean square error than the Yule-Walker estimator $\tilde{\beta}$.

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# 지수혼합 시계열 모형의 추정

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## 요약

지수혼합 시계열 모형인 EARMA(1,1) 모형이 율-워커 추정법과 조건최소제곱 추정법으로 추정되었다. 율-워커 추정량은 이동평균모수가 포함된 EARMA(1,1) 모형인 경우 유일하지 못하므로 가역 조건을 만족하는 추정량이 유일한 추정량으로 얻어졌고, 조건최소제곱 추정량은 근사추정량이 얻어졌다. 모의실험을 통하여 근사조건제곱 추정량은 율-워커 추정량 보다 평균제곱오차면에서 훨씬 좋은 추정량으로 나타났다.

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