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DETERMINING 3-D MOTION OF RIGID OBJECTS USING LINE CORRESPONDENCES [1]

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ABSTRACT

A linear method for determining three-dimensional motion of a rigid object is presented. In this method, two three-dimensional line correspondences are used. By using three-dimensional information of the features and observing that the rotation matrix is unique regardless of the translation vector, the two components of motion parameters (rotation and translation) are computed separately. Also in this paper, the solution is given without a scale factor which is necessary in other methods that use only the two-dimensional projective constraints.

1. INTRODUCTION

Computer analysis of object motion is important in many applications, e.g., object tracking, robot vision, and other image understanding systems. In order to analyze object motion by computers, the displacement of object in physical space should be measured.

Early work was mainly concentrated on two-dimensional motion analysis. Originally, studying two-dimensional motion using computers was motivated by the study of motion of clouds from satellite images (Leese *et al.* 1970). The analysis of three-dimensional motion of an object from two-dimensional images is more difficult than the analysis of two-dimensional motion. For example, rotation in three-dimensional space is defined to be about a three-dimensional line while rotation in a plane is defined to be about a point on the plane, and part of the object may disappear from the views by self-occulsion due to rotation in space. Substantial research has been conducted to estimate three-dimensional motion of objects from a sequence of two-dimensional projections of a scene. Ulman (1979) used three views of four

[1] 본 연구는 경희대학교의 연구지원에 의하여 이루어진 것임

non-coplanar points to recover three-dimensional structure of a scene under the assumption of parallel projection. Asada *et al.* (1980) also assumed parallel projection in their interpretation of three-dimensional motion of blocks. Problems become more complicated when the assumption of parallel projection is relaxed. The work by Roach and Aggarwal (1980) was the first in which the problem was formulated using central projection. The authors used an overdetermined set of equations derived from two views of six points or three views of four points to determine the movement of objects from noisy images. The method by Nagel (1981) requires five points over two frames. Recently, Tsai and Huang (1984), and Longuet-Higgins (1981) have presented the eight-point linear method to find a set of essential parameters from which the motion parameters are computed.

While all the works described above used point correspondences to determine the displacement of objects in space, Yen and Huang (1983) and Mitiche *et al.* (1986) used straight line correspondences over three frames. In Yen and Huang (1983), a seven-line iterative method was used to determine three-dimensional motion of rigid objects, and in Mitiche *et al.* (1986), a four-line over three-view system was used to determine the structure and motion of objects in space.

Most of the previous works involve solving complicated nonlinear equations and usually these equations are solved using numerical methods. This subject, avoiding nonlinear equations is the main focus of this paper. Recent development of inexpensive range acquisition systems, e.g., laser scanner, stereo vision systems, etc., makes it possible to analyze three-dimensional motion directly from range data. In this paper the avoidance of solving nonlinear equations is accomplished by using two pairs of three-dimensional line correspondences. The use of line correspondences is more advantageous than that of point correspondences in noisy situations, because the extraction of lines is less noise sensitive than that of points in general.

2. INTERPRETATION OF DISPLACEMENT EQUATION

The movement of rigid objects may be decomposed into a rotation about an axis through the origin of a coordinate system and a translation. These motion components can be determined by analyzing a sequence of time-varying images. In order to describe a three-dimensional motion of an object, one of the following equations is used in general.

$$\vec{V} = R\vec{V} + \vec{T} \tag{1}$$

$$\vec{V}' = R(\vec{V} + \vec{T}) \tag{2}$$

$\vec{V}$ and $\vec{V}'$ are position vectors of an object point at time instances of t_1 and t_2 respectively, R is a rotation matrix about an axis passing through the origin of the coordinate system, and $\vec{T}$ is a translation vector. Basically, equation (1) and (2) result in the same movement over time interval $t_2 - t_1$ if $\vec{V}$ and $\vec{V}'$ in both equations are the same points, respectively. However, the interpretations of two equations may be different. In equation (1), the point is first rotated by R about an axis passing through the origin of the coordinate system and translated by $\vec{T}$, while in equation (2), the point is first translated by $\vec{T}$ and rotated by R. Hereafter, the discussions will be with equation (1) for convenience.

Let us consider a single point in space. Then, there are infinite number of ways (sets of R and $\vec{T}$) for $\vec{V}$ at t_1 to be $\vec{V}'$ at t_2. However, if we consider a set of points on a rigid object, there exists a unique set of R and $\vec{T}$ for all the points at t_1 to be matched to corresponding points at t_2. This is an important constraint to estimating motion parameters of a moving object.

Now, let us consider a different rotational axis which passes through a point $\vec{V}_c$ on the plane which passes the origin and perpendicular to the rotational axis, i.e., $\vec{V}_c \cdot \vec{n} = 0$ ($\vec{n}$: orientation vector of rotational axis), then equation (1) becomes as follows.

$$\vec{V}' = R'\vec{V} + (I - R')\vec{V}_c + \vec{T}' \tag{3}$$

From equation (1) and (3), one can see clearly that rotational matrix $R = R'$ and translation vector $\vec{T} = (1 - R')\vec{V}_c + \vec{T}'$. In other words, the rotation matrix is independent of the location of rotational axis in space, however, translation vector $\vec{T}'$ depends on the location of rotational axis in space. Therefore, we can find the rotation matrix and translation vector separately, and the rotation matrix is unique regardless of translation vector.

3. ESTIMATION OF 3-D MOTION FROM LINE CORRESPONDENCES

In this section, a method is presented for estimating three-dimensional motion of rigid objects from two three-dimensional line correspondences. Suppose two sets of nonparallel three-dimensional line correspondences are established. Here, the meaning of line correspondences is that a line L in space (not a line segment) is moved to line L' over a time interval $t_2 - t_1$. Therefore, the useful information we can get from these lines is only the direction cosines of each line. This 'infinite line' assumption is reasonable, because from two sets of range data we can hardly get exact line segment correspondences between frames because of motion and possible occulsion.

From the assumptions, we are given four line equations as follows.
At time t_1 :

$$\vec{V}_1 = \vec{V}_a + \alpha\vec{A} \tag{4}$$

$$\vec{V}_2 = \vec{V}_b + \beta\vec{B} \tag{5}$$

at time t_2 :

$$\vec{V}_1' = \vec{V}_p + \gamma\vec{A}' \tag{6}$$

$$\vec{V}_2' = \vec{V}_q + \delta\vec{B}' \tag{7}$$

where $\vec{A}$, $\vec{B}$ and $\vec{A}'$, $\vec{B}'$ are the direction cosines of two lines before and after motion respectively, and α, β, γ and δ are arbitrary real numbers.

The displacement equations of these lines over time interval $t_2 - t_1$ are as follows.

$$\vec{V}_1' = R\vec{V}_1 + \vec{T} \tag{8}$$

$$\vec{V}_2' = R\vec{V}_2 + \vec{T} \tag{9}$$

where rotation matrix R can be represented in terms of either rotation angle Θ, Φ, Ψ about X, Y, Z axes or the orientation vector of rotational axis $\vec{n}$ and angle Θ about the axis.

$$R = \begin{bmatrix} \cos\Phi\cos\Psi + \sin\Phi\sin\Theta\sin\Psi & -\cos\Phi\sin\Psi + \sin\Phi\sin\Theta\cos\Psi & \sin\Phi\cos\Theta \\ \cos\Theta\sin\Psi & \cos\Theta\cos\Psi & -\sin\Theta \\ -\sin\Phi\cos\Psi + \cos\Phi\sin\Theta\sin\Psi & \sin\Phi\sin\Psi + \cos\Phi\sin\Theta\cos\Psi & \cos\Phi\cos\Theta \end{bmatrix} \tag{10}$$

or

$$\begin{bmatrix} n_1^2 + (1 - n_1^2)\cos\Theta & n_1n_2(1 - \cos\Theta) + n_3\sin\Theta & n_1n_3(1 - \cos\Theta) - n_2\sin\Theta \\ n_1n_2(1 - \cos\Theta) - n_3\sin\Theta & n_2^2 + (1 - n_2^2)\cos\Theta & n_2n_3(1 - \cos\Theta) + n_1\sin\Theta \\ n_1n_3(1 - \cos\Theta) + n_2\sin\Theta & n_2n_3(1 - \cos\Theta) - n_1\sin\Theta & n_3^2 + (1 - n_3^2)\cos\Theta \end{bmatrix} \tag{11}$$

If we rewrite equations (8) and (9) in component forms, we have six nonlinear equations with eight unknowns. However, based on the discussions in previous subsection, we can first find a rotation matrix independently of translation vector, and next find a translation vector given rotation matrix found. In other words, we can find R and $\vec{T}$ in two steps separately, i.e., first rotate lines $\vec{V}_1$ and $\vec{V}_2$ about an axis passing through the origin until they are parallel to $\vec{V}_1'$ and $\vec{V}_2'$ simultaneously, and next translate until an arbitrary point P_1 on line $\vec{V}_1$ and P_2 on $\vec{V}_2$ meet lines $\vec{V}_1'$ and $\vec{V}_2'$ simultaneously.

3.1 Finding Rotation Matrix

Let's denote the rotation matrix R as follows, and treat each element of R as a different unknown.

$$R = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix} \tag{12}$$

Then, the first step described above can be represented as following equations.

$$\vec{A}' = R\vec{A} \tag{13}$$

$$\vec{B}' = R\vec{B} \tag{14}$$

Furthermore, we can find the direction cosines of the third line in space simply by $\vec{A} \times \vec{B}$ (note that normalization is not necessary when used in equations, because $|\vec{A} \times \vec{B}| = |\vec{A}' \times \vec{B}'|$). Let this third direction cosines be $\vec{C}$ and $\vec{C}'$ for before and after motion, respectively. Then, we have another equation

$$\vec{C}' = R\vec{C} \tag{15}$$

From equations (13), (14) and (15), we have nine linear equations with nine unknowns, however, one should note that these are three sets of three linear simultaneous equations with three unknowns each.

Once r_i's are found, for rotation matrix R represented as (10),

$$\sin\Theta = -r_6 \tag{16}$$

$$\sin\Phi = \frac{r_3}{\sqrt{(1-r_6^2)}} \tag{17}$$

$$\sin\Psi = \frac{r_4}{\sqrt{(1-r_6^2)}} \tag{18}$$

For rotation matrix R represented as (11), we can find $\vec{n}$ and from r_i's in the same manner, however in this case, we do not even need to solve any equation. Because rotational axis should be perpendicular to both $(\vec{A} - \vec{A}')$ and $(\vec{B} - \vec{B}')$, we can compute the direction of rotational axis directly.

$$\vec{n} = \frac{(\vec{A}' - \vec{A}) \times (\vec{B}' - \vec{B})}{|(\vec{A}' - \vec{A}) \times (\vec{B}' - \vec{B})|} \tag{19}$$

However, a problem arises when $(\vec{A}-\vec{A}')$ and $(\vec{B}-\vec{B}')$ are parallel to each other. This case occurs only when the rotational axis lies on the plane formed by vectors $\vec{A}$ and $\vec{B}$. In other words, it occurs when the rotational axis is perpendicular to the normal of the plane formed by vectors $\vec{A}$ and $\vec{B}$. Therefore, when the calculation of $\vec{n}$ using equation (19) fails, we can use the following equation.

$$\vec{n} = \frac{(\vec{A}\times\vec{B})\times(\vec{A}' - \vec{A})}{|(\vec{A}\times\vec{B})\times(\vec{A}' - \vec{A})|} \tag{20}$$

In equation (19), if $(\vec{A}-\vec{A}')=0$, we can use $(\vec{B}-\vec{B}')$ instead (in fact, when $(\vec{A}-\vec{A}')=0$, $\vec{n}=\vec{A}$, and when $(\vec{B}-\vec{B}')=0$, $\vec{n}=\vec{B}$, if both $(\vec{A}-\vec{A}')$ and $(\vec{B}-\vec{B}')$ are zero, the motion is pure translation).

Once the orientation vector of rotational axis is found, rotation angle is simply the angle between two planes formed by $\vec{n}$ and $\vec{A}$, and $\vec{n}$ and $\vec{A}'$. Therefore,

$$\cos\Theta = \frac{(\vec{n}\times\vec{A})\cdot(\vec{n}\times\vec{A}')}{|\vec{n}\times\vec{A}||\vec{n}\times\vec{A}'|} \tag{21}$$

In calculation of $\cos\Theta$ using equation (21), if $\vec{n}=\vec{A}$, we can use vector $\vec{B}$ and $\vec{B}'$ instead of $\vec{A}$ and $\vec{A}'$, respectively.

3.2 Determination of Translation Vector

Once a rotation matrix is found, we have two sets of parallel lines in space to be matched simultaneously by a single translation. If we have only one set of parallel lines in space, there are infinite number of ways of translation for the lines to be matched, however, if we consider two or more sets of such lines, there exists a unique translation to be matched simultaneously. From equations (4) to (9),

$$R(\vec{V}_a+\alpha\vec{A})+\vec{T}=\vec{V}_p+\gamma\vec{A}' \tag{22}$$

$$R(\vec{V}_b+\beta\vec{B})+\vec{T}=\vec{V}_q+\delta\vec{B}' \tag{23}$$

Equations (22) and (23) seem to give us six linear simultaneous equations with seven unknowns, but note that α, β, γ and δ are arbitrary real numbers. In other words, if we select an arbitrary point on each of lines $\vec{V}_1$ and $\vec{V}_2$, and if we can find the corresponding points on the lines $\vec{V}_1'$ and $\vec{V}_2'$ after translation, that translation is what we want to determine. Therefore, by substituting $\alpha=\beta=0$ in equations (22) and (23), we have six linear equations with five knowns.

$$R\vec{V}_a + \vec{T} = \vec{V}_p + \gamma \vec{A}' \tag{24}$$

$$R\vec{V}_b + \vec{T} = \vec{V}_q + \delta \vec{B}' \tag{25}$$

Solving equations (24) and (25) is straightforward.

In the case of using point correspondence, we can determine translation vector $\vec{T}$ directly using the equation (1).

4. CONCLUSION

In this paper, a simple method for determining three-dimensional motion of a rigid body from two-dimensional line correspondences has been presented. The concept that rotation matrix is unique regardless of translation makes it possible to find two components of motion separately and makes the problem simple. This concept may be useful in determining three-dimensional motion from projected data and it will be the future problem to be investigated.

For the case of one line and one point correspondences, we can always find another line in object domain which passes through the point and meets the line perpendicularly. Therefore, the problems become the same as described in this paper. In other words, two lines, or one line and one point, or three points are the minimum requirements to determine three-dimensional motion of rigid objects in space.

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