# Simulation of Moving Storm in a Watershed Using Distributed Models

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ABSTRACT/In this paper distributed models for simulating spatially and temporally varied moving storm in a watershed were developed. The complete simulation in a watershed is achieved through two sequential flow simulations which are overland flow simulation and channel network flow simulation. Two dimensional continuity equation and momentum equation of kinematic approximation were used in the overland flow simulation. On the other hand, in the channel network simulation two types of governing equations which are one dimensional continuity and momentum equations between two adjacent sections in a channel, and continuity and energy equations at a channel junction were applied. The finite element formulations were used in the overland flow model and the implicit finite difference formulations were used in the channel network model.

Macks Creek Experimental Watershed in Idaho, USA was selected as a target watershed and the moving storm on August 23, 1965, which continued from 3:30 P.M. to 5:30 P.M., was utilized. The rainfall intensity of the moving storm in the watershed was temporally varied and the storm was continuously moved from one place to the other place in a watershed. Furthermore, runoff parameters, which are soil types, vegetation coverages, overland plane slopes, channel bed slopes and so on, are spatially varied. The good agreement between the hydrograph simulated using distributed models and the hydrograph observed by ARS are shown. Also, the conservations of mass between upstreams and downstreams at channel junctions are well indicated and the spatial and temporal variability in a watershed is well simulated using suggested distributed models.

## I. Introduction

A natural watershed is a complicated system with a diversity of input data that contribute to the

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variability of the runoff response. In recent years, much effort has been devoted to develop mathematical models to predifct runoff in a watershed system. Especially, the one-dimensional finite difference method has been widely used to predict runoff or to show the applicability and limitation of the method by many researchers. In their studies they considered the nature of flow to be a one-dimensional flow problem which is a less realistic representation. To prevent a less realistic representation of a whole watershed system, several approaches are suggested and utilized in the mathematical models. The widely accepted approaches are to combine two mathematical models, which are the overland flow model and the channel flow model, and to accept the wide variation of physical parameters in a watershed system. The model, which use the spatial and temporal input data instead of the representative values and use the corresponding solution algorithms, is called as a distributed model. The distributed model is well applied in the watershed having spatially and temporally varied data such as moving storm.

Chen and Chow(1971) formulated a comprehensive distributed mathematical model to describe the mechanics of surface runoff using the hydrodynamic approach. Their mathematical formulation is much more realistic than the conventional lumped approach in terms of understanding the physics of overland flow problem. Hromadks et al.,(1987) used the two-dimensional kinematic and diffusion approximations to develop an explicit finite difference model. Their model were tested for different shapes of idealized catchments with different variations in the size, slope, friction parameters, and effective rainfall intensity. Judah(1973) used the finite element method in solving the kinematic one-dimensional overland and channel flow problems. He used linear elements to investigate the effects of the overland slope, watershed shape, and excess rainfall on the runoff hydrograph. Taylor and Al-Mashidani(1974) used the finite element method to formulate two-dimensional full dynamic equation and one-dimensional kinematic wave approximation. They compared the results by their one-dimensional model with published data using different combinations of geometric data, boundary conditions, and time steps. They reported on the flexibility of the method in dealing with the discretization and the simplicity in handling different boundary conditions. Ross(1978) used the finite element method in developing a one-dimensional storm hydrograph model to predict the flow on ungaged areas. He studied different aspects of the finite element method such as number of nodes per element, element length, different orders of shape functions, and time of execution to investigate the effect on the convergence and stability of the method. He concluded that the finite element method is a powerful and flexible method in dealing with the complexity represented in the spatial and temporal variabilities in a watershed area. Yaramanoglu(1978) developed a distributed paramerer mathematical model to simulate a storm hydrograph on a natural watershed. His conclusions were in favor of the finite element method as a suitable method in the development of a distributed mathematical model on a watershed. Ross et al.,(1979) used the Galerkin residual method to solve the kinematic equations for overland and channel flow. The purpose of the model was to determine the hydrologic impact of land-use change on a watershed area. Jayawardena and White(1979) in a two paper series, formulated a finite element distributed mathematical model to simulate surface and subsurface flow components. Comparisons with analytical and real data showed the feasibility of the method as a useful numerical tool in building a distributed catchment model. Julien et al.,(1989) developed a distributed ond-dimensional finite element model to be used later by Richardson(1989) and Moglen(1989). Moglen(1989) used the model to show that the spatial variability of input patameters is important when a watershed system is responded to give partial equilibrium hydrograph. Zhang and Cundy(1989) presented a distributed two-dimensional hydrodynamic overland flow model. They tested their model using two-dimensional surfaces with spatially variable roughness, infiltration, and microtopography.

On the other hand, several numerical solution approaches and algorithms for one dimensional unsteady flows in a single channel are suggested and used in the literature(Amein, 1968, Balloffet, 1969, Amein et al., 1970, Ellis, 1970, Fread, 1971, Chen, 1973, Liggett et al., 1975, Abbott, 1980). Solution methods for channel networks were also investigated by several researchers. Simplified solution methods and algorithms have been suggested because of the need to solve complicated and/or very large coefficient matrices. The overlapping Y-segment method suggested by Sevuk has been widely accepted (Yen and Osman, 1976, Yen, 1979). This method is based on the assumption that each segment is separate and independent from the others. In this method, the downstream effect can not propagate to the upper segments. To achieve the accuracy required in a whole system, a large number of iterations may be needed. Barkau(1985) assumed the flow from the tributaries to be lateral inflow. This assumption is efficient in the case of having small backwater effects and small momentum contributions from tributary flows. In another approach, Tucci(1978) applied the Skyline solution algorithm to simulate unsteady flows in channel networks. The Skyline solution algorithm which was developed mainly to solve the finite element equations requires a large number of decisions during the reduction pass. Therefore, to avoid the disadvantages of above solution algorithms, more efficient solution algorithms for channel network flow simulation are necessary.

In the channel network model suggested in this paper, more efficient double sweep algorithm are applied. The conventional double sweep method is frequently used in the single channel routing because of the banded matrix feature which has many solution advantages. However, simultaneous equations describing flows in channel networks lose their banded property. Therefore, the conventional double sweep matrix solution is not valid in the channel network modelling. Without the efficient solution algorithms, large computer memory storages are required to solve the simultaneous equations resulting from finite difference approximations of such systems. For a channel network involving N number of cross sections, the internal computer memory storage requirements using the conventional method for the solution of governing equations is $2N \times 2N$. The development of a solulution algorithm transforming the off-diagonal terms of the coefficient matrix to diagonal terms through recursion equations provides the same advantages as the single channel treatment. That is, the storage requirements for the coefficient matrix for network having N cross sections is reduced from $2N \times 2N$ to $2N \times 4$. The computational times required for the solution of such systems can be accordingly reduced. The channel network can be efficiently simulated using this channel network solution algorithm.

The studies about the moving storms were mainly conducted since 1967. Attmannspacher and Schultz(1967) studied the distribution of rainfall depending upon the time. Hindi and kelway(1977) studied the velocities of moving storms using the rainfall data observed in the rain gauges. The determined velocities were utilized to predict the reliability of the observed data in the moving storm area. Amorocho and Wu(1977) developed two mathematical models using rainfall data and radar data to simulate the movement of the heavy rains. Marshall(1980) analized 219 heavy rain events and indicated the relationship between the movement of heavy rain and the wind having 700mb. Niemczynowicz(1984) studied the runoff by moving storm using data observed in the Lund, Sweden. Niemczynowicz and Dahlblom(1984) studied the runoff variations by the variations of watershed characteristics, rainfall distributions, rainfall types and the dynamic properties of rainfall. Also, Niemczynowicz(1987) studied the necessities for considering the dynamic characteristics of rainstorms. Richardson(1989) analyzed the effects for runoff in case of the moving storms move from downsteam to upstream and the moving storms move from upstream to downstream. He concluded the variation of runoff at downstream end in the rising limb.

## Ⅱ. Governing Equations

The runoff of moving storm can be simulated through the overland flow model and the channel network flow model. The phenomena of the flows in the overland planes and channel network can be written with continuity equation which is based on the conservation of mass and momentum (or energy) equation which is the conservation of momentum (or energy). These equations can be written with partial differtial equations with 1-D, 2-D or 3-D forms. In the overland flow simulation, it is very necessary to reflect two dimensional flow patterns because the runoff varies significantly depending upon the slopes in the x, y directions. Also, in the channel network flow simulation, it is necessary to simulate the network using the whole system instead of using the individual channels. The overland flow equations and channel network flow equations used in the suggested distributed models, are shown in the following sections.

### 2.1 Two-Dimensional Overland Flow Equations

The flow patterns on the overland flow planes can be written with two-dimensional continuity and momentum equations. The continuity equation can be written as Eq. (1) having unit discharge in the x, y directions.

$$\frac{\partial h}{\partial t} + \frac{\partial(q_x)}{\partial x} + \frac{\partial(q_y)}{\partial y} = W \qquad (1)$$

where, h is the water depth; $q_x$ is the unit discharge in the x direction; $q_y$ is the unit discharge in the y direction; W is the excess rainfall intensity; t is time; x is the distance in the x direction; y is the distance in the y direction.

The momentum equations in the x, y directions can be written with the equations having unknowns of water depth, velocity and energy slopes. The momentum equation in the x direction can be written with Eq. (2).

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x} + W\frac{u}{h} = g(S_x - S_{fx}) \tag{2}$$

where, u is the mean velocity in the x direction; v is the mean velocity in the y direction; g is the acceleration of gravity; Sx is the channel bed slope in the x direction; $S_{fx}$ is the energy slope in the x direction. Also, the momentum equation in the y direction can be written with Eq. (3).

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial h}{\partial y} + W\frac{v}{h} = g(S_y - S_{fy}) \tag{3}$$

where, Sy is the channel bed slope in the y direction; $S_{fy}$ is the energy slope in the y direction. Choi(1991) shows that the kinematic assumptions are well accepted in the relatively steep slopes. Therefore, the Eqs. (2) and (3) can be written as Eqs. (4) and (5).

$$S_x = S_{fx} \tag{4}$$

$$S_y = S_{fy} \tag{5}$$

Also, in the kinematic assumptions, the unit discharges $q_x$, $q_y$ in the x, y directions can be written with the functions having the unknowns of $S_x$, $S_y$ and S such as Eqs. (6) and (7).

$$q_x = Ch^M S_x S^{L-1} \tag{6}$$

$$q_y = Ch^M S_y S^{L-1} \tag{7}$$

where, the channel bed slope S can be replaced with $\sqrt{(S_x)^2 + (S_y)^2}$ ; the coefficient C and superscripts M and L can be indicated as table 1.

**Table 1.** List of coefficient C and superscripts M and L

| Flow Identification | C | M | L |
|---|---|---|---|
| Laminar Flow | $8g/k_1\nu$ | 3 | 1 |
| Turbulent(Manning Formula) | 1/n | 5/3 | 0.5 |
| Turbulent(Chezy Formula) | c | 3/2 | 0.5 |

Through combining Eq. (1) with Eqs. (6) and (7), Eq. (8) can be obtained.

$$\frac{\partial h}{\partial t} + \frac{\partial(CS_x S^{L-1} h^M)}{\partial x} + \frac{\partial(CS_y S^{L-1} h^M)}{\partial y} = W \tag{8}$$

The coefficients and channel bed slopes in Eq. (8) can be rearranged with Eqs. (9) and (10).

$$\beta_x = CS_x S^{L-1} \tag{9}$$
$$\beta_y = CS_y S^{L-1} \tag{10}$$

Eq. (8) can be simplified as Eq. (11) using $\beta_x$ and $\beta_y$.

$$\frac{\partial h}{\partial t} + \frac{\partial(\beta_x h^M)}{\partial x} + \frac{\partial(\beta_y h^M)}{\partial y} = W \tag{11}$$

The partial differtial equation Eq. (11) can be rewritten with Eq. (12).

$$\frac{\partial h}{\partial t} + \beta_x M h^{M-1}\frac{\partial h}{\partial x} + h^M\frac{\partial \beta_x}{\partial x} + \beta_y M h^{M-1}\frac{\partial h}{\partial y} + h^M\frac{\partial \beta_y}{\partial y} = W \tag{12}$$

The equation (12) is two-dimensional equation indicating two directional overland flow. The equations can be analyzed by finite element approximation for the spatial derivatives and finite difference approximation for the time derivative (Marcus, 1991). Because the moving storm continues in the relatively short period, the effective rainfall intensity can be obtained by considering the infiltration only without considering the evaporation in the overland plane and evapo-transpiration through the plants. Eq. (13), which is suggested by Green and Ampt, is utilized as a infiltration equation in the model.

$$f = K\left(1 + \frac{M_o \Psi}{F}\right) \tag{13}$$

where, f is infiltration rate; K is hydraulic conductivity; $\Psi$ is wetting front capillary pressure; $M_o$ is fillable porosity; F is cummulative infiltration.

### 2.2 Channel Network Flow Equations

In the channel network, it is necessary to analyze the equations for the whole system simultaneously are necessary rather than analyzing individual channel segments independently. For analyzing the system simultaneously, two types of flow equations are necessary. First, the gradually varied unsteady channel flow between two adjacent cross sections in a channel can be described by two basic partial differential equations which are the continuity and momentum equations. Second, the continuity and energy equations which indicates the conservation of mass and energy at channel junction exist. The continuity equation between two adjacent cross sections in a channel can be written as Eq. (14).

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \qquad (14)$$

where, A is the cross sectional area; Q is the discharge; $q_l$ is the inflow or outflow discharge per unit length of the channel; x is the distance in the longitudinal direction; t is time. The momentum equation between two adjacent cross sections in a channel can be derived by considering the forces that act in the control surface and control volume. The resulting equation can be written as Eq.(15).

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\frac{\partial h}{\partial x} = gA(S_o - S_f) + q_l v_l \qquad (15)$$

where, $S_o$ is the channel bed slope; $S_f$ is the friction slope; $v_l$ is the lateral inflow velocity in the longitudinal direction. The energy equation at a channel junction can be written as tne following equation.

$$\frac{V_i^2}{2} + gh_i + gZ_i = \int \frac{dV}{dt}\, dx + \frac{V_o^2}{2} + gh_o + gZ_o + gh_f \qquad (16)$$

where, $V_i$, $h_i$, $Z_i$ indicate the velocity, water depth and channel bed elevation in the ith upstream channel and $V_o$, $h_o$, $Z_o$ indicates the velocity, water depth and channel bed elevation in the downstream channel. Also, $h_f$ indicates the head losses between ith upsteam channel and downstream channel at a channel junction. The continuity equation at a channel junction can be written as the following equation with maximum three upstream channels.

$$Q_{u1} + Q_{u2} + Q_{u3} + Q_o = 0 \qquad (17)$$

where, $Q_{u1}$, $Q_{u2}$, $Q_{u3}$ indicate the discharges in the first, second and third upstream channels. Also, $Q_o$ indicates the discharge in the downstream stream channel. The equations for the channel network simulation can be analyzed by finite difference approximation for the spatial derivatives and time derivative (Choi, 1991).

## Ⅲ. Selection of Experimental Watershed

To demonstrate the applicability of the models, an experimental watershed experiencing the moving storm event, which is characterized with having spatially and temporally varied soil, vegetation, geometric data and so on, was chosen. This watershed is named as Macks Creek Watershed and monitered by the Agricultural Research Service(ARS) to collect the data about the runoff, flood, sediment yield and sediment transport since 1960 (Robins et al, 1965). The watershed can be characterized as a small, arid, mountainous watershed. Macks Creek watershed covers 31.74㎢ area. The climates ranges from arid to temperate. The average annual precipitation ranges from less than 250 ㎜ in the northwest part of the watershed to more than 1,140㎜ at the highest elevations in the southwest. The annual snowfall varies from 20% of the annual precipitation in the lower elevations to more than 70% at the higher elevations. The average annual air temperature is more than 7℃ in the valley below 1500m elevation and about 4℃ above this elevation. The channel bed slope in the watershed ranges from 1% to 38%. Channels are well incised, with the watershed consisting of two principal channels of approximately 10㎞ each. One principal channel drains the southern part of the watershed and flow to northeast. The other principal channel drains northern part of the watershed and flow to the east. Soils are dominately gravelly, rocky and stony loams with soil depths ranging from 20㎝ to 130㎝. Permeabilities range from very slow/none to moderately rapid. The soil types in the area have been described by Stephenson (1977). The flora on Macks Creek Watershed is composed dominantly of sagebrush species but there is an impressive diversity of plants. Typical vegetation in addition to sagebrush include bitterbrush, greasewood, shadecale, spiny hopsage, horsebrush, mountain mahogany, willow, aspen, and some coniferous stands at higher elevations. The soil classification of Macks Creek Watershed is shown in Fig. 1 and the vegetation coverages in the watershed is shown in table 2.

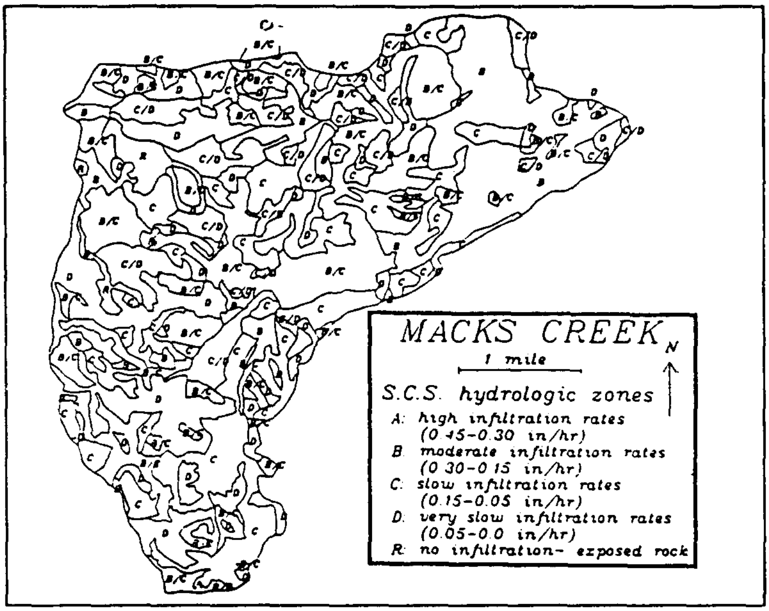


Fig. 1 Soil Classification of Macks Creek Watershed (Stephenson, 1977)

Table 2. The coverages of plants in Macks Creek watershed

| Area(%) | Coverages(%) |
|---|---|
| 35.5 | 0- 25 |
| 32.9 | 26- 50 |
| 18.0 | 51- 75 |
| 13.6 | 75-100 |

## Ⅳ. Overland Flow Simulation

The finite element model for overland flow simulation is applied to the watershed. Rainfall input data were based on the hytographs measured in the eight rain gage stations. The locations and ARS identification numbers of the rain gage stations are given in Fig. 2. A moving storm event between 15:30 and 17:30 on August 23, 1965 was selected. The rainfall intensities measured from rainfall hytographs obtained in the 8 rain gage stations varies from 19.8cm (7.8 inches) per hour at gaging station 045/04 to 0.043cm (0.017 inches) per hour at gaging station 061/25. Gaging station 055/88 measured no rainfall for this event.

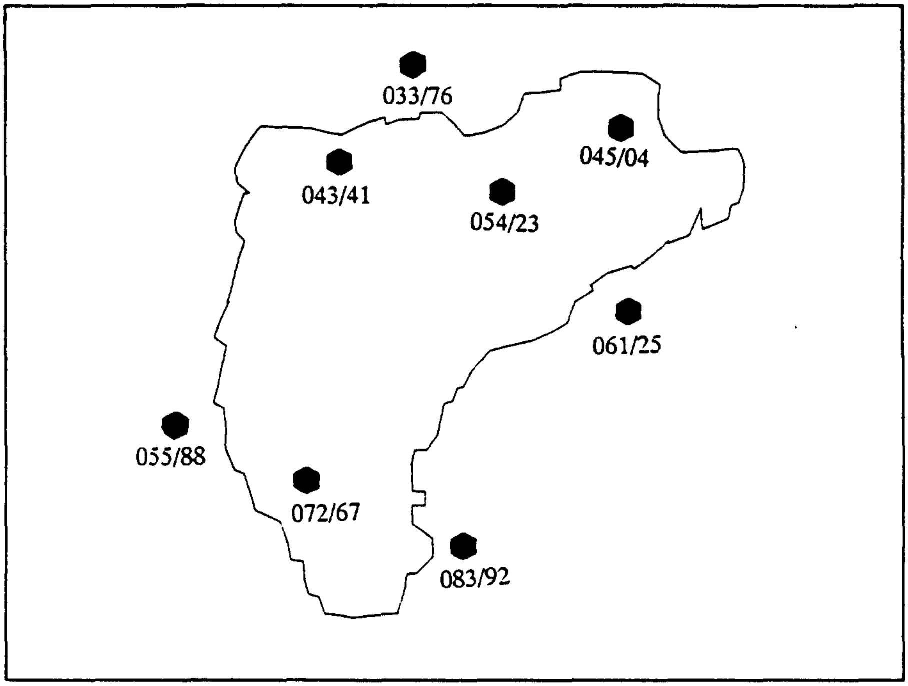


Fig. 2 Location of Rain Gage Stations

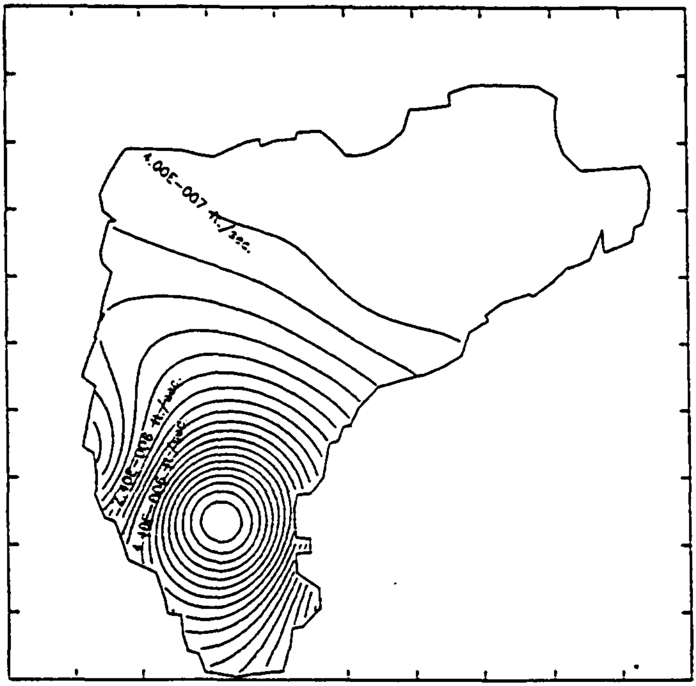


Fig 3. Rainfall intensity distribution map at time 0–14minites

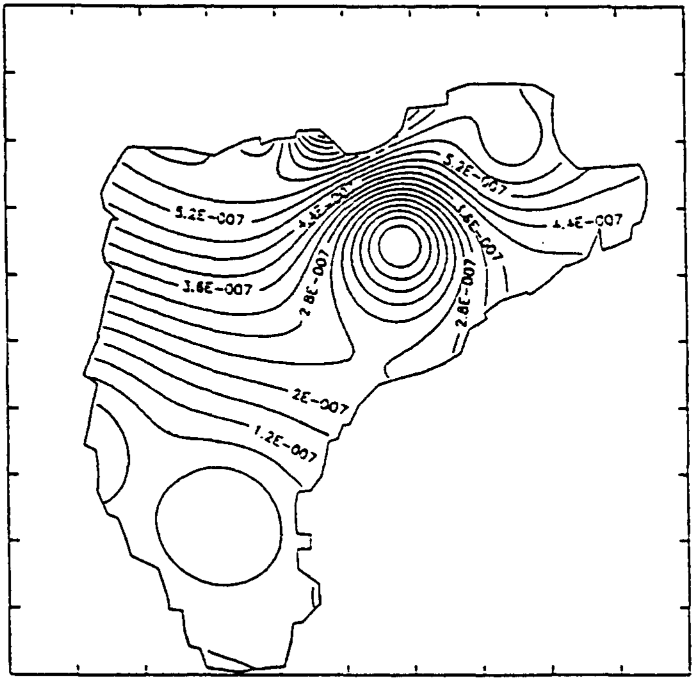


Fig 4. Rainfall intensity distribution map at time 44–58minites

The model needs the rainfall intensity at each calculation point on the finite element meshes. Since rainfall data are measured from sparse of rain gauges network, a suitable method is needed to calculate the areal distribution of rainfall on the entire watershed area. The inverse distance method is used in the the distributed rainfall intensities in the finite element meshes. Fig. 3 and Fig. 4 show the typical areal distribution of rainfall intensity for selected time intervals for simulating moving storm.

The typical hydrographs simulated using two-dimensional overland flow model are shown in Fig. 5 and and Fig. 6. The numbers shown in the legends in Fig. 5 and Fig. 6 indicate the numbers of the cross sections in the structure for channel network simulation (Fig. 7).

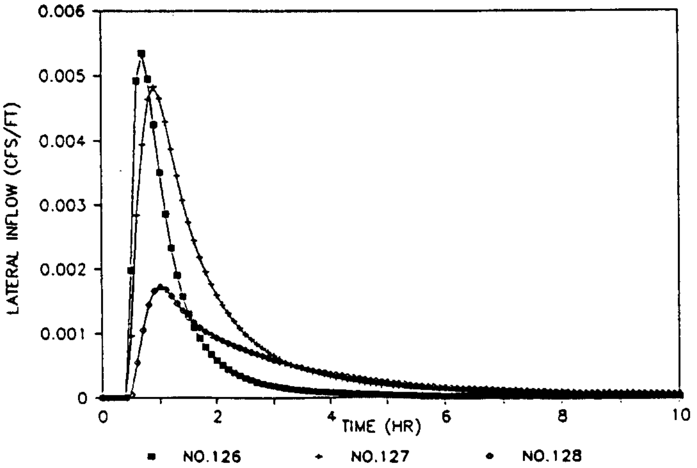


Fig. 5 The hydrographs simulated in the overland planes (1)

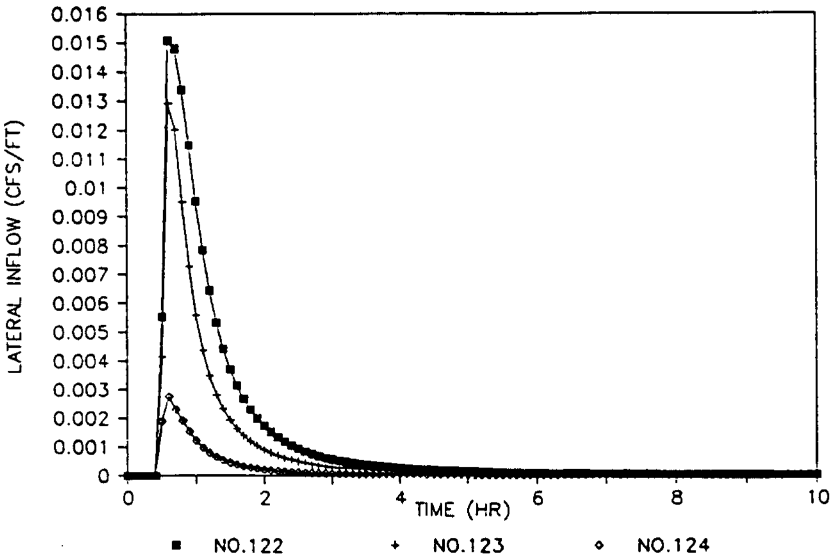


Fig. 6 The hydrographs simulated in the overland planes (2)

## V. Channel Network Flow Simulation

The channel network in Macks Creek Watershed consists with sixteen channels: ten first order channels, two second order channels, two third order channels, one fourth order channel and one fifth order channel. The schematic diagram of the Macks Creek channel network and its numbering for cross sections are shown in Fig. 7. The input data for channel network flow simulation can be classified into two groups: channel descriptional input data group and lateral inflow data group. The channel descriptional input data are composed of topographic data, base flow data, junction data, boundary data, operational coefficient data and print out option data. The incremental channel length between two adjacent cross sections varies from 142.65m(468ft) to 492.56m(1616 ft). The width of the channel ranges from 1.22m(4 ft) to 4.88m(16 ft) and a representative Manning's roughness coefficient value of 0.04 was used. The channel slope varies from 1.4% to 38.4%. Because the watershed has the highly variable channel bed slopes and the incremental lengths between adjacent two cross sections, improper initial values can cause initial numerical instability. Therefore, proper initial values(discharges) were used for preventing initial instability. A 10-hour simulation was conducted for 128 cross sections with a time increment of two minutes. A comparison between the hydrograph simulated using distributed models and the hydrograph observed by ARS in the downstream end is shown in Fig 8.

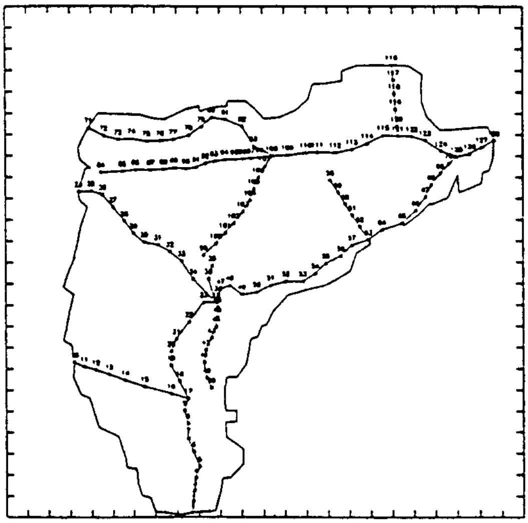

Fig. 7 Schematic Diagram of Macks Creek Channel Network

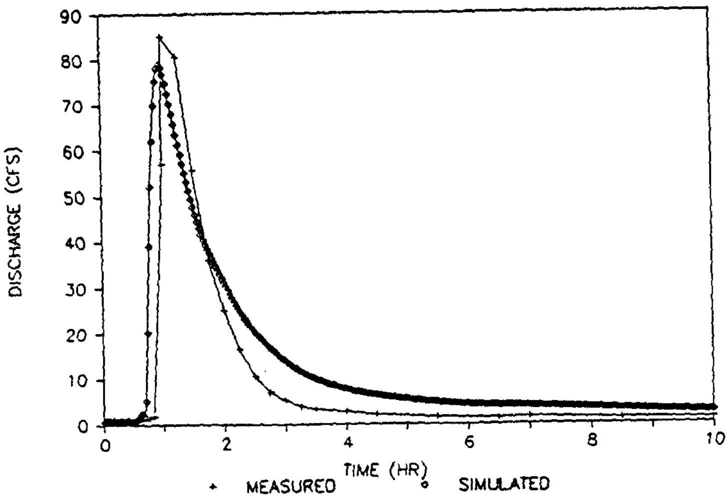


Fig. 8 Comparison between observed and simulated downstream hydrographs

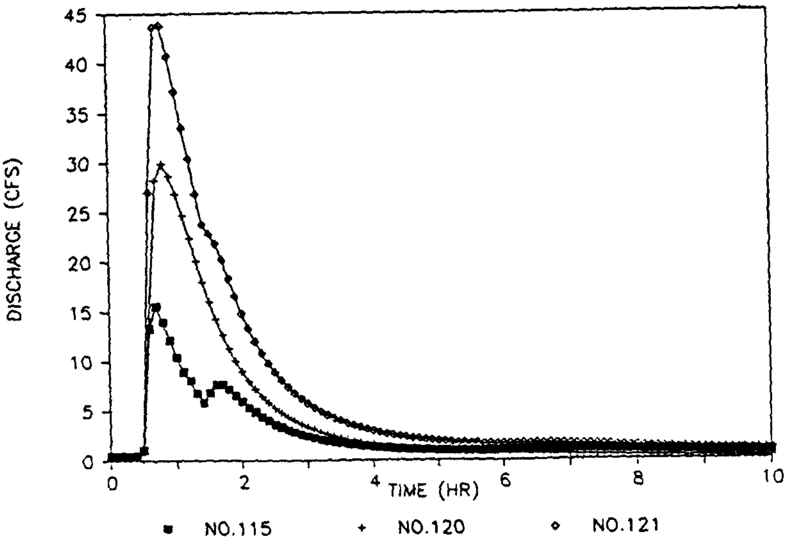


Fig. 9 Comparison of Hydrographs at a Channel Junction

The comparison in Fig. 8 indicates good agreement between simulated and observed hydrographs. The figure shows time-to-peak is almost identical between observed and simulated hydrographs. The peak discharge are approximately 2.41m$^3$/s(85 cfs) in the observed hydrograph and 2.27m$^3$/s (80 cfs) in the simulated hydrograph. The difference between the observed and simulated peak discharges is within 6%. The total discharge volume indicated in the simulated hydrograph is almost identical to that indicated in the observed hydrograph. Fig. 9 show the comparison of hydrographs between upstream and downstream sections at a channel junction. In the figure, NO. 115 and NO. 120 indicate the number of the cross sections in upstream channels at a channel junction, and NO.

121 indicates the number of the cross section in downstream channel at a channel junction. The comparison of hydrographs and the conservation of mass between upstream channels and downstream channel at channel junctions having temporally varied moving storm and spatially varied watershed data shows good agreement.

## Ⅵ. Conclusions

Distributed models, which consist of overland flow model and channel network model, were developed to simulate moving storm events on the natural watershed area. The overland flow model was consisted of two dimensional finite element formulation having kinematic wave approximation combined with the suitable resistance formula. The channel network model was consisted of one dimensional finite difference formulation based on the conservation of mass, momentum, and energy. In the channel network model, a new matrix solution algorithm was utilized to convert the matrix of coefficients resulting from the governing equations into a form exhibiting banded matrix features. The distributed models were applied to simulate the moving storm event on the natural watershed area. Macks Creek Watershed, which has the spatially varied soil type data, vegetation data, geometric data and so on, was selected as the natural watershed area. The comparison between the hydrograph simulated using the suggested distributed models with the moving storm on August 23, 1965 and the hydrograph observed by ARS shows good agreement. Using the suggested distributed models, the moving storms on the natural watershed, which consists of spatially and temporally varied data, can be well simulated.

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