水理學的 方法에 의한 土石流의 發生 豫測 및 算定 Prediction and Analysis of Debris Flow with Hydraulic Method

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Abstract The occurrence condition of debris fiow due to rainfall is given by solving the equations for fiow on a slope. The solution shows that a debris fiow will occur on a slope when the accumulated rainfall within the time of concentration exceeds a certain value determined by the properties of the slope. To estimate this critical value, the system analysis technique would be commendable. In this study, a procedure to find the critical rainfall from the rainfall data whith and without debris flows is proposed. Reliability of this method is verified by applying to the debris flows in Unzer Volcano which recently began to erupt.

Discharge of debris flow in a stream is obtained by solving the equation of continuity using the kinematic wave theory and assuming the cross sectional area to be a function of discharge. The computed hydrographs agree well with the ones observed at the rivers in Sakurajima and Unzen Volcanos. It is found from the derived equation that the runoff intensity of debris flow is in proportion to the rainfall intensity and accumulated rainfall, jointly. This gives a theoretical basis to the conventional method which has been widely used.

요 지: 遲滯時間 內의 累加降雨量이 특정 斜面傾斜를 지나게 될 때 발생하는 上石流의 生起條性은 傾斜面에서의 흐름에 대한 식을 사용하여 구 할 수 있으며, 이때의 土石流가 발생하는 限界 累加降雨量을 算定 하기 위하여 유역의 시스템적 분석 기법이 필요하다 따라서 본 연구에서는 土石流가 발생하는 限界 累加降雨量의 산정 과정을 제시 하였으며, 이 방법을 최근 폭발한 운젠 화산의 土石流에 적용하여 信賴性을 검토 하였다. 한편 土石流의 流量은 水理學的 방법인 Kinematic Wave방법을 이용하여 계산하였으며, 이때의 斷面續은 流量에 대한 函數關係에 있다고 假定 하였다. 이 방법에 의한 計算値와 사쿠라지마 및 운젠 화산 지역의 觀測値의 水文曲線은 대체로 잘 일치하였으며, 土石流의 流出強度를 降雨強度와 累加降雨量의 조합에 따른 식으로 算定하여 다른 지역에서의 土石流 計算에 이용할 수 있게 하였다.

I. Introduction

The debris flow is a shape of fast mass movement of a body of granular solids, water, and air, with flow properites varying with water and clay content, sediment size and sorting. Natural disasters caused by debris flow, as shown in Fig. 1, brings lots of sacrifices of people and wealth compared to natural disaster by land slide, erosion and so on, because debris flow may come suddenly without the sign before occurred. Therefor, the debris flow has been feared for its potential to cause heavy disaster. Studies on occurrence and intensity of debris flow are, therefore, required to prevent the

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disasters. In the past, occurrence criteria of debris flow have been defined by two parameters, cumulative rainfall from its beginning and a rainfall just before the occurrence of debris flow. But this method is not satisfactory in accuracy as well as in deciding the cumulative rainfall practically the lack of theoretical clarity.

In this study, the occurrence conditions of debris flow are analyzed to obtain the critical rainfall needed to cause a debris flow, and a mathematical model of runoff which predicts the intensity of debris flow is derived.



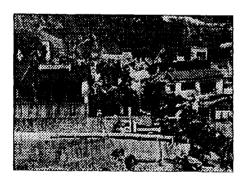


Fig. 1 Natural Disaters Caused by Debris Flow.

II. The Critical Rainfall for Occurrence of Debris Flow

1. Occurrence Condition of Debris Flow

On a slope of deposits as shown in Fig. 1, the shear stress at a point in the deposit is given.

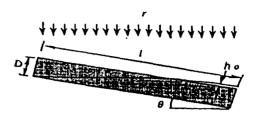


Fig. 2 Schematic sketch of a slope.

$$\tau = \{C_{\bullet}(\sigma - \rho) a + \rho (h_{\theta} + a)\} g \sin\theta \tag{1}$$

where, C* is the concentration of deposited material, σ and ρ are the densities of the deposits and water, respectively, a is the distance from the surface, ho is the depth of the surface flow, g is the gravitational acceleration, and θ is the angle of the slope. The resisting stress τ_L at the point is expressed by

$$\tau_1 = c + C \cdot (\sigma - \rho) \operatorname{ag} \operatorname{cos}\theta \operatorname{the}\phi$$
 (2)

where c is the adhesive force, and ϕ is the angle of internal friction.

Since the critical condition is $\tau = \tau L$, the critical angle of a slope θ_c for the occurrence debris flow is obtained by Eqs. (1) and (2) as

$$\tan\theta_{c} = \frac{\frac{c}{\rho \operatorname{gacos}\theta_{c}} + C \cdot \left(\frac{\theta}{\rho} + 1\right) \tan\phi}{C \cdot \left(\frac{\sigma}{\rho} - 1\right) + 1 + \frac{h_{0}}{a}}$$
(3)

By substituting ordinary values of $C_*=0.6$, $\tan\theta=1.0$, $\sigma/\rho=2.65$ and c=0 for sandy materials to Fq.(3) and considering that a and h_0 should be larger than grain size d to cause a debris flow⁽¹⁾, one obtains $\phi_c=14.8^{\circ}$. This has been supported by field data as well as flume data.

2. Critical Rainfall for Occurrence of Debris Flow

According to the theory mentioned above, a

debris flow will occur on a slope steeper than c when depth of the surface flow exceeds the grain size. There are two approaches to obtain the critical rainfall based on this theory.

One is to assume the discharge of surface flow in which the depth is equal to the grain diameter of the deposits as the critical discharge. Ashida et al. (2) derived the critical discharge Q by introducing $h_0 = d$ and $Q_c = Buh_0$ as

$$Q_c = \sqrt{\frac{8\sin\theta}{f_0 K^3} B^2 g d^3}$$
 (4)

where, B is the width of the flow, u is the velocity of surface flow, fo is the resistance coefficient, K is the ratio of ho and d close to unity, and d is the grain diameter of the deposits.

Applying the rational formula to Eq. (4), one obtain the critical rainfall intensity as

$$rT = \frac{1}{T} \int_{0}^{T} r dt \ge \frac{Bd}{fA} \sqrt{\frac{\sin\theta}{f_0 K^3} gd}$$
 (5)

where, T is the time of concentration, r is the rainfall intensity, f is the runoff coefficient, and A is the catchment area.

The other is to assume the occurrence of surface flow to be the occurrence condition of debris flow. Since irregularity of the slope surface is larger than the grain size, depth of the surface flow will exceed in some part of the slope when surface flow appears on the slope. Consequently, a debris flow will occur as soon as surface flow appears on a slope due to the heavy rainfall. The criteria for the surface flow are given as follows:

On a slope as shown in Fig. 1, the momentum and continuity equations of subsurface flow are expressed by

$$\frac{\partial(\lambda h)}{\partial t} + \frac{\partial(vh)}{\partial x} = \gamma \cos\theta \text{ and } v = k\sin\theta$$
 (6)

where, γ is the porosity, h is the depth of the subsurface flow, t is the time, v is the velocity of the flow, x is the coordinate taken in the downstream direction, and k is the hydraulic conductivity.

By solving Eq. (6) by using the kinematic wave theory, one obtains the occurrence conditions of surface flow as

$$l \ge k T \frac{\sin \theta}{\lambda}$$
 and $\lambda D \ge \int_0^T \cos \theta \, dt$ (7)

where, I is the length of the slope, and D is the depth of the deposits.

Assuming that debris flow occurs when surface flow appears on a slope, the occurrence condition of debris flow is derived from Eq.(7) as

$$rT = \frac{1}{T} \int_{0}^{T} rdt \ge \frac{Dk}{l} tan\theta$$
 (8)

The applicability of this equation was verified by the experiments(3) as shown in Fig. 3

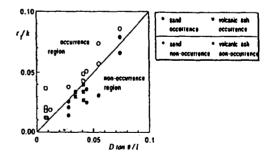


Fig. 3 Comparison between theoretical and experimental results.

In spite that Eqs. (5) and (8) are derived from the different basis, right hand sides of the equations are the same. These equations indicate that a debris flow will occur when rainfall intensity within the time of concentration exceeds a certain value determined by the properties of the slope.

3. Estimation of the Critical Rainfall

3.1 Estimating Method

Equation (8) is rewritten as

$$R(t, T) = \int_{t-T}^{t} r(\tau) d\tau \ge \frac{Dk}{l} tan\theta = R_c \quad (9)$$

where, t is the time, and Rc is the critical rainfall. Equation (9) shows that debris flow will occur when cumulative rainfall within the time of concentration exceeds a certain value related to the properties of the slope. Two parameters, the time of concentration T and critical amount of rainfall RC, should be estimated to obtain the criterion for occurrence of debris, flow. It may be possible to estimate the value of Rc by measuring the value of D, l and, however, the estimated value will not be accurate enough for practical use due to the large errors in the measurements. This is the reason that the method of system analysis will be commendable to identify the parameters.

To estimate the time of concentration and critical rainfall, T and Rc, cumulative rainfall R (t,to) defined as below is calculated.

$$R(t, t_0) = \int_{t-t_0}^{t} r(\tau) d\tau$$
 (10)

The maximum values of R(t,to) for each time, Rmax(to), are plotted against to. If there are no errors in the data as well as in the theory, the plotted lines should exceed the point (R _c,T) when debris flow occurred, and not exceed the point when debris flow did not occur as schematically illustrated in Fig. 4.

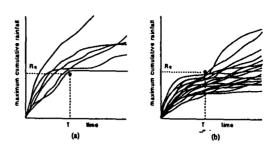


Fig.4 Cumulative rainfall when debris flow occurred(a) and not occurred(b).

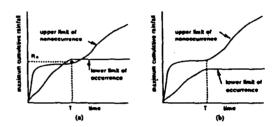


Fig. 5 Upper limit of non-occurrence and lower limit of occurrence.

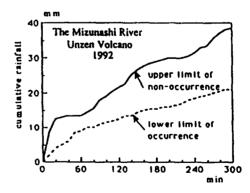


Fig. 6 Upper limit of non-occurrence and lower limit of occurrence of debris flow in the Mizunashi River, Unzen Volcano.

Consequently, the upper limit line of non-occurrence and the lower limit line of occurrence should cross at the point (R_oT) as schematically shown in Fig.4(a). Because of the errors in the data and the unsteady field conditions, however, the upper limit of non-occurrence and the lower limit of occurrence will be like two lines shown in Fig. 4(b). The point where the difference between two curves is minimum is estimated to be the time of concentration.

3.2 The critical rainfall of debris flow in the Mizunashi River

Unzen Volcano began to erupt in November 1990 after 198 years of dormancy and has been in violent activity. Continuous growth of lava dome and falls of lava rocks have resulted in frequent pyroclastic flows. As a great volume of volcaniclastic material has been deposited and scattered by the pyroclastic flows, debris flows have frequently occurred along the Mizunashi River and damaged many houses.

The cumulative amounts of rainfall were calculated using the rainfall data collected by the Unzen Meteorological Observatory, both when debris flows had occurred and when they had not. In case when debris flow occurred, the amount of rainfall up until the time of occurrence was computed, and in case without debris flow, whole data were used. In Fig. 5, the upper limit of non-occurrence and lower limit of occurrence are illustrated.

From the Fig. 5, the following are confirmed: 1) the time of concentration is estimated to be about an hour on average; 2) the occurrence of debris flows is possible when the amount of rainfall per hour rises over the limit of 9 mm; and 3) debris flows will definitely occur when this amount rises over the limit of 14 mm. At Sakurajima Volcano, which has been in violent activity in this 20 years, debris flow have been generated by rainfall from 7 to 13 mm over a period of forty minutes. By comparison, the debris flows in the Mizunashi River show the typical property of volcanic debris flow which is possible by a small amount of rainfall.

III. Runoff Analysis of Debris Flow 1.

1. Runoff coefficient of Debris Flow

Runoff coefficient of the debris flow, f, is defined as the ratio of the flow rate and the rainfall intensity as

$$f = \frac{\text{flow rate}}{(\text{rainfall intensity}) \times (\text{catchment area})} = f_s F$$
where
$$f_s = \frac{\text{flow rate}}{(\text{rainfall intensity}) \times (\text{area where debris flow has occurred})}$$
(11)

$$F = \frac{\text{(area where debris flow has occurred)}}{\text{(catchment area)}}$$

The continuity condition leads the following equation for fs as(4)

$$f_s = \frac{(1 - \lambda)}{(1 - \lambda - C)} \tag{12}$$

where, C is the concentration of debris flow. and is the porosity of the deposits. It is seen in the above equation that the range of fs is unity of infinitive, and for water flow, fs is unity as C =0. According to the experiments(3), $\lambda = 0.54$, C=0.50 and $f_s=18.00$. In usual runoff of water flow, F is considered to be unity, while in the case of debris flow, F should be less than unity varying with time.

2. Modeling of Runoff

The equation of continuity in a stream is given by

$$\frac{\partial A_s}{\partial t} + \frac{\partial Q}{\partial x} = q_s + q_{\bullet}$$
 (13)

where, A_s is the cross sectional area of the stream, Q is the discharge of the flow, and qs is the lateral inflow rate and q* is the rate of erosion of bed and bank. Lateral inflow rate is expressed by using the runoff coefficient as

$$q_s = f_s r l \cos\theta \tag{14}$$

Assuming A_s to be a function of Q, Eq. (13) is solved by use of the characteristic curve as follows:

On the characteristic curve $dx/dt = dQ/dA_{s}$,

$$Q = \int_0^L (q_s + q_{\bullet}) dx \tag{15}$$

Substituting Eq.(14) into Eq.(15) and omitting the erosion rate, one obtains

$$Q = \int_0^L f_s r \boldsymbol{l} \cos \theta \, dx \tag{16}$$

If we substitute a constant rainfall intensity r on into Eq.(16), then we obtain

$$Q = f_s r_0 \int_0^L r l \cos\theta \, dx \tag{17}$$

Considering Q=fsr₀A in this case, the following is obtained.

$$A = \int_0^L \cos\theta \, dx \text{ or } \int_0^L l \cos\theta \, \frac{dx}{A} = 1$$
 (18)

As I cos dx is a very small area of a watershed, lcos dx/A is taken to be a probability function of a slope, resulting in the following expression.

$$Q = A \int_0^{\infty} \int_0^{\infty} f_s r \phi(\eta, \boldsymbol{l}) d\eta d\boldsymbol{l}$$
 (19)

where, (η, l) is the probability function of $\eta = \lambda D$ and l.

On a slope where the condition given by Eq. (7) are satisfied, a debris flow will occur. While on a slope where rainfall intensity is less than the critical values given by Eq.(8), water flows into a stream, but no debris flow occurs on the slope.

2.1 Runoff Model for Water Flow

When all slopes are shorter and/or thicker than the critical values given by Eq.(7), no debris flow will occur in the watershed. In this case, $f_s=1$ and rainfall intensity is defined as

$$r_{T} = \frac{1}{T} \int_{0}^{T} r(t - \tau) d\tau$$
 (20)

Substitution of Eq.(20) into Eq.(19) yields

$$Q(t) = A \int_0^{\infty} r_T f(\boldsymbol{l}) dl = A \int_0^{\infty} r_T \phi(T) dT$$

where,
$$f(\boldsymbol{l}) = \int_0^\infty \phi(\eta, \boldsymbol{l}) d\eta$$
 and $\phi(T) = f(\boldsymbol{l}) \frac{d\boldsymbol{l}}{dT}$
(21)

From Eq.(21), instantaneous unit hydrograph u(t) is derived as

$$Q(t) = A \int_0^\infty r(t-\tau)u(\tau)d\tau$$
 (22)

where,
$$u(\tau) = \int \frac{1}{T} \phi(T) dT$$
 (23)

It is clarified that instantaneous unit hydrograph is a function of the time of concentration.

2.2 Runoff Model for Debris Flow

From Eq.(7), debris flow occurs on a slope where the followings are met.

$$l \ge \frac{k \sin \theta}{\lambda} (t - t_0)$$
 and $\lambda D = \int_{t_0}^{t} r \cos \theta dt$ (24)

$$Q_s(t) = \sum_{t_0 = -\infty}^{t} f_s r \phi(\eta, 1) \Delta \eta \Delta l$$

$$=f_{s}r\left\{\int_{kt\sin\frac{\theta}{\lambda}}^{\infty}\theta(\eta_{0}, l)dl\Delta\eta_{0}+\sum_{t=0}^{t}\phi(\eta_{1}, l)\Delta\eta\Delta l\right\}$$
(25)

From Eq.(26), the following relations are obtained.

$$\Delta \eta_0 = \Delta \eta = \text{rcos}\theta \Delta t \text{ and } \Delta l = \text{kt}\Delta \sin \frac{\theta}{\lambda}$$
 (27)

Assuming that debris on a slope outflows in a short period of time t, one obtains

$$\int_{t}^{t+\Delta t} q_{s} dt = \int_{t}^{t+\Delta t} f_{s} \mathbf{r} \mathbf{l} \cos\theta dt = D\mathbf{l} + \int_{t}^{t+\Delta t} \mathbf{r} \mathbf{l} \cos\theta dt$$
(28)

and

$$\Delta t = \frac{D}{(f_s - 1)r\cos\theta}$$
 (29)

The discharge of debris flow is obtained by substituting Eqs. (26) - (29) into Eq. (25) as

$$Q(t) = Ar(t) \frac{f_s}{(f_s - 1)\lambda} \left\{ \eta_0 \int_{kt \sin \frac{\theta}{\lambda}}^{\infty} \phi(\eta_0, l) dl \right\}$$

$$+\frac{\mathrm{ksin}\theta}{\lambda}\int_{0}^{t} \left(\eta, \frac{\mathrm{k}(t-t_{0})\mathrm{sin}\theta}{\lambda}\right) \mathrm{d}t_{0} \right\}$$
 (30)

For practical use, assumption that D and \boldsymbol{l} are independent each other is introduced as

$$\phi(\eta, \mathbf{l}) d\eta d\mathbf{l} = \phi_{\eta} d\eta dl$$
 (31)

where, and I are the probability function of D and I, respectively.

Since the first term of the right hand side of Eq.(30) is dominant compared with the second one, Eq.(30) can be simplified considering Eq. (31) as

$$Q(t) = Ar(t) \frac{f_s}{(f_s - 1)\lambda} \eta_0 \phi_l(\eta_0) \int_{kt \sin \frac{\theta}{\lambda}}^{\infty} \phi_l(1) d1$$
(32)

It has been found that distribution of slope length in a watershed is likely to be log-normal⁵. Results of application of Eq.(32) to debris flows in Sakurajima and Unzen Volcanoes are shown in Fig. 6.

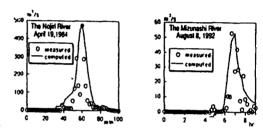


Fig. 7 Comparison between computed and observed hydrographs.

3. Prediction of Debris Flow

Equation (32) is rewritten as

$$\frac{Q(t)}{A} = r(t)\eta_0 M$$
where, $M = \frac{f_s}{(f_s - 1)\lambda} \phi_n(\eta_0) \int_{kt \sin \frac{\theta}{\lambda}}^{\infty} \phi_l(l) dl$
and $\eta_0 = \int_0^t r \cos \theta dt$ (33)

Equation(33) indicates that the runoff intensity of debris flow is in proportion to the rainfall intensity at the time r(t) and the cumulative rainfall up to that time 0, jointly. This means that a constant value of $Q_s/(AM)$ is shown as a

hyperbola on a (r(t), o) plane as schematically illustrated in Fig. 7.

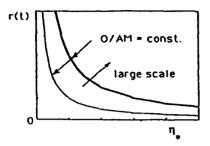


Fig. 8 Schematic sketch of diagrams for forecasting debris flow.

An empirical method which uses hyperbolalike curve(s) on the $[r(t), \eta_o]$ plane has been widely used to forecast the occurrence of debris flow. Equation(32) indicates that this conventional method has a theoretical basis and to be useful to predict the intensity of debris flow but not predict occurrence itself.

IV. Conclusions

The occurrence condition of debris flow due to heavy rainfall and runoff analysis of debris flow were studied. Results obtained are as follows:

- (1) Debris flow will occur on a slope when amount of rainfall within the time of concentration exceeds a certain value which is peculiar to the slope. The time of concentration and the critical amount of rainfall is obtained by analyzing the data of rainfall and debris flows.
 - (2) A mathematical model for runoff of de-

bris and water flows is derived. This model results in the instantaneous unit hydrograph when no debris flow occurs. The applicability of this model was verified by adopting to the debris flows in Sakurajima and Unzen Volcanoes.

(3) The derived equation for debris flow discharge gives the theoretical basis to an empirical method which uses the cumulative rainfall and rainfall intensity at the moment to forecast the occurrence of debris flow.

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