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An Exponentialization Procedure for General FMS Network of Queues with Limited Buffer

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Abstract

In this paper we develop an exponentialization procedure for the modelling of open FMS networks with general processing time at each station and limited buffer size. By imposing a reasonable assumption on the solution set, the nonlinear equation system for the exponentialization procedure is formulated as a variational inequality problem and the solution existence is examined. The efficient algorithm for the nonlinear equation system is developed using linearized Jacobi approximation method.

1. Introduction

A Flexible Manufacturing System (FMS) is an automated batch manufacturing system which is designed to produce different part types with the efficiency of mass production systems and the flexibility of job shops. The generic FMS consists of the following components[5]:

(1) A set of machines or work stations, which have some degree of flexibility, in particular they do not require significant set-up time or change-over time between successive jobs.
(2) A material handling system (MHS) that is automated and flexible, i.e. it permits jobs to move between any pair of machines so that any job routing can be followed.
(3) A network of supervisory computers and microprocessors, which i) directs the routing of jobs, ii) tracks the status of all jobs in the system, iii) passes the instruction for the processing of each operation to each station, iv) provides essential monitoring of the correct performance of operation and signals problems requiring attention.

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(4) Storage, locally at the work stations, and/or centrally at the system level.
(5) The jobs to be processed by the system.

For the evaluation and control of performance, FMSs have been approximately modelled as open networks of queues or closed networks of queues. In the open network model, the number of parts within the system is a random variable; parts have to arrive to the system (from an upstream production stage for instance). In the closed network model, the number of part within the system is a fixed constant, N, which usually represents the total number of pallets available in the system. During the production process, all N pallets are occupied; as soon as one part completes all its processing requirements and leaves the system, another is immediately relased in the system. One could find an unrealistic assumption that parts to be processed are always available whenever a pallet becomes available. In both of open network and closed network, we usually assume two strong assumptions as follows:

(1) At all stations with first-come-first-served (FCFS) queue discipline, the processing time distributions are exponential, and all parts types must have the same service rate parameter at a station.
(2) All stations have a local buffer with unlimited capacity, i.e., it can accommodate all parts circulating in the system if so necessary. In other words, parts will never be blocked at any station.

With these two restrictive assumptions, the medel can be easily analyzed by the well-known "product form" equilibrium joint queue length distribution. See[9] for closed queueing network and[12] for open queueing network. But it is also known that the model will not in general yield satisfactory performance evaluations if, with the FCFS queue discipline, the service time distributions are non-exponential[15, 17].

In this paper we develop an exponentialization procedure for the modelling of open FMS networks with general processing time at each station and limited buffer size. The resulting exponential networks[1)] can be easily analyzed by the results of previous research[1, 18].

Recently, Yao and Buzacott[19] formulated the exponentialization problem of closed FMS networks as a fixed point problem and developed an iterative solution scheme which does not guarantee the convergence of solution. We will propose a rigorous treatment of the solution procedure based on the finite-dimensional variational inequality problem in the context of general

1) Hereafter, we shall refer to an FMS network with general processing times as a general network, and refer to one with exponential processing times as an exponential network.

open FMS networks with limited buffer which is more realistic in practice.

This paper is organized as follows: A short review of the related works is given in chapter 2. The next two chapters include the description of system configuration and the concept of exponentialization. The nonlinear equation system for exponentialization procedure and its solution algorithm with a proof of solution existence are given in chapter 5 and 6. In chapter 7, a simple example is used to demonstrate the algorithm.

2. Short Review of the Literature

The research on the network of queues has focused primarily on performance evaluation. We adopt a classification of research by Bitran and Tirupati[2] for the brief review of the literature.

2.1 Exact Analysis

Exact results exist for Markovian systems. A seminal contribution in this area is the paper by Jackson[12] which provides result for equilibrium probability distributions of the number of jobs for a variety of open network system that are referred to as Jacksonian networks. With two major assumption mentioned above this restricted class of network can be described in the following framework.

(1) The arrival process is Poisson with parameter $\lambda(K)$ where K is the total number of jobs in the system.

(2) Job route is modeled as a Morkov process.

The main result is that the equilibrium distribution, if it exists, is of a prouduct form - product of marginal distribution of each station.

The classical model of closed network was done by Gordon and Newell[9]. Eventhough the normalizing constant in the equilibrium distribution requires the combinatorial computation[2], the closed network also has a product form solution. While the product form results are

2) The major difficulty in computing joint distribution is obtaining the value of normalizing constant. The number of different states is the number of ways M jobs can be placed in N work stations. For example, $M=100$, $N=10$, there are roughly 4.25*10E12 states of the network. Fortunately, we have an efficient computational technique by Buzen[3].

interesting and useful, they are difficult to implement in practice due to the very large state space. Also, in many cases, the parameters of interest are the mean values of queue lengths, waiting times etc. and not equilibrium distribution. Resier and Lavenberg[16] considered closed network for which the product form results hold. For such network, they developed a procedure (mean value analysis) to compute mean values without evaluating the equilibrium distribution. However, the above assumptions are overly restrictive in many practical situations. Since the exact results do not extend to more general network, it has led to the development of approximation scheme.

2.2 Approximation Analysis

The lack of success in obtaining exact solutions for general networks has motivated researchers to develop approximations to evaluate performance measures. This may be broadly described under the following five categories.

(1) Diffusion approximation
(2) Mean value analysis
(3) Operational analysis
(4) Decomposition analysis
(5) Exponentialization approach

Diffusion approximations are motivated by heavy traffic limit theorem and are based on the asymptotic method for approximating point processes. The work by Harrison and Reiman[11] is representative of this type of analysis. Mean value analysis is a heuristic approach similar to the work by Reiser and Laveberg[16] and is intended for closed networks. Buzen[3] was among the first to use operational analysis for computer systems. This paper focused on directly measurable quantities and testable assumpions. The analysis is distribution free and relies on flow balance and homogeneous service principles. Decomposition analysis is essentially an attempt to generalize the notion of independence and product form results for Jackson type networks to more general systems. Essentially the method involves three steps:

Step 1) Analysis of interaction between stations
Step 2) Decomposition of the network into subsystems of individual stations and their analysis
Step 3) Recomposition of the results to obtain the network performance

Works by Sauer and Chandy[17], Buzacott and Shanthikumar[4] and Bitran and Tirupati[2] are included in this category.

The exponentialization approach can be traced to numerous earlier works including Chandy et al.[6,7], Marie[15], Sauer and Chandy[17]. The idea of the approach is to tranform the network into an (approximately) equivalent exponential network. Similar approach was done by Buzacott and Shanthikumar[4]. The outline of their research is as follows: For $K=1,2,..,C$, the system is first modelled as a closed queueing network with K jobs, and the throughput of station i, $TH_i(K)$, is determined. The open network is then solved by representing the FMS as a single-server queue with state dependent service rate equals to $TH_i(K)$.

Buzacott and Yao[5] analyzed the simplified open FMS network model by equating the blocking probabilities. Because they did not specify the queues in the central storage, a very restrictive and unrealistic modeling assumption that blocked jobs will again follow the same set of predetermined routing probabilities without considering where they were blocked was imposed.

Yao and Buzacott[19] dealt with the exponentialization of closed queueing network with general processing time.

3. Description of the General FMS Network

The FMS network to be considered can be described as follows:

1) It consists of a set of N work stations. Each station i has a single server with a general service time distribution, which is characterized by its mean, v_i, and variance, σ_i^2.
2) Total number of jobs at central storage is limited to C. Jobs may be blocked on arrival to a work station which is already occupied by an other job, and blocked jobs will be recirculated ("blocked and recirculated"). In other words, they will be back to the central storage which has a separate queue for work station, join the associated queue, and get prepared for retrial.
3) A job leaving station $i(i=1,2,...,N)$ will either be fed back to the central storage and join the associated queue with porbability r_{ij} or leave the system with probability r_{iN+1}. Both the feedback and the exit transits are handled by the conveyor in a negligible time.
4) External jobs arrive at the system following a Poisson stream with rate θ. They join the associated queues in the central storage with rate $\theta_i(=r_{0i}\theta)$, where r_{0i} is the probability that an arriving job visits station i first. Whenever the total number of jobs at the central

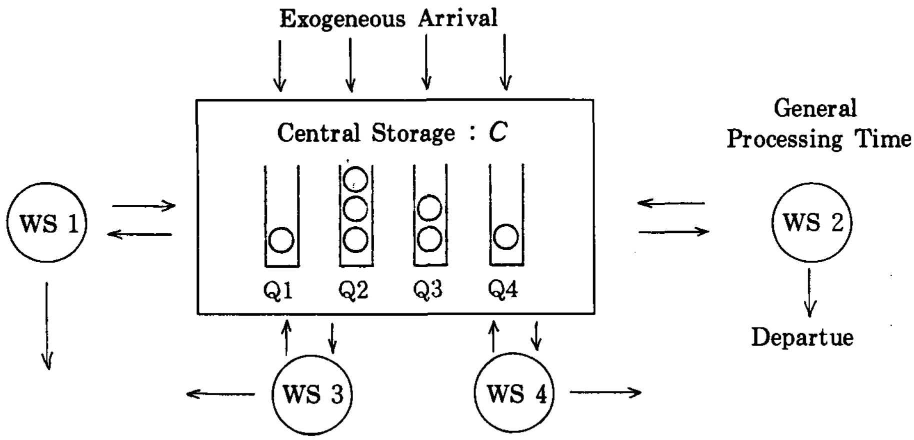
storage reaches C, external arrivals are turned away and lost. That is,

$$\theta(K)=\theta,\ \theta_i(K)=r_{0i}\theta \qquad \text{if } K<C$$

$$\theta(K)=0,\ \theta_i(K)=0 \qquad \text{if } K=C$$

Figure 1 shows the open FMS network with general service time and limited central storage capacity.

Figure 1. General FMS Network (N=4)



4. Concept of Exponentialization Procedure

The exponentialization approach to approximation used in this paper is to develop a solution technique consisting of the appropriate combination of a solvable queueing network model and an exact or approximate solution algorithm which relates the original system to the solvable queueing network model. The exponentialization procedure is based on the following definition of equivalent queueing networks.

Definition 1 Two queueing networks with an identical network structure are said to be equivalent in performance measute Ω if every two corresponding nodes (stations) in each network have same value in performance measure Ω.

It can be easily seen that the equivalence in queue length probability is the strongest one. That is, two equivalent queueing networks in performance measure "queue length probability" yield the same expected queue length and throughput. But the reverse is not true.

The exponentialization procedure is to obtain an exponential network that is equivalent to the given general network. In other words, it is to obtain a set of equivalent (approximate) exponential service rates, $\mu=\{\mu_i,\ i=1,2,...,N\}$, of the exponential network by analyzing general network through decomposition[3]. Specifically, an important performance measure such as queue length probability, expected queue length and throughput etc. is obtained for every two corresponding nodes in each network (the isolated node is analyzed for the given general network), and then by equating one to another for each station i, we get a set of N nonlinear equations of N unknown variables μ_i, $i=1,2,..,N$. Then we get a system of nonlinear equations. In this paper we use an expected queue length as a performance measure.

5. Exponentialization and Nonlinear Equation System

The idea of the exponentialization is to approximately transform the general network into an exponential network. Specifically we want to find a set of "equivalent service rates" μ that satisfies following system of nonlinear equations.

$$L_i^e(\mu)=L_i^g(\lambda_i(\mu)) \quad \forall i=1,2,...,N \tag{1}$$

where $L_i^e(\mu)$: expected queue length of station i in the exponential network which is characterized by the service rate vector μ.

$L_i^g(\lambda(\mu))$: expected queue length of isolated station i in the general service time, $(\mu_i,\ \sigma_i^2)$, and Poisson arrival with rate $\lambda_i(\mu)$.

$\lambda_i(\mu)$: Poisson arival rate at isolated station i under the condition that the complement of station i forms exponential network which is characterized by μ.

5.1 Expected Queue Length of the Exponential Network, L_i^e

Let α_i be the net exponential arrival rate at station i of the exponential network. Then α_i can be calculated as follows:

Fact 1 [8] The following equation system

$$\alpha_i(\mathrm{K})=\theta_i(K)+\sum_{j=1}^{N}\alpha_j(K)\ r_{ij} \quad \forall i=1,\ 2,\cdots,N \tag{2}$$

3) Each queue in the general network is analyzed separately interfaced with the exponential network.

has a unique solution $\alpha(K)=(\alpha_1(K),\ \alpha_2(K),\cdots,\alpha_N(K))$ with $\alpha_i(K)>0\ \forall i=1,2,\cdots,N$. In vector notation, $\alpha(K)=\Theta(K)+\alpha(K)\Gamma$ has a solution $\alpha(K)=\Theta(K)(I-\Gamma)^{-1}$.

Hereafter we use α_i for $\alpha_i(K)$. Let $n_i,\ \forall i=1,2,\cdots,N$ denotes the number of customer at station i, and $K=\sum_{i=1}^{N} n_i$ denotes the total number of customers in the central storage. From the balance equation, we obtain the equilibrium state probability vector π of exponential network as follows:

Fact 2 [1] For an open queueing network with exponential service time and limited storage capacity C, the joint equilibrium state probability is given by

$$\pi(n,\ \mu)=\ A(C)^{-1}\ \theta^K\ \prod_{i=1}^{N}(\alpha_i/\mu_i)^{n_i} \tag{3}$$

where $n=(n_1,\ n_2,\ldots,n_N)$ is a state vector of the network and $A(C)$ is chosen so that the probabilities with $K=1,2,\ldots,C$ sum to 1.

Let $P(K)$ denote the probability of finding K jobs in the system. Then $P(K)$ is obtained by summing $\pi(n,\ \mu)$ over n_i with $\sum_{i=1}^{N} n_i=K$. We get,

$$P(K)=\theta^K q(K)/A(C),\ \forall K=0,1,\cdots,C \tag{4}$$

where $q(K)=\sum_{S}\prod_{i=1}^{N}(\alpha_i/\mu_i)^{n_i}$

$$S=\{(n_1,\ n_2,\cdots,n_N)\mid \sum_{i=1}^{N} n_i=K,\ n_i\geq 0\ \forall i=1,2,\cdots,N\}$$

Since $\sum_{K=0}^{C} P(K)=1$, we get the normalizing constant

$$A(C)=\sum_{m=0}^{C}\ \theta^m q(m)\ (\because\ 1=\sum_{K=0}^{C}\ P(K)=\sum_{K=0}^{C}\ \theta^K q(K)/A(C)) \tag{5}$$

The simple recursive technique used by Buzen[3] can be applied to calculate the normalizing constant $A(C)$. It is simple to get the marginal distribtion, $\pi_i(n_i,\ \mu)$ from the joint distribution, $\pi(n,\ \mu)$. Define

$$S'=\{(n_i,\cdots,n_{i-1},n_{i+1},\cdots,n_N)\mid \sum_{j=1,j\neq i}^{N} n_j=K,\ 0\leq K\leq C-n_i,\ n_j\geq 0\ \forall j\}$$

Summing $\pi(n,\ \mu)$ over all states except the station i, we get

$$\pi_i(n_i,\ \mu)=\sum_{S'}\pi(n,\ \mu)$$

$$=A(C)^{-1}\ \theta^{n_i}(\alpha_i/\mu_i)^{n_i}\sum_{S'}\ \prod_{k=1,k\neq i}^{N}\ \theta^{n_k}(\alpha_k/\mu_k)^{n_k} \tag{6}$$

Letting $q_i(K)=\sum_{S'}\ \prod_{k=1,k\neq i}^{N}\ \theta^{n_k}(\alpha_k/\mu_k)^{n_k}$ and $A_i(K)=\theta^K q_i(K)$, then

$$\pi_i(n_i,\ \mu)=\theta^{n_i}\ (\alpha_i/\mu_i)^{n_i}\ A_i(C-n_i)/A(C)\ \forall n_i=0,1,2,\cdots,C \tag{7}$$

Now $L_i^e(\mu)$ is calculated as follows:

$$\begin{aligned} L_i^e(\mu) &= 1+\sum_{k=1}^{C} k\pi_i(k,\ \mu) \\ &= 1+\sum_{k=1}^{C} k\theta^k(\alpha_i/\mu_i)^k\ A_i(C-k)/A(C) \\ &= 1+\theta^C \sum_{k=1}^{C} k(\alpha_i/\mu_i)^k\ q_i(C-k)/A(C) \\ &= 1+\theta^C \sum_{k=1}^{C} k(\alpha_i/\mu_i)^k\ q_i(C-k)/\sum_{k=0}^{C} \theta^k\ q(k) \\ &= 1+\theta^C \sum_{k=1}^{C} k(\alpha_i/\theta\mu_i)^k\ (q_i(C-k)/q(k))\ \forall i=1,2,\cdots,N \end{aligned} \tag{8}$$

5.2 Expcted Queue Length of the General Network, L_i^g

Since the complement of station i forms an exponential network, the arrival process at the isolated staion i is Poisson, The throughput of station j, TH_j is defined as the number of service completion in a unit time. Then,

$$\lambda_i(\mu)=\sum_{j=1,j\neq i}^{N} r_{ji}TH_j+r_{0i}\theta(K)=\sum_{j=1,j\neq i}^{N} r_{ji}\ \mu_j+\theta_i(K) \tag{9}$$

Now the isolated station i can be considered as an M/G/1 queue with arrival rate $\lambda_i(\mu)$ and mean service time v_i. From the Pollaczek-Khintchine formula[14], we get

$$L_i^e(\lambda_i(\mu))=\rho_i+\rho_i^2\ ((\sigma_i^2/v_i^2)+1)/(2(1-\rho_i))\ \forall i=1,2,\cdots,N \tag{10}$$

where $\rho_i=\lambda_i(\mu)v_i$. From (8) and (10) we now have a nonlinear equation system as follows:

$$L_i^e(\mu)=L_i^g(\lambda_i(\mu)),\ \forall i=1,2,\cdots,N \tag{11}$$

6. Existence of Solution and Algorithm

Consider the following nonlinear equation system:

$$F(\mu)=L^g(\lambda(\mu))-L^e(\mu)=0,\ \mu\in X \tag{12}$$

where

$F(\mu)=\{F_i(\mu),\ i=1,2,\cdots,N\}$

$L^g(\lambda(\mu))=\{L_i^g(\lambda(\mu)),\ i=1,2,\cdots,N\}$

$L^e(\mu))=\{L_i^e(\mu),\ i=1,2,\cdots,N\}$

Since μ_i is a service rate of station i, it is natural to assume that the rate is in closed and bounded subset of R_+^N excluding 0 (it is convenient to confine the feasible set to the compact subset of R_+^N because the service rate can be neither 0 nor infinite). In fact, X is polyhedral in R_+^N in practice.

Letting the solution space X be the compact and convex subset of R^N, then we can define a variational inequality problem, $VI(X, F)$ as follows[13]:

$$F(\mu^*)^T(y-\mu^*)\geqslant 0,\ \forall y\in X \tag{13}$$

where $\mu^*\in X$ and $F{:}X\rightarrow R^N$. Since $F(\mu)$ is continuous mapping from X into R^N and X is a nonempty, compact and convex subset of R^N , there exists a solution to the problem (13)[10]. From the following theorem we can characterize the solution to the nonlinear equation system (12).

Fact 3 [13] Let μ^* be a solution to (13) and suppose that $\mu^*\in int(X)$, the interior of X. Then it is a solution of (12).

To solve the $VI(X, F)$, we use the linearized Jacobi, one of symmetric linear approximation methods. The motivation behind it is that each subproblem can be cast as an optimization problem with a separable quadratic objective function, which is easily solved by robust optimization software such as MINOS and GAMS etc.

Stronger properties on the mapping F for convergence[4] and low convergence rate are major problems in this linear approximation method. Now we describe the outline of solution algorithm to solve (12) using linearized Jacobi as follows:

Step 1) By choosing an arbitrary large number as an upper bound of service rate, the compact and convex solution X is constructed.

Step 2) Generate a sequence $\{\mu^k\}\subseteq X$ such that each μ^{k+1} solve problem $VI(X, F^k)$:

4) For the convergence of linearized Jacobi method, we need a condition in Theorem 4.2 (b)[10]. A stronger condition so called "diagonally dominant" is in proposition 4.3[10]. We know that the more diagonally dominant $\nabla F(\mu^*)$ is, the better the method can be expected to perform.

$$F^k(\mu^{k+1})^T(y-\mu^{k+1}) \geqslant 0,\ \forall y \in X \tag{14}$$

where $F^k(\mu)=F(\mu^k)+D(\mu^k)(\mu-\mu^k)$ and $D(\mu^k)$ is the diagonal part of $\nabla F(\mu^k)$. Actually, the subproblem $VI(X,F^k)$ can be solved by the follwing NLP:

$$Min_{\mu\in X}\ [F(\mu^k)-D(\mu^k)\mu^k]^T\ \mu + \frac{1}{2}\ \mu^T\ D(\mu^k)\mu \tag{15}$$

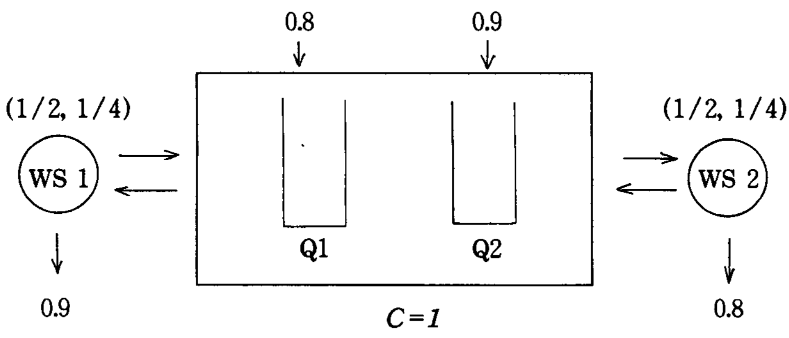
Step 3) If $\max_i(\mu_i^{k+1}-\mu_i^k) \leqslant \varepsilon$, then go to Step 4).
otherwise, go to Step 2).

Step 4) If μ^{k+1} is in interior of X, then it is a solution to (12). Otherwise we conclude that (12) does not have a solution.

7. Example

We solve a simple example to show the exponentializtion procedure and its solution algorithm. Consider the following general FMS network.

Figure 2. Example



Example) $N=2,\ C=1$

$(\nu_1,\ \sigma_1^2)=(\nu_2,\ \sigma_2^2)=(0.5,\ 0.25)$

$\Theta=(\theta_1,\ \theta_2)=(0.8,\ 0.9)$

$$\Gamma=\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix}$$

Then $\alpha=(\alpha_1,\ \alpha_2)=\Theta(I-\Gamma)^{-1}=(1,\ 1)$

$$q(0)=1,\ q(1)=(\frac{1}{\mu_1}+\frac{1}{\mu_2})$$

$$q_1(0)=1,\ q_1(1)=(\frac{1}{\mu_2})$$

$$q_2(0)=1,\ q_2(1)=(\frac{1}{\mu_1})$$

From (8),

$$L_1^e(\mu)=1+\theta(\frac{\alpha_1}{\theta\mu_1})\ (\frac{q_1(0)}{q(1)})=1+(\frac{\mu_2}{\mu_1+\mu_2})$$

$$L_2^e(\mu)=1+\theta(\frac{\alpha_2}{\theta\mu_2})\ (\frac{q_2(0)}{q(1)})=1+(\frac{\mu_1}{\mu_1+\mu_2}) \tag{16}$$

From (10),

$$\lambda_1(\mu)=0.2\mu_2+0.8$$

$$\lambda_2(\mu)=0.1\mu_1+0.9$$

$$\rho_1=0.1\mu_2+0.4$$

$$\rho_2=0.05\mu_1+0.45$$

$$L_1^g(\lambda_1(\mu))=\frac{4+\mu_2}{6-\mu_2}$$

$$L_2^g(\lambda_2(\mu))=\frac{9+\mu_1}{11-\mu_1} \tag{17}$$

From (16) and (17), we get the following nonlinear equation system.

$$F_1(\mu)=\frac{4+\mu_2}{6-\mu_2}-1-\frac{\mu_2}{\mu_1+\mu_2}=0$$

$$F_2(\mu)=\frac{9+\mu_1}{11-\mu_1}-1-\frac{\mu_1}{\mu_1+\mu_2}=0 \tag{18}$$

$$\nabla F(\mu)=\begin{bmatrix} \frac{\mu_2}{(\mu_1+\mu_2)^2}, & \frac{10}{(6-\mu_2)^2}-\frac{\mu_1}{(\mu_1+\mu_2)^2} \\ \frac{20}{(11-\mu_1)^2}-\frac{\mu_2}{(\mu_1+\mu_2)^2}, & \frac{\mu_1}{(\mu_1+\mu_2)^2} \end{bmatrix} \tag{19}$$

Choosing arbitrary large numbers, say 10E+10's, as upper bounds of $\mu=(\mu_1, \mu_2)$, we can define a compact and bounded solution space,

$$X=\{\mu \mid \mu_i \in [0,10E+10]\ \ \forall i=1,2\} \tag{20}$$

Then the NLP in Step 2) is as follows:

$$Min_{\mu\in X}\ [\ \frac{4+\mu_2^k}{6-\mu_2^k}-1-\frac{\mu_2^k}{\mu_1^k+\mu_2^k}-\frac{\mu_1^k\ \mu_2^k}{(\mu_1^k+\mu_2^k)^2}\ ,\quad \frac{9+\mu_1^k}{11-\mu_1^k}-1-\frac{\mu_1^k}{\mu_1^k+\mu_2^k}-\frac{\mu_1^k\ \mu_2^k}{(\mu_1^k+\mu_2^k)^2}\ ]^T \begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix} + \frac{1}{2}(\mu_1,\ \mu_2)\begin{bmatrix}\frac{\mu_2^k}{(\mu_1^k+\mu_2^k)^2} & 0\\ 0 & \frac{\mu_1^k}{(\mu_1^k+\mu_2^k)^2}\end{bmatrix}\begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix} \qquad (21)$$

Using MINOS 5.0 and MATMOD routine with initial point $\mu^0=(\mu_1^0,\ \mu_2^0)=(2,2)$, we get an equivalent exponential network characterized by solution $\mu^*=(\mu^*_1,\ \mu^*_2)=(3.5,1.7)$ which is in interior of X. The expected queue lengths of station 1 and 2 in the central buffer are 0.33 and 0.67 respectively. And the expected throughput of the system is 4.51 ($=3.5*0.9+1.7*0.8$).

8. Conclusion

We developed a framework of exponentalization procedure for the general network with limited central buffer. The concept of equivalent network is crucial in this procedure because users may be interested in different performance measures. Another important thing is to probe the existence of equivalent (especially exponenttal) network.

By imposing a reasonable assumption on the solution set, the nonlinear equation system for the exponentialization procedure was formulated as a variational inequality problem $VI(X,F)$ and the solution existence was examined.

The efficient algorithm for the nonlinear equation system was developed using linearized Jacobi approximation method. Furthermore, each subproblem could be cast as an optimization problem with a separable quadratic objective function, which is easily solved by MINOS 5.0 using MATMOD. Newton's method, one of the most powerful approximation methods, which needs weaker conditions for convergence can be applicable to guarantee higher rate of convergence.

The computations of large scale problems are needed for the evaluation of the exponentialization procedure and its algorithm.

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