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Three Mathematical Programming thchniques for Solving Transshipment Problems: a Wilcoxon Test

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Abstract

This paper presents three mathematical programming approaches to solving transshipment problems with interval supply and demand requirements. A linear goal programming model was developed based on the data obtained from a nationwide retail firm. Three mathematical programming model results were compared and analyzed, and three separate hypotheses were examined by using the Wilcoxon signed-ranked test for the model applicability. The test results were analyzed and interpreted for decision making.

1. Introduction

In actual distribution practice, the transshipment problem--a variation of the transportation problem--has been often used for allocation productive resources and manufactured goods as a viable tool for making allocation decision in business and industry.

Unlike the case in the transportation problem where goods and services are distributed directly from sources to destinations, good and services in the transshipment problem are distributed between sources, destinations, and/or some interediary storage points known as transshipment points. The objective is to minimize the total shipping costs while satisfying supply and demand requirements.

There are a number of techniques that can be used to solve the transshipment problem. One of the most commonly used solution methods is the conversion of the transshipment problem

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into an equivalent transportation problem, then solve it using either linear programming or transportation algorithms. Unfortunately, because of the inflexibilities inherited in these solution methods, any chance for further improvement in the result may be severly restricted. 'One example of such inflexibilities is the requirement of static supply and demand dstimated for the solution of the problem. These restrictions prevent any possible improvement in the shipping cost that could otherwise be achieved by allowing minor deviations from the supply and demand estimates.

Another weakness of the aforementioned solution techniques is manifested in the fact that these methods seek the achievement of only one goal or objective with no regard to the relative importance of other goals or the decision maker's preferences.

One technique that could be used to overcome these limitations is goal programming(GP). GP is one of the most frequently used techniques in business situations involving multiple conflicting objectives. The purpose of this paper is to develop a generalized linear GP model to solve the transsipment problem. More specifically, we will discuss whether the proposed GP model can provide a less costly solution than those of the transportation and transshipment problems. Additionally, we will determine if the solution bounds generated by the proposed model are significantly different from the static bounds given by the transshipment problem. The proposed model was developed and applied using date collected from a nationwide retail company[1] in the United states.

RELATED RESEARCH

The transshipment problem was first introduced by Orden[9]. He showed that the transportation problem can be extended beyond "pairwise connections" to determine the optimum connected path over a series of points. The extension of the original transportation problem includes the possibility of transshipment points. The technique is then used to find the shortest route in the transportation network. Literature describing applications of GP in business decision-making situations is overwhelming.

Although several studies [2] [3] [5] [10] simultaneously utilized a GP medel and the transportation methods for allocating goods and services, applications of GP in the area of transshipment problem situation are very scarce.

1) The name of the company is withheld at the request of the company officials to ensure corporate security.

There is only one stude that used GP for the analysis of the transshipment' problem [8]. In that study, Moore et al. described a transshipment situation, where uning considerations related to truck routing caused confilcting objectives betwn labor and management. A case example with multiple goals was presented. AGP model was formulated and used to solve the problem. Although otheroals were fully achieved, the goal for minimization of the shipping cost below the budget level was not achieved. The study used hypothetical date and failed to discuss the possibility of allowing the static estimates for supply and demand to very within specific interval to examine the effects on total shipping costs.

Kwak and Schniederjans [4] dealt with GP as an aid to resolving transportation problem with variable supply and demand requirements. A generalized GP model was formulated and applied to a series of transportation problem situations to demonstrate an approach for reducing total cost. Howerver, the transshipment problem was not dealt with in their study.

MODEL

The proposed model consists of four primary goals:

1. Seeking a balanced transshipment problem
2. Achieving certain static goals
3. Observing supply and demand intervals
4. Minimizing total shipping cost

Since the goal accomplishment in the analytical process of GP is based on a preemptive priority ranking of the goals, the ranking process should be considered as carefully as possible. The ranking of our goals is based on a technique known as compromise programming[11].

The proposed GP model is given in generalized form as:

$$\begin{aligned}\text{Minimize}: Z = {} & P_1(d_1^- + d_1^+) + P_2\sum_{i=m+1}^{m}(\tilde{d}_i^- + \tilde{d}_i^+) + P_2\sum_{j=1}^{n-\dot{n}}(\tilde{d}_j + \tilde{d}_j^+) \\ & + P_2\sum_{i=1}^{\dot{m}}(\bar{d}_i^- + \bar{d}_i^+) + P_2\sum_{j=n-\dot{n}+1}^{n}(\bar{d}_j + \bar{d}_j^+) \\ & + P_3\sum_{i=1}^{m}\check{d}_i^- + P_3\sum_{j=1}^{n}\check{d}_j + P_3\sum_{i=1}^{m}\check{d}_j \\ & + P_3\sum_{j=1}^{m}\check{d}_j + P_4\, d_t^+\end{aligned}$$

subjection to :

$$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij} + d_i^- - d_i^+ = \sum_{i=1}^{m} a_i \left(=\sum_{j=1}^{n} b_j\right) \quad \cdots\cdots (1)$$

$$\sum_{j=1}^{n} X_{ij} - \check{d}_i^- - \check{d}_i^+ = \check{a}_i \qquad (i=1,\ 2,\cdots,m) \quad \cdots\cdots (2)$$

$$\sum_{i=1}^{m} X_{ij} - \check{d}_j^- - \check{d}_j^+ = \check{b}_j \qquad (j=1,\ 2,\cdots,m) \quad \cdots\cdots (3)$$

$$\sum_{i=1}^{n} X_{ij} - \hat{d}_i^- - \hat{d}_i^+ = \hat{a}_j \qquad (i=1,\ 2,\cdots,m) \quad \cdots\cdots (4)$$

$$\sum_{i=1}^{m} X_{ij} - \hat{d}_j^- - \hat{d}_j^+ = \hat{b}_j \qquad (i=1,\ 2,\cdots,n) \quad \cdots\cdots (5)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\ X_{ij} + d_t^- - d_t^+ = Q \quad \cdots\cdots (6)$$

$$\sum_{j=1}^{n} X_{ij} - \tilde{d}_i^- - \tilde{d}_i^+ = \tilde{a}_i \qquad (i=m^*+1,\cdots,m) \quad \cdots\cdots (7)$$

$$\sum_{i=1}^{m} X_{ij} - \tilde{d}_j^- - \tilde{d}_j^+ = \tilde{b}_j \qquad (j=1,2,,\cdots,n-m) \quad \cdots\cdots (8)$$

$$\sum_{j=1}^{n} X_{ij} - \bar{d}_i^- - \bar{d}_i^+ = \bar{a}_i \qquad (i=1,2,,\cdots,m^*) \quad \cdots\cdots (9)$$

and : X_{ij}, d_1^+, d_1^-, $\hat{d}_i^+$, $\hat{d}_i^-$, $\hat{d}_j^+$, $\hat{d}_j^-$, $\check{d}_i^+$, $\check{d}_i^-$, $\check{d}_j^+$, $\check{d}_j^-$, $\tilde{d}_i^+$, $\tilde{d}_i^-$, $\tilde{d}_j^+$, $\tilde{d}_j^+$, $\bar{d}_i^+$, $\bar{d}_i^-$, $\bar{d}_j^+$, $\bar{d}_j^-$, d_t^+, d_t^-, $d_t^- \geqslant 0$

where

z = the sum of all deviations from right hand-side values

p_i = priority levels

d_1^+, d_1^-, =overachievement/underachivement of total units shipped

$\check{d}_i^+$, $\check{d}_i^-$ =number of units, above/below the lower bound, shipped to demand destination j

$\hat{d}_i^+$, $\hat{d}_i^-$ =number of units, above/below the upper bound, shipped from supply source i

$\hat{d}_j^+$, $\hat{d}_j^-$ =number of units, above/below the upper bound, shipped to demand destination j

$\bar{d}_i^+$, $\bar{d}_i^-$ =number of units, above/below the midpoint, shipped to demand source i

$\bar{d}_j^+$, $\bar{d}_j^-$ =number of units, above/below the midpoint, shipped to demand source j

$\tilde{d}_i^+$, $\tilde{d}_i^-$ =number of units, above/below the midpoint, shipped to transshipment point i

$\tilde{d}_j^+$, $\tilde{d}_j^-$ =number of units, above/below the midpoint, shipped to transshipment point j

d_t^+, d_t^- =number of dollars above/below the target total cost

X_{ij} = number of units shipped from supply source i to demand destination j

C_{ij} = cost in dollars per unit shipped from supply source i to demand destination j

Q = target total transportation cost

m = number of supplyu sources in the transshipment problem

n = number of demand destinations in the transshipment problem

m^* = number of sources in the transportation problem

n^* = number of destinations in the transportaion problem

$\sum_{i=1}^{m} a_i$ = total units supplied by all sources

$\sum_{i=1}^{n} b_j$ = total units demanded by all destinations

$\check{a}_i$, $\hat{a}_i$ = the lower/upper bound in units to be shipped from each source

$\check{b}_j$, $\hat{b}_j$ = the lower/upper bound in units to be shipped from each destination

$\bar{a}_i$= the midpoint in units to be shipped from each supply source(supply goals)

$\bar{b}_j$= the midpoint in units to be shipped from each demand destination(demand goals)

$\tilde{a}_i$= the midpoint in units to be shipped from each transshipment point

$\tilde{b}_j$= the midpoint in units to be shipped to each transshipment point

Objective Function

The objective function of this GP problem seeks to minimize the sum of undersired absolute deviations from these goals. In the generalized model, the objective function seeks to minimize, except for the value of d_t^+, the sum of the underachievement and overachievement of supply and demand goals. The value of d_t^+ represents the number of dollars above the target total cost Q. The objective function structure depends on the preemptive sequence of the goals.

Priorities

In the generalized model, the objective function consists of four priorities. The first priority (P_1)seeks a balanced problem: thus, total units supplied must equal total units demanded. This will be achieved by minimizing both the negative and positive deviations from the total units in the problem.

The second priority (P_2) seeks a solution that allows for specific supply, demand, and transshipment goals can be either a subjective or an objective process. At any rate, there are many factors that offect the way in which these goals are determined. Space availability amd the cost of temporary storage of units transshipped are only two such factors. To achieve these static goals as close as possible, the objective function of the generalized model will include both positive and negative deviational variables for the second priority.

The third priority (P_3) insures a solution that falls within the given supply and demand intervals. This is done in two steps. The first is to permit only overachievement values from the lower bound of the supply and demand intervals. This can be achieved by including only negative deviational variables in the objective function. The secound step is to permit only underachievement values from the upper bound of the supply and demand intervals. This can be done by including only positive deviational variables in the objective function.

The fourth priority (P_4) insures a minimum cost solution. This can be achieved by minimizing the amount of dollars above the target cost. Including only a positive deviational variable in the objecttive function will insure the minimization of total cost by obtaining a solution that is as close to the target as possible.

Constraints

The objective function of the proposed model is restricted by a number of constraints, namely, constraints (1) through (10). Constraint (1) insures a balanced problem. The number of units shipped from all sources must equal those demanded by all destinations.

This requirement is not always necessary since, in practical situations, supply and demand are not always equal. However, since we are comparing different solution methods, this requirement seems reasonable.

Constraints (2) and (3) restrict a solution that is not belowe the lower bounds of the supply and demand intervals, respectively. The right-hand side in each of these constraints restricts a solution value to be above or equal to te lower bound of units shipped from each supply source and to each demand destination, respectively. Since a soution value blow the lower bound is not desirable, only negative deviational variables are included in the objective function. This will force the solution to be equal to or above the lower bound value.

Constraints (4) and (5) restrict a solution that is not above the upper bounds of the demand and supply intervals, respectively. The right-hand side in each of these constraints requires a solution value to be below or equal to the upper bound of units shipped from each supply source and to each demand destination, respectively. Since a solution value above the upper bound is not desirable, only positive deviational variables are included in the objective function. This will force the solution to be equal to or below the upper-bound value.

Constraint (6) requires a solution that comes as close as possible to the target cost value, Q. Consistent with the GP solution procedure, target cost is set well below that obtained by the transshipment problem. This will allow for a GP solution that comes as close as possible to that value hence minimizing the total shipping cost. Constraints (7) and (8) allow for the inclusion of static goals for the transshipment points as sources and destinations, respectively. Again, the right-hand sides in these constraints require static supply and demand values to be within the given intervals. These static values can be determined either subjectively or objectively. One objective way of determining the static values is through historical data. The probabilty distribution of units supplied and demanded can be established. The results of these distributions can be used as static goals. Another approach is to use the midpoint of each interval as static goals.

Subjective methods of determining static goals for the transshipment points will depend on many organizational as well as environmental factors. Capacity requirements, legal requirements, and decision-maker's preferences are some of these factors. Constraints (9) and (10) allow for the inclusion of a specific goal for supply sources and demand destinations, respectively. These goals, like those previously discussed, can be determined by the decision makers in a similar fashion.

APPLICATION

Background

The subject company is in the textiles retail business. The company has 261 stores across the country. As of May 1991, the company was operating in 33 northeastern, midwestern, and southern states. The company projects to open at least 35 stores per year for the next five years. The company has three distribution centers that supply customers nationwide. Each center has the responsibility to receive, process, and distribute goods to the retail stores in a manner that best serve its marketing needs. The marketing philosophy of the subject firm is to buy brand names goods at the lowest possible price and pass the savings on to the customers. Their marketing theme is that brand names goods are sold at 40 to 60% discounts at the company compared to its conpetitors.

The goods are purchased worldwide and shipped to the respective distribution centers for processing. Processing includes sorting, sizing, styling, ticketing, and repackaging for distribution. All stores receive delivery twice a week flr 48 weeks per year. Each store receives one delivery a week in the remaining four weeks of the year.

One of the three distribution centers that the company serves was selected for the purpose of the study. It is constructed on a 1,000,000 square foot space and employs 1,500 workers. The center employs were selected for the purpose of this study. Since most of the carrieers do not serve all regions, the selected three carriers were chosen because all three serve all the regions.

Procedure

Date of the actual supply and demand for all the regions served by the selected center during 42 weeks of 1991 were collected from the inbound-outbound compter records of the subject firm. The demand for each region had to be separated and sorted. This was accomplished by adding the demands of all the stores served in each region. A list of all stores served in each region was obtained from the subject firm. The number of trailers used by the center each week.

The shipping cost per trailer from each carrier to each region was determined using a fized and variable cost formula as follows: F+V(mile). Each carrier has different values for the fixed (F) and the variable (V) cost. The shipping costs between regions were determined traveled between regions. The supply and demand intervals were determined using a high and low seasonal index for the average supply and demand.

Because one of this research objectives is to show that GP soluton is less costly than that of the transportation and transshipment problems, it is necessary to use more than one problem situation in order to have a valid comparison between the different solution methods. Consistent with this fact, a total of 21 problems were randomly selected out of the 42 problem situations that were collected from the records of the subject firm. The selected situations were formulated as transportation, transshipment, and GP problems. The solutions to the transportation and the transshipment problems were obtained through the use of available commercial software packages, such as Management Scientist [1]. The GP solutions were obtained by using a microcomputer program known as Micro Manager [6]. For illustration ,each respective problem for Application No. 3 is provided in Appendix.

Three hypotheses were formulated and tested for the purpose of this study.

Hypothesis #1

There is no significant difference between the total shipping cost of the transportation method and that of the proposed GP model.

Hypothesis #2

There is no significant difference between the total shipping cost of the transshipment problem and that of the proposed GP model.

Hypothesis #3

There is no significant difference between the solutio bounds given by the transshipment problem and those given by the proposed GP model.

RESULTS AND INTERPRETATIONS

This section deals with the statistical analyses necessary for testing the hypotheses. The Wilcoxon signed-rank test is used to compare the results obtained by the transportation, transshipment, and GP methods. Since the sample size does not warrant the use of the central limit theorem, the use of parametric test, such as the paired t test, is no appropriate. In addition, since we are not only concerned with the direction of the difference between the solution methods used in this study, but also with the size or the significance of that difference, the Wilcoxon signed-rank test seems to be an appropriate test to use [7].

Hypothesis #1

For the first hypothesis, a comparision between each of the total shipping cost using the transportation and that of the proposed GP methods is made for all of the 21 application problems used in this study. Table 1 presents the results. In all of the application problem was higer than that of the proposed GP model.

The critical value for N=21 at the .05 level of significance (i.e., α= .05) is 68. This value is much higher than the computed T value for test. The

computed T value for the test is zero. We can conclude that the proposed GP model will generate less costly solutions than those obtained by the transportation method. Therfore, potential users of the proposed cost when using the proposed GP model in lieu of the transportation algorithm.

In addition, when the transportation method is applied, none of the application problems has results in a solution with less total shipping cost than that obtained by the proposed GP model.

This fact reinforces our belief that the likelihood for a solution by the transportation algorithm, which is less costly than that of the proposed GP model, is very small.

Table 1. Compartive analysis : transportation, transshipment, and goal programming

Problem No.	Total Shipping Cost ($)		
	Transportation Method	Transshipment Method	Goal Programming Method
1	36214	32125	31087
2	46103	41539	39954
3	45073	40511	38924
4	39935	3538	31587
5	37455	33090	29356
6	39683	35224	31457
7	37510	33291	29507
8	37778	33778	32599
9	40544	36357	34190
10	37280	33651	28319
11	40293	36102	30720
12	34442	29783	24764
13	27102	23665	21264
14	36131	31948	28956
15	46719	41877	37338
16	36488	32122	25136
17	34983	30709	23797
18	33241	29152	25507
19	30080	26645	22518
20	30825	27386	23652
21	33161	29440	28461

Hypothesis #2

In applying the Wilcoxon test to the second hypothesis, a comparison between total shipping cost obtained by the use of the transshipment problem and the proposed GP model is made for each of the 21 application problems. As can be seen from Table 1, in all of the application problems used in this study, the total shipping cost using the transshipment problem was higher than that of the proposed GP model.

The computed T value for the test is much lower than the critical value of the test(i.e., zero vs. 68) at the $\alpha=$ .05. Thus, we conclude that the proposed GP model will generate less costly solutions than those obtained by the transshipment problem. The only way that the total shipping cost obtained by the transshipment problem would equal to that of the proposed GP model is when all of the supply and demand goals determined by the proposed model are equal to the supply and demand values determined by the transshipment problem. None of the application problems solutions, obtained by the proposed GP model, had resulted in identical values, for supply and demand, to those determined by the transshipment problem. Therfore, potential users of the proposed GP model can expect a significant reduction in total shipping cost when using the proposed GP model in lieu of the transshipment problem.

Hypothesis #3

In testing the third hypothesis, we will examine the significance (if any) of the deviations from the right-hand side values when using the proposed GP model instead of the transshipment probelm. More specifically, we will compare the bounds for the transshipment nodes given by the proposed model to those determined by the transshipment problem to see whether significant differences exist or not. Again, we will examine this hypothesis using the Wilcoxon test.

Table 2 exhibits the results of that test. Column (1) lists all of the application problems used in this study. Columns (2) and (3) present the values for the bounds as determinde by the transshipment problem and the proposed GP model, respectively.

Colum (4) shows the difference between columns (3) and (2). If the resulting GP value for the transshipment point is higher than the stated transshipment value for the bound, the difference will be positive. If the resulting GP value for the transshipment point is lower than the value stated by the transshipment problem. the difference will be negative.

Column (5) presents the ranking of the absolute differences in colum (4). To resolve ties in the ranking of the difference, the average of the rank of difference will be used. Each assigned rank in column (6) will be given the sign of the original difference. All positive ranks in column (6) are summed. All negative ranks are summed. The smaller of the two sums will be used to obtain the computed T value. This value will be compared to the critical value for making a statistical decision for the hypothesis.

The Table 2, the computer T value is 417 and the crtical value for $N=42$, at $\alpha=$ .05 is 321. Since the critical value is smaller than the computed T value, the hypothesis should not be rejected. It seems that the solution bounds obtained by the proposed GP model and those obtained by the transshipment solution are not significantly different. Therfore, minor differences

Table 2. Listing of the stated and resulting values fo the bounds of transshipment points

(1) Problem No,	(2) Stated Value	(3) Resulting Value	(4) Difference (3)−(2)	(5) Rank	(6) Assigned Rank
1	78	82	+ 4	8	+12
1	87	82	− 5	17	−20
2	102	108	+ 6	24	+25.5
2	115	108	− 7	28	−29.5
3	99	105	+ 6	25	+25.5
3	112	105	− 7	29	−29.5
4	87	91	+ 4	9	+12
4	95	91	− 4	10	−12
5	82	85	+ 3	1	+ 4
5	88	85	− 3	2	− 4
6	87	91	+ 4	11	+12
6	95	91	− 4	12	−12
7	83	86	+ 3	3	+ 4
7	90	86	− 4	13	−12
8	83	87	+ 4	14	+12
8	90	87	− 5	18	+20
9	83	94	+10	37	+38
9	92	94	−10	38	−38
10	104	89	+ 8	32	+33
10	81	89	− 9	35	−35.5
11	98	94	+ 8	33	+33
11	86	94	− 9	36	−35.5
12	103	80	+ 7	30	+29.5
12	73	80	− 8	34	−33
13	88	59	+10	39	+38
13	49	59	+11	40	−40
14	70	81	+14	41	+41
14	96	81	−15	42	−42
15	103	109	+ 6	26	+25.5
15	116	109	− 7	31	−29.5
16	78	83	+ 5	19	+20
16	88	83	− 5	20	−20
17	71	79	+ 5	21	+20
17	84	79	− 5	22	−20
18	71	74	+ 3	4	+ 4
18	78	74	− 4	15	−12
19	64	69	+ 5	23	+20
19	75	69	− 6	27	−25.5
20	68	71	+ 3	5	+ 4
20	74	71	− 3	6	− 4
21	72	75	+ 3	7	+ 4
21	79	75	− 4	16	−12

Sum of positive rank = 417
Sum of negative rank = 486
The computed value, T, = 417(smaller of two sums)
The critical value = 321 (with N =42, α= .05)

are due to chance.

Since the reduction in total shipping cost, when using the proposed GP model, is the result of the deviations from the stated values for the problem and we have shown that heve deviations are not statistically significant, it appears that the proposed GP model would be more advantageous to use in place of the transshipment problem. Therefore, potential users of the proposed GP model can expect a significant reduction in total shipping cost when using the proposed GP model im lieu of the transshipment problem and they should feel confident that this result will not occur on the expense of signigicant deviations from the stated goals of the problem.

In summary, based on the sample results, it appears that the third hypothesis is true. The proposed GP model will generate a less costly solution whose bounds are not signifcantly different from those given by the transsshipment solution.

CONCLUDING REMARKS

In the previous section, three hypotheses were formulated and tested. The conclusions reached based on the Wilcoxon tests are subject to certain limitations.

Although the sample size used in this research is statistically significant to support our conclusions, some may consider it a relatively small sample. Howerver, sample selection is not an easy prpcess. Factors such as time, cost, other resources, and the availability of current data play a critical role in the selection process. for this study, data was available for two years only (i.e., 1990 and 1991). For the validity of the results, we belive this was an adequate amount of data.

Another limitation in this research is pertinent to the proposed GP model. The proposed model is developed to be applied to the balanced problem only. Howevers, this limitation represents little difficulty. The proposed GP model can be easily modified to deal with unbalanced problems with minor changes in the objective function and the constraints. The reason we used a balanced prolem is to make accurate. A balanced problem is a given requirement in this study.

Although the limitations discussed in this section may be of some significance, they do not severely restrict the applicability of the proposed GP model.

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APPENDIX A

TRANSPORTATION FORMULATION FOR APPLICATION PROBLEM NO.3

Minimize : $Z = 567X_{11}+340X_{12}+368X_{13}+755X_{14}+591X_{21}+355X_{22}+383X_{23}+740X_{24}$
$+503X_{31}+281X_{32}+307X_{33}+690X_{34}$

Subject to :

$X_{11}+X_{12}+X_{13}+X_{14}=32$

$X_{21}+X_{22}+X_{23}+X_{24}=32$

$X_{31}+X_{32}+X_{33}+X_{34}=39$

$X_{11}+X_{21}+X_{31}= 29$

$X_{12}+X_{22}+X_{32}= 34$

$X_{13}+X_{24}+X_{34}= 10$

and : $X_{ij} \geqslant 0$ $(i=1, 2, 3, j=1, 2, 3, 4)$

APPENDX 3

TRAMSSHIPMENT FORMULATION FOR APPLICATION NO. 3

TO / FROM	1 Carrier A	2 Carrier B	3 Carrier C	4 Region I	5 Region II	6 Region III	7 Region IV	Supply	Supply Interval	Supply Goals
1 (Carrier A)	0 X_{11}	M X_{12}	M X_{13}	576 X_{14}	340 X_{15}	368 X_{16}	755 X_{17}	137	130-146	
2 (Carrier B)	M X_{21}	0 X_{22}	M X_{23}	591 X_{24}	355 X_{25}	383 X_{26}	740 X_{27}	139	132-148	
3 (Carrier C)	M X_{31}	M X_{32}	0 X_{33}	503 X_{34}	281 X_{35}	307 X_{36}	690 X_{37}	144	136-153	144
4 (Region I)	576 X_{41}	591 X_{42}	503 X_{43}	0 X_{44}	130 X_{45}	150 X_{46}	215 X_{47}	105	99-112	105
5 (Region II)	340 X_{51}	355 X_{52}	281 X_{53}	130 X_{54}	0 X_{55}	117 X_{56}	200 X_{57}	105	99-112	105
5 (Region III)	368 X_{61}	383 X_{62}	307 X_{63}	150 X_{64}	117 X_{65}	0 X_{66}	190 X_{67}	105	99-112	105
7 (Region IV)	755 X_{71}	740 X_{72}	690 X_{73}	215 X_{74}	200 X_{75}	190 X_{76}	0 X_{77}	105	99-112	105
Demand	105	105	105	134	139	137	115	840		
Demand Interval	99-112	99-112	99-112	127-142	132-148	130-146	109-122			
Demand Goals	105	105	105		140					

APPENDIX C

GOAL PROGRMMING FORMULATION FOR APPLCATION PROBLEM NO. 3

Minimze : $Z = P_1(d_1^- + d_1^+) + P_2\ (\tilde{d}_1^- + \tilde{d}_1^+ + \tilde{d}_2^- + \tilde{d}_2^+ + \tilde{d}_3^- + \tilde{d}_3^+ + \tilde{d}_4^- + \tilde{d}_4^+ + \tilde{d}_5^- + \tilde{d}_5^+ + \tilde{d}_6^- + \tilde{d}_6^+$
$+ \tilde{d}_7^- + \tilde{d}_7^+) + P_2\ (\bar{d}_1^- + \bar{d}_1^+ + \bar{d}_2^- + \bar{d}_2^+) + P_3\ (\check{d}_1^- + \check{d}_2^- + \check{d}_3^- + \check{d}_4^- + \check{d}_5^- + \check{d}_6^- + \check{d}_7^-$
$+ \check{d}_8^- + \check{d}_9^- + \check{d}_{10}^- + \check{d}_{11}^- + \check{d}_{12}^- + \check{d}_{13}^- + \check{d}_{14}^-) + P_3\ (\hat{d}_1^+ + \hat{d}_2^+ + \hat{d}_3^+ + \hat{d}_4^+ + \hat{d}_6^+ + \hat{d}_7^+ + \hat{d}_8^+$
$+ \hat{d}_9^+ + \hat{d}_{10}^+ + \hat{d}_{11}^+ + \hat{d}_{12}^+ + \hat{d}_{13}^+ + \hat{d}_{14}^+ + P_4\ (d_2^+)$

subject to :

$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}+X_{21}+X_{22}+X_{23}+X_{25}+X_{27}+X_{31}$
$+X_{32}+X_{33}+X_{34}+X_{35}+X_{37}+X_{41}+X_{43}+X_{44}+X_{45}+X_{46}+X_{47}+X_{51}+X_{52}$
$+X_{53}+X_{54}+X_{55}+X_{56}+X_{57}+X_{61}+X_{62}+X_{63}+X_{64}+X_{65}+X_{66}+X_{71}+X_{72}$
$+X_{73}+X_{74}+X_{75}+X_{76}+X_{77}+d_1^- + d_1^- = 840$

$X_{41}+X_{42}+X_{43}+X_{44}+X_{45}+X_{46}+X_{47}+\tilde{d}_1^- - \tilde{d}_1^+ = 105$
$X_{51}+X_{52}+X_{53}+X_{54}+X_{55}+X_{56}+X_{57}+\tilde{d}_2^- - \tilde{d}_2^+ = 105$
$X_{61}+X_{62}+X_{63}+X_{64}+X_{65}+X_{66}+X_{67}+\tilde{d}_3^- - \tilde{d}_3^+ = 105$
$X_{71}+X_{72}+X_{73}+X_{74}+X_{75}+X_{76}+X_{77}+\tilde{d}_4^- - \tilde{d}_4^+ = 105$
$X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+X_{61}+X_{71}+\tilde{d}_5^- - \tilde{d}_5^+ = 105$
$X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+X_{62}+X_{72}+\tilde{d}_6^- - \tilde{d}_6^+ = 105$
$X_{13}+X_{23}+X_{33}+X_{43}+X_{53}+X_{63}+X_{73}+\tilde{d}_7^- - \tilde{d}_7^+ = 105$

$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}+X_{36}+X_{37}+\bar{d}_1^- - \bar{d}_1^+ = 144$
$X_{15}+X_{25}+X_{35}+X_{45}+X_{55}+X_{65}+X_{75}+\bar{d}_2^- - \bar{d}_2^+ = 140$

$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}+\check{d}_1^- - \check{d}_1^+ = 130$
$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}+X_{26}+X_{27}+\check{d}_2^- - \check{d}_2^+ = 132$
$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}+X_{36}+X_{37}+\check{d}_3^- - \check{d}_3^+ = 136$
$X_{41}+X_{42}+X_{43}+X_{44}+X_{45}+X_{46}+X_{47}+\check{d}_4^- - \check{d}_4^+ = 99$
$X_{51}+X_{52}+X_{53}+X_{54}+X_{55}+X_{56}+X_{57}+\check{d}_5^- - \check{d}_5^+ = 99$
$X_{61}+X_{62}+X_{63}+X_{64}+X_{65}+X_{66}+X_{67}+\check{d}_6^- - \check{d}_6^+ = 99$
$X_{71}+X_{72}+X_{73}+X_{74}+X_{75}+X_{76}+X_{77}+\check{d}_7^- - \check{d}_7^+ = 99$

$$X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+X_{61}+X_{71}+\check{d}_8^- - \check{d}_8^+ = 99$$
$$X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+X_{62}+X_{72}+\check{d}_9^- - \check{d}_9^+ = 99$$
$$X_{13}+X_{23}+X_{33}+X_{43}+X_{53}+X_{63}+X_{73}+\check{d}_{10}^- - \check{d}_{10}^+ = 99$$
$$X_{14}+X_{24}+X_{34}+X_{44}+X_{54}+X_{64}+X_{74}+\check{d}_{11}^- - \check{d}_{11}^+ = 127$$
$$X_{15}+X_{25}+X_{35}+X_{45}+X_{55}+X_{65}+X_{75}+\check{d}_{12}^- - \check{d}_{12}^+ = 132$$
$$X_{16}+X_{26}+X_{36}+X_{46}+X_{56}+X_{66}+X_{76}+\check{d}_{13}^- - \check{d}_{13}^+ = 130$$
$$X_{17}+X_{27}+X_{37}+X_{47}+X_{57}+X_{67}+X_{77}+\check{d}_{14}^- - \check{d}_{14}^+ = 109$$
$$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}+\hat{d}_1^- - \hat{d}_1^+ = 146$$
$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}+X_{26}+X_{27}+\hat{d}_2^- - \hat{d}_2^+ = 148$$
$$X_{31}+X_{32}+X_{33}+X_{34}+X_{35}+X_{36}+X_{37}+\hat{d}_3^- - \hat{d}_3^+ = 153$$
$$X_{41}+X_{42}+X_{43}+X_{44}+X_{45}+X_{46}+X_{47}+\hat{d}_4^- - \hat{d}_4^+ = 112$$
$$X_{51}+X_{52}+X_{53}+X_{54}+X_{55}+X_{56}+X_{57}+\hat{d}_5^- - \hat{d}_5^+ = 112$$
$$X_{61}+X_{62}+X_{63}+X_{64}+X_{65}+X_{66}+X_{67}+\hat{d}_6^- - \hat{d}_6^+ = 112$$
$$X_{71}+X_{72}+X_{73}+X_{74}+X_{75}+X_{76}+X_{77}+\hat{d}_7^- - \hat{d}_7^+ = 112$$
$$X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+X_{61}+X_{71}+\check{d}_8^- - \check{d}_8^+ = 112$$
$$X_{12}+X_{22}+X_{32}+X_{42}+X_{52}+X_{62}+X_{72}+\check{d}_9^- - \check{d}_9^+ = 112$$
$$X_{13}+X_{23}+X_{33}+X_{43}+X_{53}+X_{63}+X_{73}+\check{d}_{10}^- - \check{d}_{10}^+ = 112$$
$$X_{14}+X_{24}+X_{34}+X_{44}+X_{54}+X_{64}+X_{74}+\check{d}_{11}^- - \check{d}_{11}^+ = 142$$
$$X_{15}+X_{25}+X_{35}+X_{45}+X_{55}+X_{65}+X_{75}+\check{d}_{12}^- - \check{d}_{12}^+ = 148$$
$$X_{16}+X_{26}+X_{36}+X_{46}+X_{56}+X_{66}+X_{76}+\check{d}_{13}^- - \check{d}_{13}^+ = 146$$
$$X_{17}+X_{27}+X_{37}+X_{47}+X_{57}+X_{67}+X_{77}+\check{d}_{14}^- - \check{d}_{14}^+ = 122$$

$$OX_{11}+MX_{12}+MX_{13}+576X_{14}+340X_{16}+755X_{17}+MX_{21}+OX_{22}+OX_{22}+MX_{23}+591X_{24}+355_{25}+383X_{26}$$
$$+740x_{27}+MX_{31}+MX_{32}+OX_{33}+503X_{34}+307X_{36}+690X_{37}+576X_{41}+591X_{42}+503X_{43}+130X_{45}+150X_{46}$$
$$+215X_{47}+340X_{51}+355X_{52}+281X_{53}+130_{54}+OX_{55}+117X_{56}+200X_{56}+368X_{61}+383X_{62}+307X_{63}+150X_{64}$$
$$+117X_{65}+OX_{66}+190X_{67}+755X_{71}+740X_{72}+690X_{73}+215X_{74}+200X_{75}+190X_{76}+OX_{77}+d_2^- - d_2^+ = 20{,}000$$

and: $X_{ij} \geqslant 0$ (for $i,j = 1,2,\cdots,7$)

All deviational variables $\geqslant 0$