Korean J. Cryst., Vol. 5, No. 1, pp. 7~13, 1994.

변형 및 온도 변화 존재시 단결정에서의 빛의 거동

조동원, 김기수
쌍용중앙연구소

Light Propagation in a Strained and Heated Crystal

Dong-Won Cho and Ki-Soo Kim
Ssang Yong Research Center

요 약

구조물에 매설된 광센서나, 기판위에 필름상으로 성장시켜 제조한 광디바이스등에서 보이는 바와 같이 변형이나 온도변화가 존재할 경우 이방성을 가진 일반적인 결정내에서의 빛의 진행의 변화를 검토하였다. 이방성을 가지는 결정내에서의 빛의 진행을 나타내는 굴절율을 계산하였고 변형이나 온도변화가 존재할 경우 이 복굴절율들의 변화를 표시하였다. 그리고 등방성 매질로 모델을 단순화하였을 경우에 빛의 진행을 검토하여 선행연구자들의 결과와 비교 검토하였다.

Abstract

Light propagation in an anisotropic crystal in the presence of strains and temperature change is investigated. This phenomenon appears in an embedded optical fiber sensor inside the structure or in an optical film on the substrate for optical devices. The refractive indices which represent the light propagation in an anisotopic crystal are calculated and the changes of these refractive indices in the presence of strains and temperature change are also calculated. The calculations for the light propagation in an isotropic medium with the simplified model are performed and the results are compared with previous investigators.

1. Introduction

Light propagation in stressed or heated media has become common phenomenon after advent of the embedded fiber optic sensors[1-9]. It is well known that embedded fiber optic sensors are exposed to strain fields. Thin films in optical devices are also under stresses due to the difference of the thermal expansion coefficients between the films and the substrates because processing temperatures are usually much higher than operating temperatures. Therefore, the thin films in the optical devices are kinds of strained crystals. People also have utilized strain effects to the light propagation in polarization maintaining fibers. The inserted inclusions in the cladding of polarization maintaining fiber cause thermal strains after the fabrication process due to the difference of the thermal expansion coefficients between the inclusions and the optical fiber. Due to the strains in the core, the optical fiber has polarization for the light propagation.

There have been several models for the light propagation in strained bodies proposed by several investigators, but none of them covers the generally anisotropic crystals in the case of strains and temperature presence. Previously we showed the strain effects to the isotropic optical fiber[10,11]. In this investigation, we process a general model for the general cases based on simple crystal physics, then we narrow down to specific cases like isotropic materials.

2. Principal Refractive Indices

The parameter which typically shows the optical properties of the crystal is the refractive index. The refractive index n in an isotropic medium is simply represented by

$$n=\sqrt{k} \qquad \text{...... (1)}$$

where k is a dielectric constant.

In an anisotropic crystal, it is noted that refractive indices are not the tensor properties while dielectric constants are the tensor properties. The refractive indices are at the principal axes.

$$n_k=\sqrt{k_{kk}} \qquad \text{...... (2)}$$

These refractive indices represents the material properties since the dielectric constants are the material properties. If the light travels to one of the crystal axis direction, the actual refractive indices are useful. However, if the light travels in an arbitrary directions, the principal refractive indices are useful. Moreover, if the light travels in an arbitrary direction, the actual refractive indies for the light propagation are different from the principal refractive indices. They must be dependent to the propagation direction.

3.. Refractive Indices for The Light Propagation

We assume that temperature and the strains are uniform and constant inside the crystal, For a coherent transverse light wave travelling in a certain direction set by x_1, the intensity is[13)]

$$I=J\int_0^{\frac{2\pi}{\omega}} D\cdot D\,dt \qquad \text{....... (3)}$$

where J is a proportional constant, w is the angular frequency, and t is the time.. $\boldsymbol{D}$ is the displacement field vector given by[13)]

$$D=A_pS_P\sin(wt-k_px_1)+A_qS_q\sin(wt-k_qx_1) \qquad \text{...... (4)}$$

x_p and x_q are the axes for the polarization directions ("fast" and "slow" axes) perpendicular to the light propagation direction x_1 · s_p and s_q are unit vectors in the x_p and x_q directions (Figure 1). A_p and A_q are

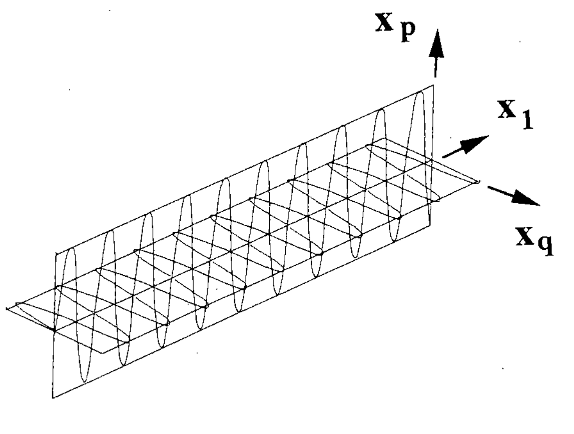

Figure 1. Light propagation in X_1 direction

the amplitudes in the x_p and x_q directions. k_p and k_q are the wave propagation constants in the x_p and x_q directions, and are defined as

$$k_p = \frac{2\pi n_p}{\lambda_0} \qquad k_q = \frac{2\pi n_q}{\lambda_0} \qquad \text{...... (5)}$$

λ_0 is the wavelength of light in vacuum, n_p and n_q are the refractive indices in the x_p and x_q directions, and are the two optical parameters which represent the light propagation properties in the propagation direction inside the crystal. The equation shows that the light wave always has two polarized modes with two different refractive indices. Our objective is to find the relationships between n_p, n_q and the strains and the temperature inside the crystal.

To establish the needed relationships, we start with the wave equation of the form[13)]

$$s \times (s \times BD) + \frac{1}{(n)^2} D = 0 \qquad \text{...... (6)}$$

where s is the unit vector in the direction of propagation. In our problem, where the light propagates in the x_1 direction, the components of s are

$$s_1 = 1, \quad s_2 = 0, \quad s_3 = 0 \qquad \text{......(7)}$$

B is the dielectric impermeability tensor which will be defined in the next section and n is the refractive index may have two different values n_p and n_q. Because we are considering a transverse wave travelling in the x_1 direction, the x_1 component of $\mathbf{D}$ is zero (D_1=0).

Equation (6) can be rearranged to yield[13)]

$$\begin{bmatrix} B_2 - \frac{1}{n^2} & B_4 \\ B_4 & B_3 - \frac{1}{n^2} \end{bmatrix} \begin{Bmatrix} D_2 \\ D_3 \end{Bmatrix} = 0 \qquad \text{......(8)}$$

Since D_2 and D_3 are not zero, the determinant in Eq. (8) must be equal to zero. From this condition, we have

$$\frac{1}{(n)^2} = \frac{(B_2+B_3) \pm \sqrt{(B_2-B_3)^2 + 4B_4^2}}{2} \qquad \text{....(9)}$$

We denote by n_p and n_q the two solutions of Eq. (9)

$$n_p = \frac{1}{\sqrt{b_1 + b_2}} \qquad n_q = \frac{1}{\sqrt{b_1 - b_2}} \qquad \text{......(10)}$$

where

$$b_1 = \frac{B_2 + B_3}{2} \quad b_2 = \sqrt{\left(\frac{B_2 - B_3}{2}\right)^2 + B_4^2} \qquad \text{..(11)}$$

Sometimes, it is to introduce the concept of average and difference of refractive indices which are defined by

$$n_{avg} \equiv \frac{n_p + n_q}{2} \qquad n_{dif} \equiv \frac{n_p - n_q}{2} \qquad \text{......(12)}$$

The above equations express the unknowns n_p, n_q or n_{avg}, n_{dif} in terms of B_2, B_3, and B_4.

4.. Dielectric Impermeability

In the absence of strains and temperature change, we define a tensor **B** the components of which in the principal axes of the crystal are[14)]

$$B_1 = \frac{1}{k_{11}} \qquad B_2 = \frac{1}{k_{22}} \qquad B_3 = \frac{1}{k_{33}}$$

$$B_4 = B_5 = B_6 = 0 \qquad \text{......(13)}$$

If the coordinate system is changed, B can be calculated by simple tensor transformation as is in Appendix. n_p and n_q are always calculated by Eq. (10) and Eq. (11). In the presence of strains and temperature difference, we establish **B** by employing the following analogy.

In a material subjected to an electric field ε_k and strains e_j, the change in the dielectric impermeability $\triangle B_i$ can be expressed as[14)]

$$\triangle B_i = H_{ik}\varepsilon_k + p_{ij}e_j$$

$$(i, j = 1, 2 \ldots 6, \; k = 1, 2, 3) \qquad \text{......(14)}$$

where p_{ij} is Pockel's photoelastic constant. p_{ij} is a six by six matrix[14]. H_{ik} is the electro-optical coefficient. $\triangle$ means the change from the reference value to the value in the presence of strains and temperature. In our problem, the change in B_i results from a change in temperature $\triangle T$ and from the imposed strain e_j. Thus, analogous to Eq. (14), we write W_i is a proportional constant.

$$\triangle B_i = W_i \triangle T + p_{ij} e_j \qquad ......(15)$$

The stress-strain relation inside the crystal is[12]

$$\sigma_i = Q_{ij}(e_j - \alpha_j \triangle T) \text{ or } (e_i - \alpha_i \triangle T) = S_{ij}\sigma_j \qquad ......(16)$$

where Q_{ij} is the stiffness matrix, S_{ij} is the compliance matrix and α_j is the thermal expansion coefficient. By substituting Eq. (16) into Eq. (15) we obtain

$$\triangle B_i = W_i' \triangle T + \pi_{ij}\sigma_j \qquad ......(17)$$

where

$$W_i' = W_i + p_{ij}\alpha_j \quad \pi_{ij} = p_{im}S_{mj}$$
$$(i, j, m = 1, 2 \ldots 6) \qquad ......(18)$$

By combining Eqs. (16) and (17), we obtain the following expression for $\triangle B_i$

$$\triangle B_i = W_i' \triangle T + p_{ij}(e_j - \alpha_j \triangle T) \qquad ......(19)$$

π_{ij} and σ_j are independent of the temperature. Thus W_i' can be expressed as (see Eq. 17)

$$W_i' = \left(\frac{\partial B_i}{\partial T}\right)_{\sigma_j = const} \qquad ......(20)$$

By approximating $B_i\ (i = 1, 2 \ldots 6)$ with its values which exist inside the crystal in the absence of strains and temperature (see Eq. 13), in the principal axes we obtain

$$W_i = \begin{bmatrix} -\frac{2}{n_1^3}\frac{dn_1}{dT} \\ -\frac{2}{n_2^3}\frac{dn_2}{dT} \\ -\frac{2}{n_3^3}\frac{dn_3}{dT} \\ 0 \\ 0 \\ 0 \end{bmatrix}_{\sigma_j = const} \qquad ......(21)$$

By the proper transformation of coordination system, W_i can be usable form.

5. The Change in Refractive Indices With Temperature and Strain

In the reference state ($\triangle T=0$ and $e_1=e_2 \ldots e_6=0$), the refractive indices are

$$n_{p_0} = \frac{1}{\sqrt{b_1 + b_2}}, \quad n_{q_0} = \frac{1}{\sqrt{b_1 - b_2}} \qquad ......(22)$$

where b_1 and b_2 are given in Eq. (11). It is noted that the symbol $\triangle$ which represents the change in the parameter from its value in the reference state to its value in the presence of strains and temperature. From Eq. (10) we have

$$\triangle n_p = \frac{1}{\sqrt{(b_1 + \triangle b_1) + (b_2 + \triangle b_2)}} - \frac{1}{\sqrt{(b_1 + b_2)}}$$
$$\triangle n_q = \frac{1}{\sqrt{(b_1 + \triangle b_1) - (b_2 + \triangle b_2)}} - \frac{1}{\sqrt{(b_1 - b_2)}} \qquad ......(23)$$

Expanding Eq. (23) in a Taylor series and retaining only the first terms of the series, we obtain

$$\triangle n_p = \frac{1}{\sqrt{b_1 + b_2}}\left[1 - \frac{1}{2}\left(\frac{\triangle b_1 + \triangle b_2}{b_1 + b_2}\right) + \ldots\right] - \frac{1}{\sqrt{b_1 + b_2}} \approx -\frac{1}{2}\frac{\triangle b_1 + \triangle b_2}{(b_1 + b_2)^{\frac{3}{2}}}$$
$$\triangle n_p = \frac{1}{\sqrt{b_1 - b_2}}\left[1 - \frac{1}{2}\left(\frac{\triangle b_1 - \triangle b_2}{b_1 - b_2}\right) + \ldots\right] - \frac{1}{\sqrt{b_1 - b_2}} \approx -\frac{1}{2}\frac{\triangle b_1 - \triangle b_2}{(b_1 - b_2)^{\frac{3}{2}}} \qquad ......(24)$$

In these expressions, b_1 and b_2 are the dependent variables of B_i which are also dependent variables of $\triangle T$ and e_i. If the material constants such as the dielectric constants k_{kk}, the thermal expansion coefficients α_i, the elastic constants Q_{ij}, thermal coefficient of refractive indices dn_i/dT and the

Pockel's photo-elastic constants pij are known, then $\triangle B_i$ due to the change in temperature and strains can be calculated with the aid of computers.

6. Refractive Indices in Isotropic Materials

In an isotropic material, the material constants have the same values or the same tensor form regardless of coordinate systems. Therefore all the calculation becomes much simple. In the reference state, regardless of direction, the refractive indices are all same ($n_{po}=n_{qo}=n_o$). The photo-elastic constants p_{ij} have only two differemt independent variables (p_{11}, p_{12}). After assuming isotropic crystal, substitution of Eqs. (11) and (19) into Eq. (24) yields

$$\triangle n_p = -\frac{n_o^3}{2}\ [\ (p_{11}+p_{12})e_h + p_{12}e_1 - \{(p_{11}+2p_{12})\alpha + \frac{2}{n_o^3}\frac{dn_o}{dT}\}\triangle T + \frac{(p_{11}-p_{12})}{2}e_s\ ]$$

$$\triangle n_p = -\frac{n_o^3}{2}\ [\ (p_{11}+p_{12})e_h + p_{12}e_1 - \{(p_{11}+2p_{12})\alpha + \frac{2}{n_o^3}\frac{dn_o}{dT}\}\triangle T - \frac{(p_{11}-p_{12})}{2}e_s\ ] \quad ......(25)$$

The above equations are expressed in terms of the hydrostatic e_h and the maximum shear e_s strains in the cross-sectional plane which are given by

$$e_h \equiv \frac{e_2+e_3}{2} \qquad e_s \equiv \sqrt{(e_2-e_3)^2+e_4^2} \quad ......(26)$$

The changes in n_{avg} and n_{dif} are obtained by combining Eqs .(12) and (25). The results are

$$\triangle n_{avg} = -\frac{n_o^3}{2}\ [\ (p_{11}+p_{12})e_h + p_12e_1 - \{(p_{11}+2P_{12})\alpha + \frac{2}{n_o^3}\frac{dn_o}{dT}\}\triangle T]$$

$$\triangle n_{dif} = -\frac{n_o^3}{4}\ (p_{11}-p_{12})e_s \quad ......(27)$$

The above two expressions provide the needed relationships between the refractive indices, the strains, and the temperature change.

7. Special Cases

We consider three special cases. In all cases the crystal is an opitcally isotropic and elastically isotropic material like glass.

In the first problem, a stress is applied only in one direction ($\sigma_1^{m,\infty} \neq 0$, all other stress components are zero). The strain components are[15]

$$e_1 = e_1,\ \ e_2 = -\nu e_1,\ \ e_3 = -\nu e_1 \qquad e_4 = e_5 = e_6 = 0 \quad ......(28)$$

Thus, the hydrostatic and maximum shear strains become (see Eq. 26)

$$e_h = -\nu e_1 \qquad e_s = 0 \quad ......(29)$$

In the absence of a temperature change ($\triangle T$=0), Eqs.(27) and (29) yield

$$\triangle n_{avg} = -\frac{n_o^3}{2}\{(1-\nu)p_{12} - \nu p_{11}\}e_1 \quad ......(30)$$

This result is the same as that of Butter and Hocker[3] and Bertholds and Dandliker[4,5]

In the second problem, there is only temperature induced strains inside the crystal. For a temperature change $\triangle T$, the stains are

$$e_1 = e_2 = e_3 = \alpha\triangle T \qquad e_4 = e_5 = e_6 = 0 \quad ......(31)$$

By combining Eqs.(27)and (31), we obtain

$$\triangle n_{avg} = \frac{dn_o}{dT}\triangle T \quad ......(32)$$

This is the same result which has been reported by Hocker[6] and by Lee, et al.[7].

Third, and last, we consider the case when only e_1, e_2, and e_3 exist in the crystal, and e_4, e_5 and e_6 are zero. Under this condition, the hydrostatic and maximum shear strains become

$$e_h = \frac{e_1+e_3}{2} \qquad e_s = e_2 - e_3 \quad ......(33)$$

For these strains (at zero temperature difference, $\triangle T$=0), the average refractive index is

$$\triangle n_{avg} = -\frac{n_o^3}{2}\{(p_{11}+p_{12})e_h + p_{12}e_1\} \quad ......(34)$$

This result is the same as that obtained by Sirkis and Haslach[2].

8. Concluding Remark

The model developed in this investigation desrribes the light propagation in the crystal in the presence of strains and temperature change. The model provides the relationship between the changes in the strains and the temperature inside the crystal and the refractive indices of the crystal. The special cases in this investigation support the validity of the model.

This model clarified the actual refractive indices for the light propagation and can be applied to all kinds of media in the presence of strains and temperature change.

References

1. Sirkis, J.S., "Phase-Strain-Temperature Model for Structurally Embedded Interferometric Optical Fiber Strain Sensors with Applications," *SPIE Proceedings,* Vol. 1588, (1991), pp. 26-42.
2. Sirkis, J.S. and Haslach, H.W., "Complete phase-Strain Model for Structurally Embedded Interferometric Optical Fiber Sensors," *J. Intelligent Material Systems and Structures,* Vol. 2, (1991), pp. 3-24.
3. Butter, C.D. and Hocker, G.B., "Fiber Optics Strain Gauge," *Applied Optics*, Vol. 17, (1978), pp. 2867-2869.
4. Bertholds, A. and Dandliker, R., "Deformation of Single-Mode Optical Fibers Under Static Longitudinal Stress," *J. Lightwave Technology,* Vol. LT-5, (1987), pp.895-900.
5. Bertholds, A. and Dandliker, R., "Determination of the Individual Strain-Optic Coefficient in Single-Mode Optical Fibers," *J. Lightwave Technology,* Vol. 6, (1988), pp. 17-20.
6. Hocker, G.B., "Fiber=Optic Sensing of Pressure and Temperature," *Applied Optics*, Vol. 18, (1979), pp.1445-1448.
7. Lee, C.E., Atkins, R.A. and Taylor, H.F., "Performance of a Fiber-Optic Temperature Sensor from-200 to 1050℃," *Optics Letters*, Vol. 13, (1988), pp. 1038-1040.
8. Yeh, Y., Lee, C.E., Atkins, R.A., Gibler, W.N. and Taylor, H.F., "Fiber Optic Sensor for Substrate Temperature Monitoring." *J. Vcuum Science Technolgy,* A8(4), (1990), pp. 3247-3250.
9. Murphy, K.A., Koob, C.E., Plante, A. J., Desu, S. and Claus, R. O., "High Temperature Sensing Applications of Silica and Sapphire Optical Fibers," *SPIE Proceedings*, Vol. 1370, (1990), pp. 169-178.
10. Kim, K.S., "A Model of Embedded Fiber Optic Fabry-perot Temperature and Strain Sensors," Ph. D. Thesis, Stanford University, (1993).
11. Kim, K.S., Kollar, L., and Springer G.S., "A Model of Embedded Fiber Optic Fabry-Perot Temperature and Strain Sensors," *J. Composite Materials* Vol. 27, (1993), pp. 1618-1662.
12. Tsai, S.W. and Hahn, H.T., Introduction to Composite Materials, Technomic Publshing, Lancaster, PA, (1980).
13. Yariv, A., and Yei, P., Optical Waves in Crystals, John Wiley & Sons, New york, (1984).
14. Nye, J. F., Physical Properties of Crystals, Clarendon Press, Oxford, (1985).
15. Timoshenko, S.P., and Goodier, J.N., Theory of Elasticity, McGraw Hill, New York, (1982).

Appendix. Transformations

The stree tensor is transformed from B coordinate system to another A coordinate system by[12)]

$$\sigma_i^A = T_{ij}\sigma_j^B \quad (i, j=1, 2...6) \qquad ......(A.1)$$

where

$$T_{ij}=\begin{bmatrix} \cos^2\theta & \sin^2\theta & 0 & 0 & 0 & \rho_1\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & 0 & 0 & 0 & -\rho_1\cos\theta\sin\theta \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 & \sin\theta & -\cos\theta & 0 \\ -\rho_2\cos\theta\sin\theta & \rho_2\cos\theta\sin\theta & 0 & 0 & 0 & \cos^2\theta-\sin^2\theta \end{bmatrix} \qquad ......(A.2)$$

θ is the angle between the two coordinate systems. For stress transformation, ρ_1=2 and ρ_2=1. The transformations of the strain e_i and the dielectric impermeability B_i follow the same rule as the stress transformation (see Eq. A. 1), with ρ_1=1 and ρ_2=2.

The transformation of thermal expansion coefficients is

$$\alpha_i^A = T_{ij}\alpha_j^B \quad (i, j=1, 2...6) \qquad ....(A.3)$$