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˙Stationary Dual-Porosity Fractal Model of Groundwater Flow in Fractured Aquifers
균열대수층내 지하수 유동에 관한 정상류의 이중공극 프락탈 모델

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Abstract/요약

The stationary dual-porosity model is not sufficient to describe the hydraulic characteristics of fractured aquifers as the groundwater flow in fractured aquifers is often controlled by the fractal geometry of fractures.

This study deals with new stationary dual-porosity fractal model. This model simulates pseudo-steady state flow from matrix block to fissure in the fractal aquifer. Furthermore, it considers storage capacity and well loss effect at the production well. Type curves for different flow dimensions with different drainage factors are plotted.

This new model has been applied to experimental data. The result of the interpretation shows a good accordance between the theoretical model and the observed data.

균열 암반내 지하수 유동은 프락탈 성질의 균열 분포에 영향을 받으므로 정상류 이중공극 모델로는 균열 암반 대수층의 수리지질학적 성질을 설명하기가 어렵다.

본 연구에서는 새로운 정상류 이중공극 프락탈 모델이 제안되었다. 본 모델은 블록으로부터 균열로의 정상류를 포함하는 프락탈 대수층을 모식화한 것이다. 아울러 본 모델은 양수정의 우물저장효과와 우물손실효과를 고려하는 것이다. 여러가지 흐름의 차원과 여러가지 값의 배출계수에 대한 표준곡선들이 만들어졌다.

이 새로운 모델을 야외 자료에 적용시킨 결과, 이론곡선과 관측자료가 잘 일치됨을 보여 주었다.

INTRODUCTION

The characteristics of ground water flow in the fractured media have been studied since several decades(Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Streltsova, 1976; Boulton and Streltsova, 1977a; Boulton and Streltsova, 1977b; Gringarten and Witherspoon, 1972; Ferris et al., 1962; Jenkins and Prentice, 1982; Cinco-Ley et al., 1981; Moench, 1984). The dual-porosity concept was, initially, proposed by Barenblatt et al.(1960)). The dual-porosity model is classified into two categories : one assumes pseudo-steady state flux from a block to a fissure(Barenblatt et al., 1960; Warren and Root, 1963) and the other assumes unsteady state flux from a block to a fissure(Kazemi, 1969; Boulton and Streltsova, 1977b). The dual-porosity theory is applicable where a fractured system behaves as a dual-porosity medium. The dual-porosity medium is composed of units of fissures with high hydraulic conductivity and low specific storage, and of units of blocks with low hydraulic conductivity and high specific storage.

All the models mentioned above can only treat integral flow dimensions, except Cinco-Ley et al.(1981), while flow in fractured rocks is controlled by the fracture network which often has a fractal geometry(Allégre, 1982; Thomas, 1987; Velde, 1991). The basic concept of fractal theory is that a phenomenon will be repeated on different scales in the same way. A major parameter is the fractal dimension which is used as a measure of the nature of the phenomenon. Ground water flow analysis by fractal theory has been introduced into hydrodynamics by Barker(1988) and Chang and Yortsos(1988). Bangoy et al.(1992) successfully applied the fractal theory to field data. Hamm and Bidaux (1994) proposed a fractal model of flow with leakage from aquitard. The theory is a generalization of Hantush equation (1956). Moench(1984) developed a new dual-porosity model with fracture skin. Barker(1988) proposed a global concept of dual-porosity fractal theory. One of the important points in the fractal theory is non-integer flow.

In this paper, the authors propose stationary dual-porosity fractal model which can simulate pseudo-steady state flow from a block to a fissure in the fractal media. It also considers wellbore storage and skin effects at the pumped well. It can be easily utilized for the multi-well and multi-rate pumping system composed of several production and observation wells.

MATHEMATICAL FORMULATION

The stationary dual-porosity fractal model may be applicable especially to the system of fractures with high hydraulic conductivity and low specific storage and blocks with low hydraulic conductivity and high specific storage (Fig.1). We write K_f and S_{sf} as the hydraulic conductivity and the specific storage of fissures, and K' and S_S' as those of blocks. These parameters are supposed that each "point" is occupied simultaneously by the two porosities which extend to the totality of the transverse extension of the reservoir. That may be understood by the representative elementary volume or REV(Bear, 1972, p.19-20) such that it covers a sufficient quantity of each of the porosities. Hence, the REV should be large enough to compare to the scale of fracturation. It is not necessary to know b′ (the transverse extension of the matrix block) as the assumption of Warren-Root model.

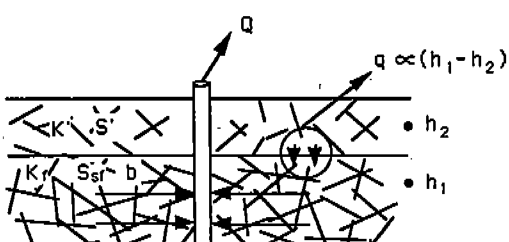


Fig.1 Stationary dual-porosity fractal model.

It is a generalized model of Warren and Root(1963).

In the dual-porosity fractal medium at pseudo-steady state condition, the flux in the matrix block may be written :

$$\frac{K'}{r^{n-1}}\frac{\partial}{\partial r}\left(r^{n-1}\frac{\partial s_2}{\partial r}\right) = S_S'\frac{\partial s_2}{\partial t}+v \qquad (1)$$

where

K' : hydraulic conductivity of the matrix block[LT^{-1}]

S_S' : specific storage of the matrix block (dimensionless)

s_2 : drawdown in the matrix block[L]

r : radial distance from the production well, along the flow line[L]

t : time since pumping started[T]

v : rate of drainage from the fracture to the matrix block per element of volume[T^{-1}]

n : fractal dimension of flow.

Supposing that the hydraulic conductivity of the matrix block is small, the leftside terms of the equation (1) is negligible comparing to the terms of the light-side. Consequently, equation (1) becomes :

$$v = -S_S' \frac{\partial s_2}{\partial t} \qquad (2)$$

For the steady-state condition,

$$v = \chi K' (s_2 - s_1) \qquad (3)$$

s_1 ; drawdown in the fracture[L]

Here, χ is the geometry factor of fractured medium[L^{-2}].

From equations (2) and (3),

$$\chi K' (s_2 - s_1) = -S_S' \frac{\partial s_2}{\partial t} \qquad (4)$$

Similary, the equation of the drawdown for the fracture is written as :

$$\frac{K_f}{r^{n-1}} \frac{\partial}{\partial r}\left(r^{n-1} \frac{\partial s_1}{\partial r} \right) = S_{Sf} \frac{\partial s_1}{\partial t} - v \qquad (5)$$

K_f ; hydraulic conductivity of the fracture[LT^{-1}]

S_{Sf}; specific storage of the fracture (dimensionless)

We assume that the fractal system obeys Darcy's law. Consequently, the change of storage in the well can be given by :

$$W_S \frac{\partial s_w}{\partial t} = Q + K_f b^{3-n} \alpha_n r_w^{n-1} \left(\frac{\partial s_1}{\partial r} \right)_{r=r_w} \qquad (6)$$

where Q is the discharge rate [L^3/T], s_w is the drawdown at the production well [L], s_f is the skin factor(dimensionless), W_s is the storage capacity [L^2] of the production well, r_W is the radius of the production well, b is the transverse extension of the fractures to the flow path and $\alpha_n = 2\pi^{n/2}/\Gamma(n/2)$. Here, $\Gamma(n/2)$ is a gamma function. The drawdown at the abstraction well will be different from that at radius r_W if there is a skin loss :

$$s_W(t) = s_1(r_W, t) - s_f r_W \left(\frac{\partial s_1}{\partial r} \right)_{r=r_W} \qquad (7)$$

If we assume that water level is stable before pumping and the lateral extension of the aquifer is infinite, the initial conditions and the boundary conditions are :

$s_W(r,0) = s_1(r,0) = s_2(r,0) = 0$ for all the distance r

$s_1(\infty,t) = s_2(\infty,t) = 0$ for all the time t

Let us define the following dimensionless parameters :

$$t_D = \frac{4K_f t}{(S_{Sf} + S_S')r_W^2} \qquad (8)$$

$$r_D = r/r_W \qquad (9)$$

$$\lambda = \frac{\chi K' r_W^2}{K_f} \qquad (10)$$

$$\omega = \frac{S_{Sf}}{S_{Sf} + S_S'} \qquad (11)$$

$$s_{1D} = \frac{4\pi^{n/2} K_f b^{3-n}}{Q r_W^{2-n}} s_1 \qquad (12)$$

$$s_{2D}=\frac{4\pi^{n/2}K_f b^{3-n}}{Q r_w^{2-n}}s_2 \quad (13)$$

$$W_{sD}=\frac{W_S}{\pi^{n/2}b^{3-n}r_w^{n}(S_{Sf}+S_s')} \quad (14)$$

In consequence, we can write equations (4), (5), (6) and (7) in dimensionless forms :

$$4(1-\omega)\frac{\partial s_{2D}}{\partial t_D}=\lambda(s_{1D}-s_{2D}) \quad (15)$$

$$\frac{1}{r_D^{n-1}}\frac{\partial}{\partial r_D}\left(r_D^{n-1}\frac{\partial s_{1D}}{\partial r_D}\right)=4\omega\frac{\partial s_{1D}}{\partial t_D}+4(1-\omega)\frac{\partial s_{2D}}{\partial t_D} \quad (16)$$

$$W_{sD}\frac{\partial s_{wD}}{\partial t_D}=1+\frac{1}{2\Gamma(n/2)}\left(\frac{\partial s_{1D}}{\partial r_D}\right)_{r_D=1} \quad (17)$$

$$s_{wD}(t_D)=s_{1D}(1,t_D)-s_f\left(\frac{\partial s_{1D}}{\partial r_D}\right)_{r_D=1} \quad (18)$$

Transforming the equations (15)-(18) in the space of Laplace gives

$$4p(1-\omega)\bar{s}_{2D}=\lambda(\bar{s}_{1D}-\bar{s}_{2D}) \quad (19)$$

$$\frac{d^2\bar{s}_{1D}}{dr_D^2}+\frac{n-1}{r_D}\frac{d\bar{s}_{1D}}{dr_D}-4p(1-\omega)\bar{s}_{2D}=4p\omega\,\bar{s}_{1D} \quad (20)$$

$$pW_{SD}\,\bar{s}_{wD}=\frac{1}{p}+\frac{1}{2\Gamma(n/2)}\left(\frac{d\bar{s}_{1D}}{dr_D}\right)_{r_D=1} \quad (21)$$

$$\bar{s}_{wD}(p)=\bar{s}_{1D}(1,p)-s_f\left(\frac{d\bar{s}_{1D}}{dr_D}\right)_{r_D=1} \quad (22)$$

Here, $\bar{s}_{wD}$, $\bar{s}_{1D}$ and $\bar{s}_{2D}$ denote the Laplace transform of s_{wD}, s_{1D} and s_{2D}, respectively. p is the parameter of the Laplace transform. Solving the equations (19)-(22) gives(Hamm, 1994) :

$$\bar{s}_{wD}=\frac{1}{p\left[pW_{sD}+\frac{1}{2\Gamma(n/2)}\frac{1}{K_{V-1}^{V}(\sigma)+s_f}\right]} \quad (23)$$

$$\bar{s}_{1D}=\frac{1}{p}\frac{1}{\left[pW_{sD}+\frac{1}{2\Gamma(n/2)}\frac{1}{K_{V-1}^{V}(\sigma)+s_f}\right]}\frac{r_D^{V}K_V(\sigma r_D)}{K_V(\sigma)+s_f\sigma K_{V-1}(\sigma)} \quad (24)$$

$$\bar{s}_{2D}=\frac{\lambda}{4p(1-\omega)+\lambda}\bar{s}_{1D} \quad (25)$$

where

$$\sigma^2=\frac{4p[4p\omega(1-\omega)+\lambda]}{4p(1-\omega)+\lambda} \quad (26)$$

$$K_{V-1}(z)=\frac{K_V(z)}{zK_{V-1}(z)} \quad (27)$$

and

$$\nu=1-n/2 \quad (28)$$

The Eqs. (23), (24) and (25) can be inverted into the real plane, using Stehfest algorithm(1970). $K\nu(z)$ is calculated by Amos package(1986). Setting wellbore storage to zero to Eq.(24), we obtain :

$$\bar{s}_{1D}=\frac{2\Gamma(n/2)}{p}\frac{r_D^{V}K_V(\sigma r_D)}{\sigma K_{V-1}(\sigma)} \quad (29)$$

Then letting the well radius tend to zero, we obtain :

$$\bar{s}_{1D}=\frac{2^{1+V}}{p}\frac{r_D^{V}K_V(\sigma r_D)}{\sigma^V} \quad (30)$$

THEORETICAL CURVES OF THE MODEL

Fig.2 shows some selected theoretical curves of the model with different values of λ for the production well and for the dimensions 1.0, 1.5, 2.0 and 2.5. The curves are composed of three parts. The first part represents wellbore storage effect followed by flow in the fracture. The second part, the transition zone, reflects the progressive contribution of water from the matrix block. Finally, the third part shows the response of the total system of both the fissure and the matrix block. If the value of λ is large, the total system appears quickly and the drawdown is small.

Fig.3 presents selected theoretical curves of the model with different values of λr_D^2 for the observation well in the fissure, and for the dimensions 1.0, 1.5, 2.0 and 2.5. As the case of the production well, the curves are composed of three parts. During the early period of dimensionless time(t_D/r_D^2), the flux derives from fractures. So, in this period, the drawdown is characterized by K_f

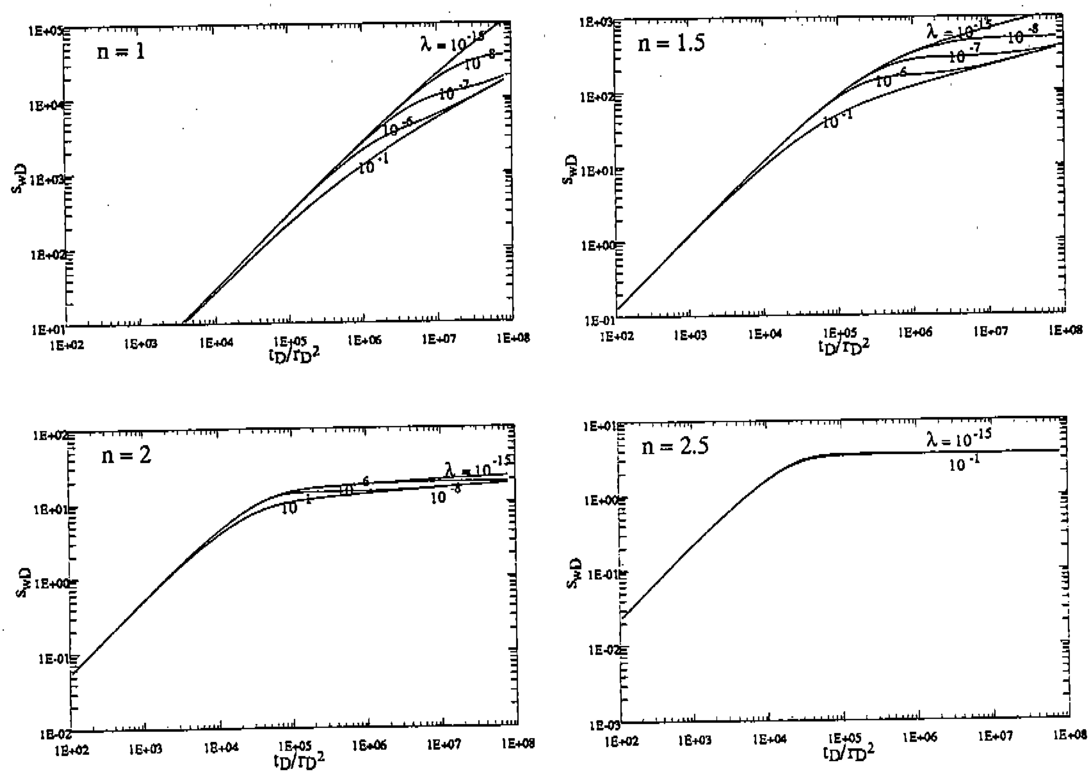


Fig.2 Selected theoretical curves of the model with different values of λ for the production well and for the dimensions 1, 1.5, 2 and 2.5. $W_S=0.03m^2$ is used.

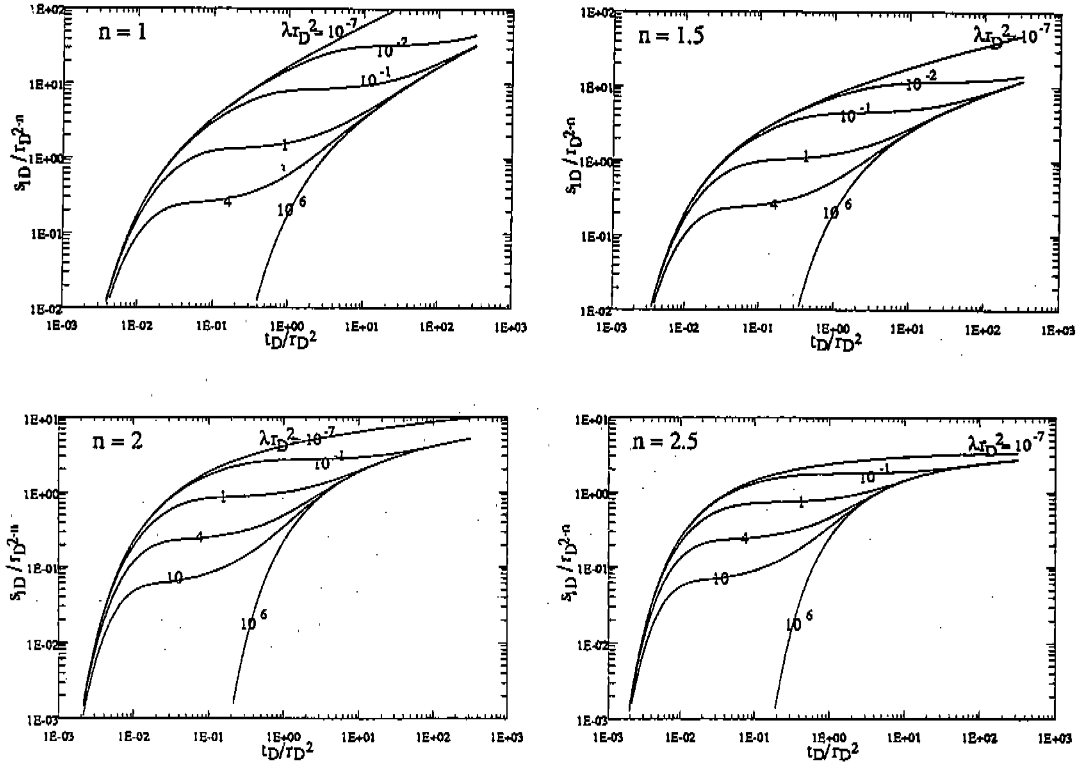


Fig.3 Selected theoretical curves of the model with different values of λr_D^2 for the observation well in the fracture, and for the dimensions 1, 1.5, 2 and 2.5.

and S_{sf}. During the intermediate period (transition period), the matrix block provides water progressively so that the rate of drawdown diminishes. The length of this period depends on ω. For a certain dimensionless distance(r_D), the start of the transition and the level of stabilization of drawdown is dependent on λr_D^2. During the final period, both the fracture and the matrix block give water simultaneously. In this period, the drawdown is characterized by a hydraulic conductivity equivalent to K_f and a specific storage equivalent to the sum of S_{sf} and S_s'. Thus, the upper limit of the curves represents the system of fissures and the lower limit displays the total system of the fracture and the matrix block.

Fig.4 shows selected theoretical curves of the model with different values of λr_D^2 for the observation well in the matrix block, and for the dimensions 1.0, 1.5, 2.0 and 2.5. When the values of λr_D^2 are large, the curves approach those of generalized radial flow model(Barker, 1988). On the contrary, when the value of λr_D^2

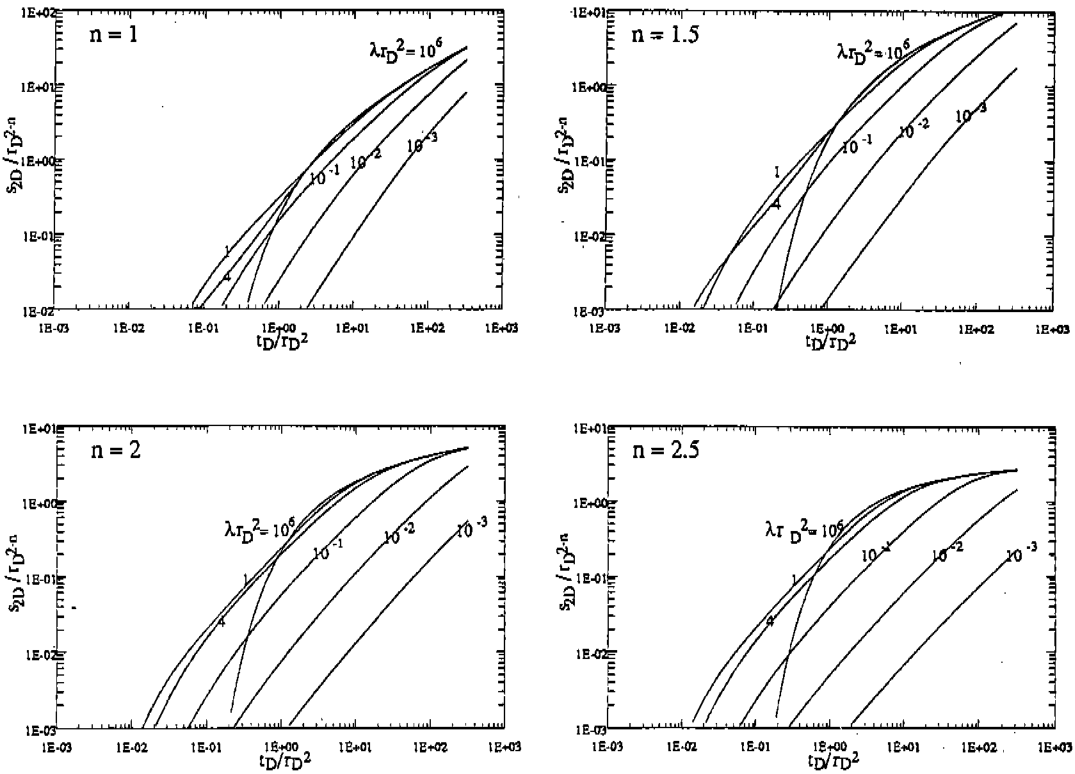


Fig.4 Selected theoretical curves of the model with different values of λr_D^2 for the observation well in the matrix block, and for the dimensions 1, 1.5, 2 and 2.5.

is small, the curve becomes a straight line in the first part and rejoins the curve of the fissure.

APPLICATION TO EXPERIMENTAL DATA

The model was applied to typical experimental data. The pump test site is situated in the Geochang area(Lee et al., 1989). The geology of the site is composed of granites, andesites and sedimentary rocks of the cretaceous age. Furthermore, acidic dykes and alluviums exist. The pumped well is located in the granite. The well is 502m in depth with casing until 250m depth. According to gamma-ray logs, it appears to have weathered zones and alluviums up to 60m in depth, a moderately weathered zone between 60m and 250m, and fresh and fractured rock between 250m and the bottom of the hole.

The pump test was conducted with a

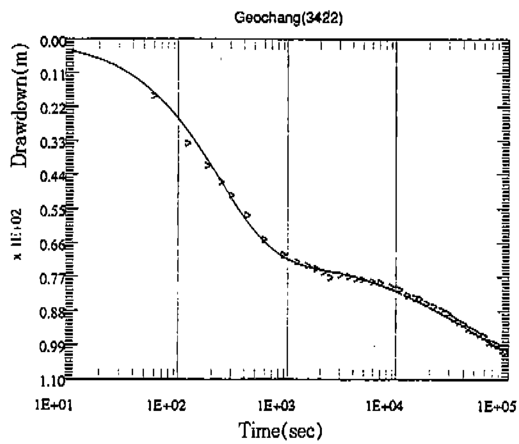
Table 1. Calculated hydraulic parameters.

Site	$K_f b^{3-n}$ $[L^{4-n}T^{-1}]$	K_f/S_{sf} $[L^2T^{-1}]$	$K'b'^{3-n}$ $[L^{4-n}T^{-1}]$	K'/S_s' $[L^2T^{-1}]$	n	λ	s_f	W_s $[L^2]$	PI $[L^2T^{-1}]$
Geochang	3.0×10^{-4}	4.6	5.7×10^{-5}	1.8×10^{-3}	1.8	3.5×10^{-6}	4.74	0.03	1.3×10^{-4}

discharge rate of 1.04×10^{-2} m^3s^{-1} for 1,535minutes(Lee et al., 1989). The drawdown was measured at the production well(Table 1).

For the pump test analysis, we have fixed the value of the specific storage in the fractures as 5×10^{-6} m^{-1} and in the matrix block as 5×10^{-5} m^{-1}. As a result of the analysis, we have determined the values of the hydraulic parameters as in Table 1. The dimension of flow is determined as 1.8. The diffusivity of the matrix block is much smaller than that of the fractures owing to much smaller hydraulic conductivity compared to specific storage in the matrix block. Also, $K'b'^{3-n}$ is larger than $K_f b^{3-n}$ because b' is much greater than b.

As shown in Fig.5, the theoretical curve shows a good agreement with the drawdown data.



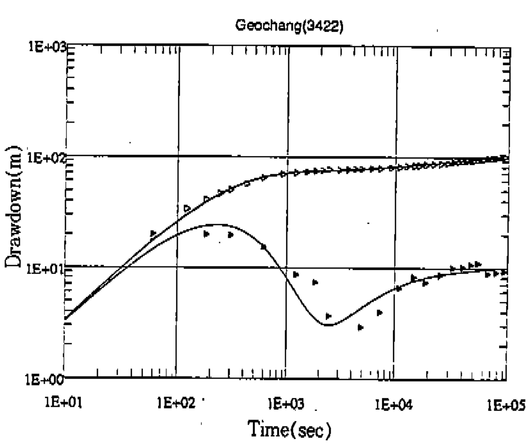


Fig.5 Theoretical curves and observed curves at the pumping well, Geochang site. Void triangle and black triangle represent observed drawdowns and derivatives of the observed drawdowns, respectively. Corresponding curves represent theoretical draw-downs and derivatives of the theoretical draw-downs. PI signifies the specific yield.

CONCLUSION

The stationary dual-porosity fractal model leads us to understand the hydraulic characteristics of fractured aquifers more reasonably than the Warren-Root model because the fractured aquifer often behaves as a fractal media. So, it is a more versatile tool to analyze pump test data from fractal or fractured media. It can considers wellbore storage and well loss effect at the production well. It can also be easily applied to the multi-well and multi-rate pumping system composed of several production and observation wells.

An application of the model to field data in Geochang area gives 1.8 of the flow dimension.

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