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과도현상 해석을 위한 시간영역에서의 등가축약법 : 프로니 해석기법을 이용한 등가 구동점 임피던스 모델의 구성

Time Domain Reduction Method for Electromagnetic Transients Study: Equivalent Driving-Point Impedance Model Using Prony Analysis

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Abstract—This paper presents a method of obtaining transmission network equivalents from the network's response to the pulse excitation signal. Proposed method is based on Prony signal analysis and transfer function identification technique. As a result Thevenin-type of discrete-time filter model can be generated. It can reproduce the driving point impedance characteristic of the network.

Key Words : Network Equivalent, Electromagnetic Transients, Prony Analysis

1. Introduction

It is required in electromagnetic transients studies to represent the complex power system in great detailed manner[1~2]. Whereas routine load-flow and fault level studies in power system are widely carried out for networks of several hundred nodes, it is seldom feasible to base electromagnetic transient analysis on this scale of explicit network representation. Thus almost invariably, it is required to represent network, or given part of network, in a reduced form, and the use of proper equivalent will greatly save the overall study time.

The conventional reduction methods, presented in[1-2], are all the method of construction equivalent that approximates the frequency response of the network. In these frequency domain methods the system to be reduced is represented by an approximated rational network function and the network function is determined in s[1] or jw-domain [2] via iterative nonlinear curve fitting procedure. Thus there is necessary transforming the network function into the time domain equivalent that is composed of lumped parameter R, L, C elements.

In this paper, a time-domain method of obtaining transmission network equivalents from the network's response to a pulse excitation is presented. Proposed method is based on Prony signal analysis and transfer function identification technique. As a result, Thevenin-type of discrete-time filter model can be generated. It can reproduce the driving-point impedance characteristics of the network. Furthermore proposed model can be implemented in an easy and direct manner into the EMTP or any other time-domain digital simulator. In the paper, the proposed reduction method will be described only for the single-phase equivalent model. However, the same approach can be extended to generate 3-phase equivalent easily by modal transform technique[1~2].

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2. Basic Concept

In the system shown in Fig.1, it is the signals at

Fig. 1 External system and Study system.

the boundary bus that are of great importance, while those in the external system are of no interest. Therefore, it is required that the equivalent can reproduce the system's response accurately at the boundary bus, over a broad frequency range, to the signals incoming from the study subsystem. If the external system is linear and time-invariant (LTI), the system that has the same impulse response or transfer function will duplicate the transient response of the external system. And the response of the system due to some excitation signal contains enough information necessary for the identification of its transfer function. In power system the driving-point impedance Z(s) or admittance function Y(s) is the commonly used transfer function.

3. Transfer Function Identification Method By Prony Analysis

Prony analysis is a technigue of analyzing signals to determine modal, damping, phase, and magnitude informations containted in the signal [3]. It is an extension of Fourier analysis in that damping information as well as frequency information is obtained. The Prony analysis gives an optimal fit in the least-squared-error(LSE) sense to a signal y(t) in the form.

$$y(t)=\sum_{i=1}^{n+m} B_i e^{\lambda_i t} \tag{1}$$

Eq.(1) means equivalently in z-plane

$$Y(z)=G(z)U(z)=\sum_{i=1}^{n+m} \frac{B_i z}{z-\lambda_i} \tag{2}$$

The n+m distinct eigenvalues, λ_i's, and signal residues, B_i's, are identified by Prony analysis. In this equation G is the system transfer function of order n, U is the z transform of the order m excitation signal, and Y is that of the output signal. It is important to note that Prony analysis results in a residue and eigenvalue decomposition of an output signal not the transfer function. But, as shown below, the analysis can be used to obtain transfer functions for known input excitation signal.

Suppose in Eq.(2) that the single rectangular pulse of height U_o is applied at time k=1 and the time k=d+1 is the first sample value for which the input is no longer being applied. In this case, d is the pulse duration. The z transform of this input is

$$U(z)=\frac{U_o}{z-1}(1-z^{-d})=U_o\sum_{i=1}^{d} z^{-i} \tag{3}$$

Applying this input to a linear transfer function in partial-fraction expansion form

$$G(z)=\sum_{i=1}^{n} \frac{R_i z}{z-\lambda_i} \tag{4}$$

yields the output signal Y(z)

$$Y(z)=G(z)\cdot U(z)=U_o\sum_{j=1}^{d} z^{-j}\sum_{i=1}^{n} \frac{R_i z}{z-\lambda_i} \tag{5}$$

The inverse z-transform gives the following sampled output signal

$$y[k]=U_o\sum_{j=1}^{d}\left\{\sum_{i=1}^{n} R_i(\lambda_i)^{(k-j)} u_s(k-j)\right\} \tag{6}$$

where $u_s(k-j)$ is the unit step function. This expression is actually a discrete time convolution of the sampled input signal with the system impulse response. When the input is no longer being applied to the system i.e. $k \geq d$ and $u_s(k-j)=1$, the output can be expressed as

$$y[k]=U_o\sum_{i=1}^{n} R_i(\lambda_i)^{k}\sum_{j=1}^{d}(\lambda_i)^{-j} \quad (k\geq d)$$

$$=U_o\sum_{i=1}^{n} R_i(\lambda_i)^{k}(\lambda_i{}^{-1})\left\{\frac{1-\lambda_i{}^{-d}}{1-\lambda_i{}^{-1}}\right\} \tag{7}$$

By equating Eq.(2) and Eq.(7) the transfer function residue R_i can be obtained from signal residue B_i as

$$R_i=\frac{B_i}{U_o}\left\{\frac{\lambda_i-1}{1-\lambda_i{}^{-d}}\right\} \tag{8}$$

3.1 Choice of the excitation signal

Since the objective of the injected excitation signal is to excite all modes of the external system within a selected broad range of frequencies, the choice of an appropriate excitation signal is important. Short-duration pulse signal can be such an excitation signal and pulse of duration 1 is the ideal excitation signal, i.e., impulse signal.

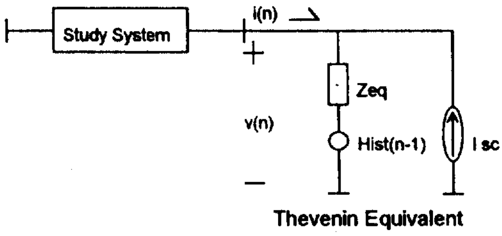

Fig. 2 Equivalent impedance representation for the external system

In this paper, voltage response to the current pulse excitation was used for driving-point impedance function identification. Since the power system, in generally, is inductive the voltage response to the current excitation will have smaller DC offset term, which may cause bias error in estimated results.

The response data for Prony analysis was calculated by EMTP. In this case, sampling time step Δt determines the frequency range to be considered according to Nyquist's criterion. Injected current pulse was generated using two unit step source in EMTP as

$$i(k\Delta t)=u_s[k\Delta t]-u_s[(k-1)\Delta t] \quad k\geq 0 \tag{9}$$

3.2 Procedure for the transfer function identification

The following steps outline the system excitation and transfer function identification procedure.

1. Disconnect the study subsystem from the external system.
2. Turn off all the sources in the external system.
3. Inject the excitation pulse signal into the boundary bus and compute the corresponding response by EMTP.
4. Identify the transfer function with calculated response using Prony analysis as described above.

4. Equivalent Impedance Model

If we choose the current as the input excitation, the output corresponds to the voltage. In this case the transfer function, estimated in z-plane, will be the driving-point impedance function, $Z_d(z)$.

$$Z_d(z)=\sum_{k=1}^{n}\frac{R_k z}{z-\lambda_k}=\sum_{k=1}^{n}\frac{R_k}{1-\lambda_k z^{-1}} \tag{10}$$

To avoid complex arithmetic operation in Eq.(10), Ep.(10) is rearranged below by combining the complex conjugate terms together as

$$Z_d(z)=\sum_{k=1}^{p}\left\{\frac{R_k}{1-\lambda_k z^{-1}}+\frac{R_k^*}{1-\lambda_k^* z^{-1}}\right\}$$
$$=\sum_{k=1}^{p}\left\{\frac{d_k+e_k z^{-1}}{1+a_k z^{-1}+b_k z^{-2}}\right\} \tag{11}$$

where

$$a_k=-(\lambda_k+\lambda_k^*)=-2\mathrm{Re}(\lambda_k) \tag{12}$$
$$b_k=\lambda_k\cdot\lambda_k^* \tag{13}$$
$$d_k=R_k+R_k^*=2\mathrm{Re}(R_k) \tag{14}$$
$$e_k=-(R_k\lambda_k+R_k^*\lambda_k^*)=-2\mathrm{Re}(R_k\lambda_k) \tag{15}$$

Now let

$$V_k(z)=Z_k(z)\cdot I(z)=\left\{\frac{d_k+e_k z^{-1}}{1+a_k z^{-1}+b_k z^{-2}}\right\}\cdot I(z) \tag{16}$$

then Eq.(16) can be expressed in time-domain

$$v_k(n)+a_k v_k(n-1)+b_k v_k(n-2)$$
$$=d_k i(n)+e_k i(n-1) \tag{17}$$

The difference equation Eq.(17) can be rewritten in compact form as

$$v_k(n)=d_k i(n)+hist_k(n-1) \tag{18}$$
$$hist_k(n-1)=e_k i(n-1)-a_k v_k(n-1)$$
$$-b_k v_k(n-2) \tag{19}$$

By the definition of the driving-point impedance

$$V(z)=Z_d(z)\cdot I(z)=\sum_{k=1}^{p}Z_k(z)I(z)=\sum_{k=1}^{p}V_k(z) \tag{20}$$

equivalently in time-domain

$$v[n]=\sum_{k=1}^{p}v_k[n] \tag{21}$$

we can obtain the resultant equation via collecting all the delay terms together and denote them by Hist(n-1) as follows

$$v(n)=Z_{eq}\,i(n)+Hist(n-1) \tag{22}$$
$$Z_{eq}=\sum_{k=1}^{p}d_k \tag{23}$$
$$Hist(n-1)=\sum_{k=1}^{p}hist_k(n-1) \tag{24}$$

This is the equation of a discrete-time Thevenin equivalent to the zero-input external system (Fig. 2).

A single-phase equivalent for the external system can be put together by connecting the Thevenin equivalent in parallel with an appropriate Norton current source as denoted in Fig.2.

The phasor value of the Norton current source can be specified by finding the steady-state shotr-circuit current for the boundary bus with all the sources in external system remaining active.

5. Validation of Equivalent

In order to validate the proposed method

- comparison of the step responses between the full representation and equivalent model
- calculation of the open-end voltages of the two models due to the energization of the transmission line

were carried out. The single-phase test system considered is shown in Fig.3 where the study system consists of the open-ended 200 Km transmission line.

To generate the equivalent, voltage response of the external system to the current pulse excitation was calculated using EMPT. In the simulation, the lines are all represented by distributed parameter line model and sampling time step Δt was 5.0×10^{-5} sec. With this Δt, frequency range up to 10kHz can be considered. In the estimated result by the Prony TFI method, the model order n, in Eq.(10), was 64 and the relative rms error was about 5.4e-6, which was defined by

$$error=\sqrt{\sum_{k=1}^{N}(v[k]-\bar{v}[k])^2}\Big/\sqrt{\sum_{k=1}^{N}v[k]^2} \qquad (25)$$

where $v[k]$ is the EMTP output and $\bar{v}[k]$ is estimated model output. Consistency of the results was not affected by the number of the sampled points, N, from about 200.

A comparison of the responses of the external system and that of the equivalent due to the step input at boundary bus is shown in Fig.4. In Fig.5, that of the line energization voltages at bus 4 is shown. The shown results demonstrate the ability of the developed equivalent to reproduce the original network's transient behavior.

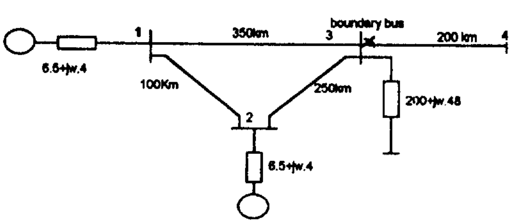

Fig. 3 Test network

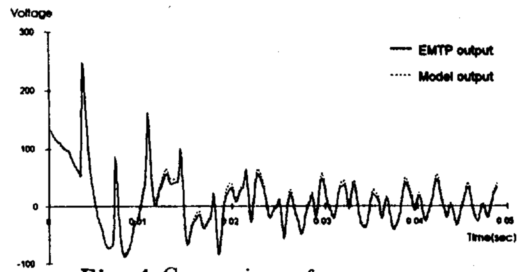

Fig. 4 Comparison of step responses

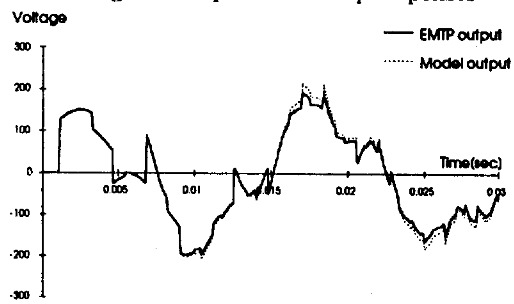

Fig. 5 Comparison of line energization voltages.

6. Conclusion

A time-domain reduction method for electromagnetic transients analysis is presented. The time-domain response of the external system to a short duration pulse excitation signal is utilized to identify a discrete-time equivalent filter model for the system. The proposed equivalent model is determined by the transfer function identification technique based on the Prony analysis and showd a good agreement to the original system.

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