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Estimation of Unknown Projection DATA Based on the Bandwidth of Projection DATA

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=**Abstract**=

In the case of the image reconstruction from unknown projection data such as imaging the object with opaque obstructions, conventional reconstruction algorithms may reconstruct a degraded image. In this paper, a new method for the estimation of the unknown projection data based on known projection data and the bandwidth of projection data is proposed. The proposed method successfully estimates the unknown projection data through iterative transformation between projection space and frequency space using the known projection data and the bandwidth of the projection data. Computer simulation shows that the proposed method significantly improves image quality and convergence behavior over conventional algorithms. In addition, the proposed method is successfully applied to ultrasound attenuation CT using a sponge phantom.

Key words : Computed tomography, Ultra sound attenuation CT, Unknown projection data.

1. INTRODUCTION

In computed tomographic imaging systems (CT), unknown projection problems often arise when projection data are only partially known. Specifically, some of projection data are unknown due to the opaque obstructions such as bones or gases within a body in the case of the ultrasound attenuation CT. The unknown projection data blur the resulting image with severe artifacts. This problem can alleviated by properly estimating the unknown projection data. One of the typical method for such an estimation is an iterative reconstruction reprojection (IRR) [1~13]. The IRR iteratively performs reconstruction and reprojection by a convolution-backprojection (CB) algorithm and a square pixel algorithm, respectively. In the IRR, an object is transformed back and forth between object (image) space and projection space using the known projection data and the estimated unknown projection data. The IRR, however, suffers from high frequency artifacts embedded in the reconstructed image due to discontinuity between the known projection data and the unknown projection data. Also, the IRR performs interpolating operation in the image space during the reconstruction and the reprojection. The errors associated with the interpolation may degrade the reconstructed image.

For the estimation of the unknown projection data, a new method based on both the known projection data and the bandwidth of projection data is proposed. Specifically, the unknown projection data successively improve accuracy in their estimation through iterative transformation between projection space and frequency space using the known projection data and the bandwidth of the projection data. Also, for the bandwidth of the projection data, it is necessary to obtain the entire spectrum of the projection data. However, the projection data are only partially known, so that the unknown projection data have to be

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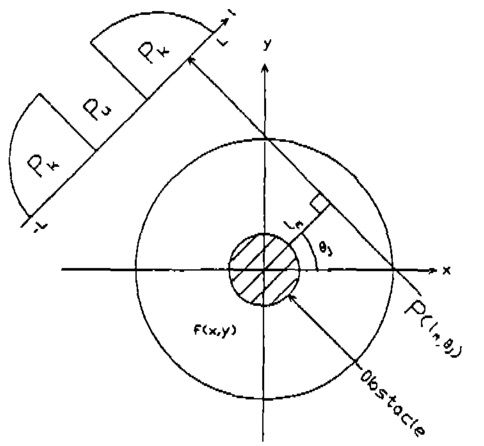

Fig. 1 Projection data for an object with obstacle

postulated. The postulation is made for the spectral density of the known projection data not to be severely interfered with by the spectral density of the postulated unknown projection data. The spectrum of the projection data is determined from the postulated unknown projection data and the known projection data.

Simulation and experiment results are presented, and image quality of the proposed method shows a substantial improvement compared to the existing IRR algorithm. In addition, the proposed algorithm was applied to ultrasound attenuation CT using a sponge phantom.

2. ESTIMATION OF UNKNOWN PROJECTION DATA

2. 1 Preliminary, IRR

Consider, for an example, that a region of unknown projection due to an obstacle exists in projection space as shown Fig. 1.

In order to estimate the unknown projection data P_u, the iteration in the IRR starts with the known projection data P_k and the initially guessed unknown projection data P_u^0. After the projection data are filtered by $|w|$-filter, the initial object image f^0 is obtained by the backprojection in a bounded object space. From this initially reconstructed object image, the next estimate of the unknown projection data P_u^1 is obtained by the reprojection. With the known projection data P_k together with the new estimate P_u^1, the next object image f^1 is calculated. These procedures are repeated until $|f^{i+1}-f^i|$ becomes less than a specified convergence criterion.

2. 2 The proposed method

In the proposed method, the unknown projection data P_u, are estimated as follows. Firstly, for the bandwidth of projection data, it is necessary to obtain the entire spectrum of the projection data. However, the projection data are only partially known, so that the unknown projection data have to be postulated. The postulation is made for the spectral density of the known projection data not to be severely interfered with the spectral density of the postulated unknown projection data. Specifically, the unknown projection data can be obtained from the known projection data using the interpolating function with low frequency component. The spectrum of the projection data is obtained from the postulated unknown projection data and the known projection data. If the spectrum of the projection data is not band-limited, the bandwidth σ of the projection data is determined with respect to its energy spread over the spectrum. Secondly, to estimate the unknown projection data P_u, the iteration starts with the known projection data P_k, and the some initial unknown projection data P_u^0. The total projection data G^0 combining the known projection data P_k and the unknown projection data P_u^0 are transformed as

$$\hat{G}^0(w,\theta)=\int_{-L}^{L} G^0(l,\theta)\,e^{-jwl}\,dl. \qquad (1)$$

$\hat{G}^0(w,\theta)$ is σ-bandlimited as

$$\hat{P}^1(w,\theta)=\hat{G}^0(w,\theta)W_\sigma(w),\ \ W_\sigma(w)=\begin{cases}1, & |w|<\sigma\\ 0, & |w|>\sigma.\end{cases} \qquad (2)$$

$\hat{P}^1(w,\theta)$ is inverse-transformed as

$$P^1(l,\theta)=\int_{-\sigma}^{\sigma}\hat{P}^1(w,\theta)\,e^{jwl}\,dw. \qquad (3)$$

$P^1(l,\theta)$ is modified by replacing the projection data of the known region by $P_k(l,\theta)$ as

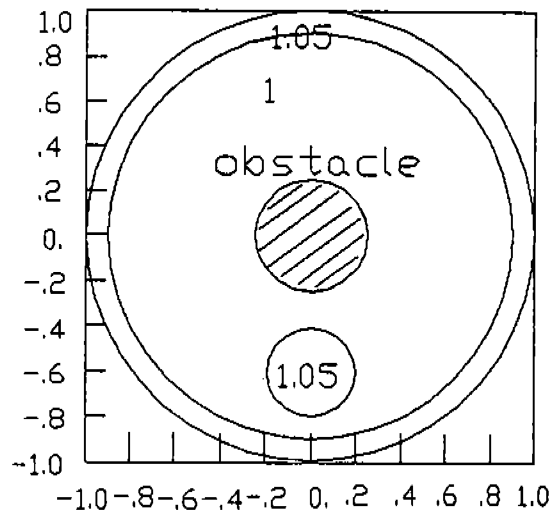

Fig. 2 Test phantom I.

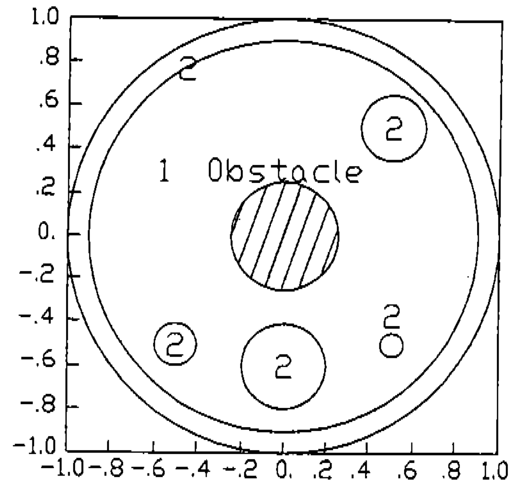

Fig. 3 Test phantom II.

$$G^1(l,\theta)=\begin{cases}P^1(l,\theta), l \text{ in the unknown region}\\ P_K(l,\theta), l \text{ in the known region}\end{cases} \quad (4)$$

These procedures are repeated until $|P^i-G^i|$ becomes less than a specified convergence criterion.

If the bandwidth of the projection data has been properly chosen, the proposed method reduces the mean square value of the error $P(w,\theta)-P(w,\theta)$ in each iteration step. $P(w,\theta)$ denotes the spectrum of the projection data $P(l,\theta)$ without the unknown region. Indeed, from eqns. 2 and 4 and Parseval's formula, I conclude that

$$\frac{1}{2\pi}\int_{-\sigma}^{\sigma}|\hat{P}(w,\theta)-\hat{P}^i(w,\theta)|^2$$
$$=\int_{-\infty}^{\infty}|P(l,\theta)-P^i(l,\theta)|^2dl$$
$$>\int_{-\infty}^{\infty}|P(l,\theta)-G^i(l,\theta)|^2$$
$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}|\hat{P}(w,\theta)-\hat{G}^i(w,\theta)|^2dw$$
$$>\frac{1}{2\pi}\int_{-\sigma}^{\sigma}|\hat{P}(w,\theta)-\hat{P}^{i+1}(w,\theta)|^2dw \quad (5)$$

3. SIMULATION AND EXPERIMENT

The proposed method and the IRR method are evaluated by a simulation with test phantoms in Fig. 2 and 3. The phantom in Fig. 2 has smaller signal components in high spatial frequencies than the phantom in Fig. 3. The projection data consists of 64 views and 65 rays per view. All images were reconstructed on a square array of 64×64 by the CB using a Shepp-Logan filter. To obtain the spectrum of projection data, unknown projection data is postulated from known projection data using a cubic-spline interpolating function. And the bandwidth of projection data is determined at a frequency which covers 99 percentage of the total energy in the spectrum of the projection data. The error called distance is defined as

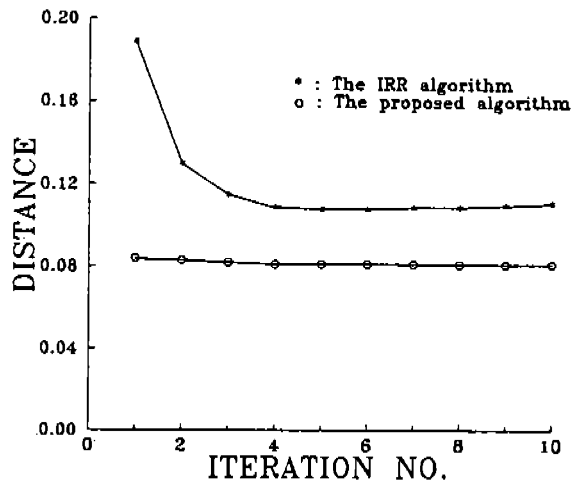

Fig. 4 Distances for the test phantom I.

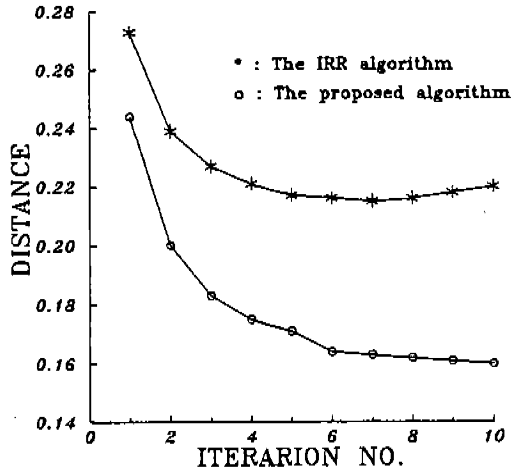

Fig. 5 Distances for the test phantom II.

$$Distance = \left(\sum_{i=1}^{N_p} \sum_{j=1}^{N_p} |f_{ij} - \hat{f}_{ij}|^2 \Big/ \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} |f_{ij} - \bar{f}\,|^{1/2} \right) \quad (6)$$

where $\hat{f}_{ij}$ is an reconstructed image at the (i, j) pixel, f_{ij} is an ideal image at the (i, j) pixel, and $\bar{f}$ is the mean of the ideal image.

For the test phantoms, Fig. 4 and 5 comparatively show the distances for the proposed method and the existing IRR, where the proposed method shows much smaller distances than the existing IRR for the entire iteration steps. Especially, in the proposed method, the rate of convergence for the phantom I is better than that for the phantom II, since the bandwidth of the projection data for the phantom I is more accurate than that for the phantom II. For comparison, Fig. 6 (a) shows the original image for the test phantom II, and Fig. 6 (b) and 6 (c) comparatively show the images reconstructed by the proposed method and the existing IRR for the test phantom II, respectively, each being obtained after ten iterations. It is observed that the image reconstructed by the proposed method is a better replica of the original image. Especially, artifacts such as streaks are noticeably reduced.

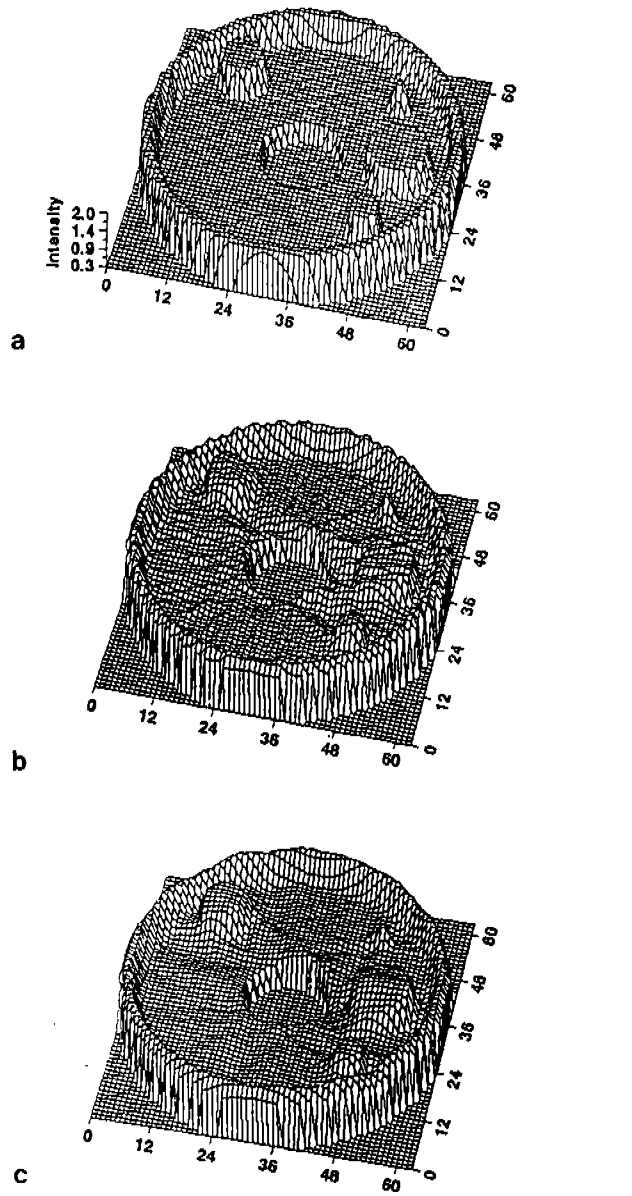

Fig. 6 Images for the test phantom II. **(a)** Original image. **(b)** Image reconstructed by the IRR method. **(c)** Image reconstructed by the proposed method.

Also, the proposed method was evaluated by the ultrasound attenuation CT where the unknown projection problem arises often due to many obstacles such as bones or gases in body. A sponge phantom with five holes, which is widely used in the ultrasound experiments because of its similar acoustic characteristics with tissue, was scanned by a precisely controlled mechanical scanner. The ultrasonic wave was generated by a weakly focused 2.25 MHZ transducer with a bandwith of 0.9 MHZ and the transmitted wave was received by another transducer with the same chracteristics as the transmitter. Both the sponge phantom and two transducers were immersed in a water bath and scanned linearly by a stepping motor at 51 linear positions

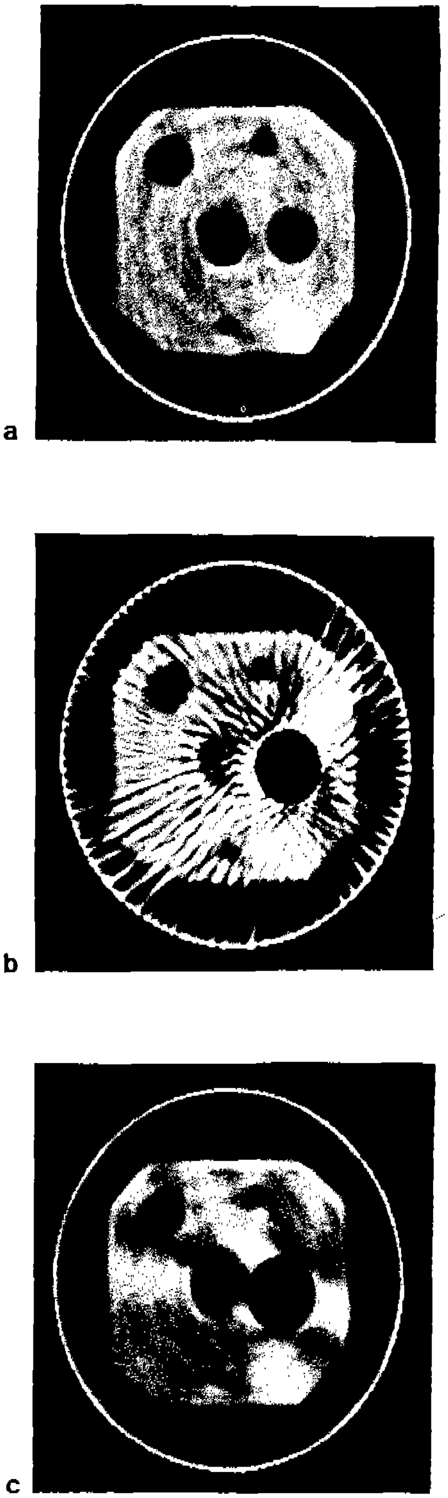

Fig. 7 Ultrasonic attenuation tomograms of a sponge. **(a)** Image reconstructed by a CB algorithm for an object without obstacles. **(b)** Image reconstructed by the existing IRR for the object with an obstacle located at the right-most hole in (a). **(c)** Imagereconstructed by the proposed IRR.

for each of 27 views around the object over 180 degrees. After digitization of the received signals with a 100MHz rate by the waveform recorder, ultrasonic attenuation tomogram is reconstructed by the peak value method [14].

Fig. 7 (a) shows the reconstructed image by a CB for an object having no obstacles. Fig. 7 (b) and 7 (c) show the reconstructed images by the proposed method and the existing IRR for the object having an obstacle located at the right-most hole in Fig. 5 (a), respectively, each being obtained after five iterations. It is observed that the image reconstructed by the proposed method shows a substantial reduction of artifacts, specifically, the errors with high spatial frequencies.

4. CONCLUSION

A new method for the estimation of unknown projection data based on known projection data and the bandwidth of projection data has been proposed and evaluated by simulation and experiment. The proposed method successfully estimates the unknown projection data through iterative transformation between projection space and frequency space using the known projection data and the bandwidth of the projection data. Through computer simulation and experiment, it has been shown that the proposed method performs the good estimation of the unknown projection data. It is to be added that the proposed method can also be applied to the truncated projection problem in CT.

REFERENCES

1. K. C. Tam and V. P. Mendez, *"Tomographical imaging with limited angle input"*, J. Opt. Soc. Am., 71, 1981.
2. K. C. TAM and V. P. MENDEZ, *"Limits to image reconstruction from restricted angular input"*, IEEE Trans. Nuclear Science, NS-28, 1981.
3. M. Nassi, W. R. Brody, B. P. Medoff, and A. Mascovski, *"Iteractive reconstruction-reprojection: an algorithm for limited data cardiac-computed tomography"*, IEEE Trans. Biomed. Eng., BME-29, pp. 331-341, 1982.
4. J. S. Choi, K. Ogwa, M. Nakajima, and S. Yuta, *"A reconstruction algorithm of body section with opaque obstructions"*, IEEE Trans. Son. Ultrason., SU-29, pp. 143-150, 1982.
5. M. Nassi, *"Iterative reconstruction-reprojection"*, IEEE Trans. Biomedical Engineering, BME-29, 1982.

6. K. M. Hanson and G. W. Wecksung, "*Bayesian approach to limited angle reconstruction in computed tomography*", *J. Opt. Soc. Am., 73, 1983.*
7. *B. P. Medoff, W. R. brody, M. Nassi, and Macovski, "Iteractive convolution backprojection algorithms for image reconstruction from limited data"*, J. Opt. Soc. Am., 73, 1983.
8. K. Ogata, M. Nakajima, and S. Yuta, "*A reconstruction algorithm from truncated projections*", IEEE Trans. Medical Imaging, MI-3, 1984.
9. J. H. Kim, K. Y. Kwak, S. B. Park, and Z. H. Cho, "*Projection space iteration reconstruction-reprojection*", IEEE Trans., Medical Imaging, MI-4, pp. 139-143, 1985.
10. K. H. Park and S. B. Park, "*Maximum entropy image reconstruction for an object with opaque obstructions*", IEEE Trans., Medical Imaging, MI-6, pp. 308-312, 1987.
11. K. H. Park, "*Improved iterative reprojection-reconstruction*", Electronic Technology Reports of Kyungpook National University, 16, pp. 75-78, 1987.
12. K. H. Park, B. M. Jung, and Y. H. Ha, "*Image reconstruction from incomplete data using new acquisition method of projection data*", trans. KITE, 25(12), pp. 57-63, 1988.
13. K. H. Park, "*Iterative reconstruction reprojection reducing the discontinuity of projection data*", Electronic Technology Reports of Kyungpook National University, 14(2), pp. 56-62, 1993.
14. K. A. Dines and A. C. Kak, "*Ultrasonic attenuation tomography of soft tissue*", Ultrason. Imaging, pp. 16-33, 1979.