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STABILITY ANALYSIS OF TWO–STAGE STOCHASTIC PROGRAMMING PROBLEMS

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I. Introduction

Consider a following two-stage stochastic programming problem as follows:

$$\begin{aligned} \text{minimize} \quad & g(x) + E_\xi\{\min \ d'y\} \\ \text{subject to} \quad & Ax = b \\ & Dx + Wy = \xi \\ & x \geq 0, y \geq 0, \end{aligned}$$

where A is an $m \times n$ matrix, D is an $\bar{m} \times n$ matrix, W is an $\bar{m} \times \bar{n}$ matrix, x is an $n-$vector, y and d are $\bar{n}-$vectors, b is an $m-$vector, $g : \mathbf{R}^n \to \mathbf{R}$ is a convex function, ξ is a random vector defined on the probability space $(\Xi, \Im, \mu)$, $\Xi \subset \mathbf{R}^{\bar{m}}$ is a Borel set, $\Im$ is a $\sigma-$algebra of all the Borel subsets of Ξ, and μ is a Borel probability measure.

We use the notation ξ to denote a random vector of dimension $\bar{m}$, as well as the specific values assumed by this random variable. Let us denote the second stage problem by

$$Q(x, \xi) = \min\{d'y \mid Wy = \xi - Dx, y \geq 0\}.$$

Then we obtain the following programming problem:

$$\begin{aligned} \text{minimize} \quad & g(x) + \int_{\mathbf{R}^{\bar{m}}} Q(x, \xi)\mu(d\xi) \\ \text{subject to} \quad & Ax = b \\ & x \geq 0. \end{aligned}$$

Now we introduce some definitions and theorems.

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DEFINITION 1. A set K is a *convex polyhedron* if it is the intersection of finite number of closed half-spaces.

Define $\wp(\mathbf{R}^{\bar{m}})$ to be the set of all Borel probability measures on $\mathbf{R}^{\bar{m}}$. We define the bounded Lipschitz metric β on $\wp(\mathbf{R}^{\bar{m}})$ as follows:

$$\beta(\mu,\nu) = \sup\Big\{ \Big| \int_{\mathbf{R}^{\bar{m}}} g(\xi)\mu(d\xi) - \int_{\mathbf{R}^{\bar{m}}} g(\xi)\nu(d\xi) \Big| \;:\; g : \mathbf{R}^{\bar{m}} \to \mathbf{R},\ \|g\|_{BL} \le 1 \Big\},$$

for any μ and $\nu \in \wp(\mathbf{R}^{\bar{m}})$, where

$$\| g \|_{BL} = \sup_{\xi \in \mathbf{R}^{\bar{m}}} | g(\xi) | + \sup_{\xi \neq \tilde{\xi}} \frac{| g(\xi) - g(\tilde{\xi}) |}{d(\xi, \tilde{\xi})} < \infty,$$

and

$$d(\xi,\tilde{\xi}) = \sqrt{(\xi_1 - \tilde{\xi}_1)^2 + \cdots + (\xi_{\bar{m}} - \tilde{\xi}_{\bar{m}})^2},$$

for $\xi = (\xi_1, \cdots, \xi_{\bar{m}}),\ \tilde{\xi} = (\tilde{\xi}_1, \cdots, \tilde{\xi}_{\bar{m}})$.

Define for $q > 1,\ c > 0$,

$$\wp(\mathbf{R}^{\bar{m}} : q, c) = \Big\{ \mu \in \wp(\mathbf{R}^{\bar{m}}) \mid \int_{\mathbf{R}^{\bar{m}}} \| \xi \|^{2q} \mu(d\xi) \le c \Big\}.$$

In the following section, we study the stability of problem (2) when we perturb the probability measure $\mu \in \wp(\mathbf{R}^{\bar{m}} : q, c)$. This is an extension of the result in [3]. In W. Römisch and R. Schultz's paper, the objective function $g(x)$ is linear. But we consider the objective function $g(x)$ is a convex function. And we consider multiple objective functions by using the weight parameter and we also observe the stability of the problem with respect to the weight parameter and probability measure.

Let us consider the following problem with a parameter $t \in T$.

$$p(t) \;:\; \min\{ f(x,t) : x \in M(t) \},$$

where T is a metric space with distance function $d(\cdot,\cdot)$, M is a closed-valued multifunction from T into $\mathbf{R}^n$, and $f : \mathbf{R}^n \times T \to \mathbf{R}$ is a continuous function. Given $Q \subset \mathbf{R}^n$, for any $t \in T$, we have

$$\begin{aligned}
M_Q(t) &= M(t) \cap clQ,\\
\varphi_Q(t) &= \inf\{f(x,t) \mid x \in M_Q(t)\},\\
\psi_Q(t) &= \{x \in M_Q(t) \mid f(x,t) = \varphi_Q(t)\},
\end{aligned}$$

where clQ denotes the closure of Q. We call φ_Q the optimal value function with respect to cl$Q$ and ψ_Q the optimal set mapping with respect to cl$Q$.

DEFINITION 2. Given $t^o \in T$, a nonempty set $X \subset \mathbf{R}^n$ is called a *complete local minimizing set* (CLM) for $f(.,t^o)$ on $M(t^o)$ if there is an open set Q containing X such that $X = \psi_Q(t^o)$.

DEFINITION 3. $M : T \to \mathbf{R}^n$, is said to be *closed* at t^o if and only if

$$\begin{aligned}
&t^k \to t^o,\ x^k \to x^o, \text{ as } k \to \infty, \quad \text{and}\\
&x^k \in M(t^k) \Rightarrow x^o \in M(t^o).
\end{aligned}$$

DEFINITION 4. A multifunction M from T into $\mathbf{R}^n$ is said to be *pseudo-Lipschitzian* at (x^o,t^o), where $t^o \in T$ and $x^o \in M(t^o)$, if there are neighborhoods $U = U(t^o)$ and $V = V(x^o)$ and a positive real number l such that both

$$\begin{aligned}
M(t) \cap V &\subset M(t^o) + l \cdot d(t,t^o) \cdot B_n \text{ and}\\
M(t^o) \cap V &\subset M(t) + l \cdot d(t,t^o) \cdot B_n
\end{aligned}$$

hold for all $t \in U$, where B_n is the closed unit ball in $\mathbf{R}^n$, and

$$X + \beta \cdot B_n = \{x + \beta \cdot u \mid x \in X, u \in B_n\},$$

for $X \subset \mathbf{R}^n$ and $\beta \in \mathbf{R}$.

THEOREM 1. ([1]) *Consider the parametric program $p(t)$. Fix $t^o \in T$ with the following conditions:*
(C1) There exists a bounded open subset V of $\mathbf{R}^n$ and a nonempty subset X of V such that $X = \psi_V(t^o)$.
(C2) The multifunction M is closed-valued and closed at t^o.
(C3) M is a pseudo-Lipschitzian at each pair $(x^o, t^o) \in \psi_V(t^o) \times \{t^o\}$.
(C4) There exist real numbers $p \in (0,1]$, $L_f > 0$ and $\delta_f > 0$ such that

$$| f(x, t^o) - f(y, t) | \leq L_f(\| x - y \| + d(t, t^o)^p)$$

for each $x, y \in \text{cl}V$ and each t satisfying $d(t, t^o) < \delta_f$.

Then the following properties hold:

(a) The multifunction ψ_V is upper semicontinuous at t^o, i.e. for each $\epsilon > 0$ there exists $\eta > 0$ such that

$$\psi_V(t) \subset \psi_V(t^o) + \epsilon B_n, \quad \text{when } d(t, t^o) < \eta.$$

(b) There exist positive real numbers δ_ρ and L_ρ such that $\psi_V(t) \neq \emptyset$ is a CLM set for $f(., t)$ on $M(t)$ and such that

$$| \varphi_V(t) - \varphi_V(t^o) | \leq L_\rho d(t, t^o)^p \quad \text{whenever } d(t, t^o) < \delta_\rho.$$

2. Stability analysis.

In the following, we study the stability of problem (2) when we perturb the probability measure $\mu \in \wp(\mathbf{R}^{\bar{m}} : q, c)$. Denote $F(x, \mu) = g(x) + \int_{\mathbf{R}^m} Q(x, \xi)\mu(d\xi)$.

We also denote a feasible solution set to (2) by

$$K = \{x \mid Ax = b,\ x \geq 0\} \cap \{x \mid \forall \xi, \exists y \geq 0 \text{ such that } Wy = \xi - Dx\},$$

and define a multi-valued function $M : \wp(\mathbf{R}^{\bar{m}} : q, c) \longrightarrow \mathbf{R}^n$ by $M(\mu) = K$ for any μ. Then K is a convex polyhedron([4]).

Given $V \subset \mathbf{R}^n$ and $\mu \in \wp(\mathbf{R}^{\bar{m}} : q, c)$, we set

$$M_V(\mu) = M(\mu) \cap \text{cl}V,$$
$$\rho_V(\mu) = \inf\{F(x, \mu) \mid x \in M_V(\mu)\},$$
$$\psi_V(\mu) = \{x \in M_V(\mu) \mid F(x, \mu) = \rho_V(\mu)\}.$$

Fix some $\mu_o \in \wp(\mathbf{R}^{\bar{m}} : q, c)$ and assume the following hypotheses:
$(H1)$ $\psi(\mu_o)$ is nonempty and bounded.
$(H2)$ $W \in L(\mathbf{R}^{\bar{n}}, \mathbf{R}^{\bar{m}})$ such that for all $v \in \mathbf{R}^{\bar{m}}$, $\{y \in \mathbf{R}^{\bar{n}} \mid Wy = v,\ y \geq 0\} \neq \emptyset$ and $\{u \in \mathbf{R}^{\bar{m}} \mid W'u \leq d\} \neq \emptyset$.
$(H3)$ g satisfies Lipschitz condition.

LEMMA 1. *Let $B \subset \mathbf{R}^n$ nonempty and compact. Let $q > 1$ and $c > 0$. Fix some $\mu_o \in \wp(\mathbf{R}^{\bar{m}} : q, c)$. Assume that $(H1)$, $(H2)$, and $(H3)$ are satisfied. Then there exists $L_F > 0$ such that*

$$| F(x,\mu) - F(y,\mu_o) | \leq L_F(\| x - y \| + \beta(\mu,\mu_o)^{1-1/q}),$$

for all $x, y \in B$ and all $\mu \in \wp(\mathbf{R}^{\bar{m}} : q, c)$.

Proof.

$$\begin{aligned} | F(x,\mu) - F(y,\mu_o) | \leq & \ | g(x) - g(y) | \\ & + \left| \int_{\mathbf{R}^m} Q(x,\xi)\mu(d\xi) - \int_{\mathbf{R}^m} Q(y,\xi)\mu(d\xi) \right| \\ & + \left| \int_{\mathbf{R}^m} Q(y,\xi)\mu(d\xi) - \int_{\mathbf{R}^m} Q(y,\xi)\mu_o(d\xi) \right| . \end{aligned}$$

By $(H3)$, there exists constant b such that $| g(x) - g(y) | \leq b \| x - y \|$, for all $x,\ y \in B$. Let $\bar{r} > 0$ satisfy $\max\{\| d \| + \| \xi - Dx \|,\ \| d \| + \| \xi - Dy \|\} \leq \bar{r}$, and then, by Proposition 24 in [5] and Lemma 3.2 in [3], we get $| Q(x,\xi) - Q(y,\xi) | \leq e \cdot \bar{r} \cdot \| D \| \| x - y \|$, for some constant $e \geq 0$. Therefore

$$\left| \int_{\mathbf{R}^m} (Q(x,\xi) - Q(y,\xi))\mu(d\xi) \right| \leq e \cdot \bar{r} \cdot \| D \| \| x - y \| .$$

Take $L_1(t) = e \cdot \bar{r} \cdot t$, and by Theorem 2.1, [2], for $q > 1$,

$$\begin{aligned} & \left| \int_{\mathbf{R}^m} Q(y,\xi)\mu(d\xi) - \int_{\mathbf{R}^m} Q(y,\xi)\mu_o(d\xi) \right| \\ \leq & h(1 + M_q(\mu) + M_q(\mu_o))\beta(\mu,\mu_o)^{1-1/q}, \end{aligned}$$

where $h = \max\{4, 2e\|d\| + 10|Q(y,0)|\}$ and $M_q(\mu) = (\int_{\mathbf{R}^{\bar m}} L_1(\| \xi \|^2)^q \mu(d\xi))^{1/q}$. Therefore

$$\begin{aligned} &| F(x,\mu) - F(y,\mu_o) | \\ \leq & b \| x-y \| + e \cdot \bar{r} \cdot \| D \| \| x-y \| \\ & \quad + h(1 + M_q(\mu) + M_q(\mu_o))\beta(\mu,\mu_o)^{1-1/q} \\ \leq & L_F(\| x-y \| + \beta(\mu,\mu_o)^{1-1/q}), \end{aligned}$$

where $L_F = \max\{b + e \cdot \bar{r} \cdot \| D \|, h(1 + M_q(\mu) + M_q(\mu_o))\}$.

THEOREM 2. *Fix some $\mu_o \in \wp(\mathbf{R}^{\bar m} : q, c)$ and assume that $(H1)$, $(H2)$ and $(H3)$ are satisfied. Then it follows that*
(a) for each $\epsilon > 0$ there exists some $\eta > 0$ such that

$$\psi(\mu) \subset \psi(\mu_o) + \eta \cdot B_n,$$

whenever $\mu \in \wp(\mathbf{R}^{\bar m} : q, c)$ and $\beta(\mu,\mu_o) < \eta$.
(b) There exists positive reals δ_ρ and L_ρ such that $\psi_V(\mu) \neq \emptyset$

$$| \rho(\mu) - \rho(\mu_o) | \leq L_\rho \cdot \beta(\mu,\mu_o),$$

whenever $\mu \in \wp(\mathbf{R}^{\bar m} : q, c)$ and $\beta(\mu,\mu_o) < \delta_\rho$.

Proof. Take $(T,d) = (\wp(\mathbf{R}^{\bar m} : q, c), \beta)$, a metric space. We check the conditions of Theorem 1.
$(C1)$:Since $F(x,\mu_o)$ is a convex function, $\psi(\mu_o)$ is a global local minimizing set and hence $\psi(\mu_o)$ is a CLM set. And since $\psi(\mu_o)$ is bounded, $(C1)$ is hold. $(C2)$ and $(C3)$ are trivially hold. $(C4)$ is proved in Lemma 1.

Now we consider multiple r-objective two-stage stochastic programming problem as follows:

$$\begin{aligned} &\text{minimize } \; g_i(x) + \int_{\mathbf{R}^{\bar m}} Q(x,\xi)\mu(d\xi) \\ &\text{subject to } Ax = b \\ &\qquad\qquad\quad x \geq 0, \end{aligned}$$

where $g_i(x)$, $i = 1, \cdots, r$, is a convex function, $Q(x, \xi) = \min\{d'y \mid Wy = \xi - Dx,\ y \geq 0\}$, and all other vectors are the same as in problem (1).

We approach problem (3) by the weighted sum of the r functions using the weights λ_i, $i = 1, \cdots, r$. Hence the problem becomes

$$\begin{aligned} \text{minimize } & \sum\nolimits_{i=1}^{r} \lambda_i g_i(x) + \int_{\mathbf{R}^{\bar{m}}} Q(x,\xi)\mu(d\xi) \\ \text{subject to } & Ax = b \\ & x \geq 0, \end{aligned}$$

where $\lambda_i \geq 0$, $\sum_{i=1}^{r} \lambda_i = 1$.

We want to study the stability of problem (4) when we perturb weight parameter λ and probability measure μ. We denote the objective function by $F(x, \lambda, \mu)$. Let s be a random variable defined on the sample space $\Omega = \{s_1, s_2, \cdots, s_r\}$ and $s_i \in \mathbf{R}$. We may consider $\Lambda = \{\lambda \in \mathbf{R}^r \mid \sum_{i=1}^{r} \lambda_i = 1, \lambda_i \geq 0\}$ as a discrete probability measure space on Ω with $p(s_i) = \lambda_i$. Define a usual metric on a parameter space $\Lambda = \{\lambda = (\lambda_1, \cdots, \lambda_r) \mid \lambda_i \geq 0, \sum_{i=1}^{r} \lambda_i = 1\}$ by

$$d(\lambda, \lambda_o) = \sqrt{(\lambda_1 - \lambda_{o1})^2 + \cdots + (\lambda_r - \lambda_{or})^2},$$

for λ, $\lambda_o \in \Lambda$. Then (Λ, d) is a metric space and for any $q > 1$,

$$\sum_{i=1}^{r} \lambda_i \cdot | s_i |^q \leq \sum_{i=1}^{r} \lambda_i \cdot s^q,$$

where $s = \max\{| s_i | \mid 1 \leq i \leq r\}$. We may consider $\Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$ as a product of two probability measure spaces. Define a metric $\tilde{d}$ on $\Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$ by

$$\tilde{d}((\lambda, \mu), (\lambda_o, \mu_o)) = d(\lambda, \lambda_o) + \beta(\mu, \mu_o)^{1-1/q}.$$

Then $(\Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c), \tilde{d})$ is a metric space. We also denote a feasible solution set to (4) by

$$K = \{x \mid Ax = b, x \geq 0\} \cap \{x \mid \forall \xi, \exists y \geq 0 \text{ such that } Wy = \xi - Dx\},$$

and define a multi-valued function $M : \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c) \to \mathbf{R}^n$ by taking

$$M(\lambda, \mu) = K, \text{ for any } (\lambda, \mu) \in \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c).$$

Given $V \subset \mathbf{R}^n$ and $\lambda \in \Lambda$, we set

$$M_V(\lambda, \mu) = M(\lambda, \mu) \cap clV,$$

$$\rho_V(\lambda, \mu) = \inf\{F(x, \lambda, \mu) \mid x \in M_V(\lambda, \mu)\},$$

$$\psi_V(\lambda, \mu) = \{x \in M_V(\lambda, \mu) \mid F(x, \lambda, \mu) = \rho_V(\lambda, \mu).$$

Fix some $(\lambda_o, \mu_o) \in \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$ and assume the following hypotheses:

$(H1)$ $\psi_V(\lambda_o, \mu_o)$ is nonempty and bounded.

$(H2)$ $W \in L(\mathbf{R}^{\bar{n}}, \mathbf{R}^{\bar{m}})$ such that for all $v \in \mathbf{R}^{\bar{m}}$, $\{y \in \mathbf{R}^{\bar{n}} \mid Wy = v,\ y \geq 0\} \neq \emptyset$ and $\{u \in \mathbf{R}^{\bar{m}} \mid W'u \leq q\} \neq \emptyset$.

$(H3)$ g_i satisfies Lipschitz condition for each i.

LEMMA 2. *Fix some $(\lambda_o, \mu_o) \in \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$ and assume that $(H1)$, $(H2)$ and $(H3)$ are satisfied. Let $B \subset \mathbf{R}^n$ be nonempty and compact. Let $q > 1$ and $c > 0$. Then there exists $L_F > 0$ such that*

$$| F(x, \lambda_o, \mu_o) - F(y, \lambda, \mu) | \leq L_F(\| x - y \| + \tilde{d}((\lambda, \mu), (\lambda_o, \mu_o))),$$

for all $x,\ y \in B$ and all $(\lambda, \mu) \in \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$.

Proof.

$$\begin{aligned} & | F(x, \lambda_o, \mu_o) - F(y, \lambda, \mu) | \\ \leq & \left| \sum_{i=1}^{r} \lambda_{oi} g_i(x) - \sum_{i=1}^{r} \lambda_i g_i(y) \right| \\ & + \left| \int_{\mathbf{R}^{\bar{m}}} (Q(x, \xi) - Q(y, \xi)) \mu_o(d\xi) \right| \\ & + \left| \int_{\mathbf{R}^{\bar{m}}} Q(y, \xi) \mu_o(d\xi) - \int_{\mathbf{R}^{\bar{m}}} Q(y, \xi) \mu(d\xi) \right| \end{aligned}$$

By $(H3)$, there exists $b_i \geq 0$ such that

$$| g_i(x) - g_i(y) | \leq b_i \| x - y \|,$$

for all $x,\ y \in B$ and for all $i = 1, \cdots, r$. Let $\bar{b} = \max\{b_1, b_2, \cdots, b_r\}$. And since each g_i satisfies Lipschitz condition, g_i is bounded on B. Let $m_i \geq 0$ such that $| g_i(x) | \leq m_i$. Then

$$\begin{aligned}
&| \sum_{i=1}^{r} \lambda_{oi} g_i(x) - \sum_{i=1}^{r} \lambda_i g_i(y) | \\
&\leq | \sum_{i=1}^{r} \lambda_{oi}(g_i(x) - g_i(y) | + | \sum_{i=1}^{r} (\lambda_{oi} - \lambda_i) g_i(y) | \\
&\leq \sum_{i=1}^{r} | g_i(x) - g_i(y) | + \| \lambda_o - \lambda \| \sum_{i=1}^{r} | g_i(y) | \\
&\leq r \cdot \bar{b} \| x - y \| + (m_1 + m_2 + \cdots + m_r) \| \lambda_o - \lambda \| .
\end{aligned}$$

By the same way in Lemma 1, we have

$$| \int_{\mathbf{R}^m} (Q(x,\xi) - Q(y,\xi)) \mu_o(d\xi) | \leq e \cdot \bar{r} \cdot \| D \| \| x - y \| .$$

$$\begin{aligned}
&| \int_{\mathbf{R}^m} Q(y,\xi) \mu_o(d\xi) - \int_{\mathbf{R}^m} Q(y,\xi) \mu(d\xi) | \\
&\leq h \cdot (1 + M_q(\mu) + M_q(\mu_o)) \beta(\mu, \mu_o)^{1-1/q}.
\end{aligned}$$

Hence

$$\begin{aligned}
&| F(x, \lambda_o, \mu_o) - F(y, \lambda, \mu) | \\
&\leq (r \cdot \bar{b} + e \cdot \bar{r} \cdot \| D \|) \| x - y \| + L_F{}'(\| \lambda_o - \lambda \| + \beta(\mu, \mu_o)^{1-1/q}), \\
&\quad \text{where } L_F{}' = \max\{m_1 + \cdots + m_r,\ h(1 + M_q(\mu) + M_q(\mu_o))\} \\
&\leq L_F(\| x - y \| + \tilde{d}((\lambda, \mu), (\lambda_o, \mu_o)), \\
&\quad \text{where } L_F = \max\{r \cdot \bar{b} + e \cdot \bar{r} \cdot \| D \|, L_F{}'\}.
\end{aligned}$$

So we have the following theorem.

THEOREM 3. *Consider problem (4). Fix some* $(\lambda_o, \mu_o) \in \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$ *and assume that* $(H1)$, $(H2)$ *and* $(H3)$ *are satisfied. Then it follows that*

(a) *for each* $\epsilon > 0$, *there exists* $\eta > 0$ *such that*

$$\psi_V(\lambda, \mu) \subset \psi_V(\lambda_o, \mu_o) + \epsilon B_n,$$

whenever $(\lambda, \mu) \in \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$ *and* $\tilde{d}((\lambda, \mu), (\lambda_o, \mu_o)) < \eta$.
(b) *There exist positive reals* δ_ρ *and* L_ρ *such that* $\psi_V(\lambda, \mu) \neq \emptyset$

$$| \rho_V(\lambda, \mu) - \rho_V(\lambda_o, \mu_o) | \leq L_\rho \tilde{d}((\lambda, \mu), (\lambda_o, \mu_o)),$$

whenever $(\lambda, \mu) \in \Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c)$ *and* $\tilde{d}((\lambda, \mu), (\lambda_o, \mu_o)) < \delta_\rho$.

Proof. Take $(T, d) = (\Lambda \times \wp(\mathbf{R}^{\bar{m}} : q, c), \tilde{d})$, a metric space. We check the conditions of Theorem 1.
$(C1)$, $(C2)$ and $(C3)$ are clear. $(C4)$: This is proved in Lemma 2.

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