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Multi-product Lot Quantity Verification : A Weighing InspectionApproach

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Abstract

This paper presents an alternative inspection method for counting items of a lot(or kit) in production lines or distribution centers. In this inspection, lots are weighed instead of counting all items of the lots in order to reduce the effort required for the 100% manual counting inspection. Inspection errors of this inspection procedure are analyzed and the impact of the variability of item weights on inspection errors are investigated. Two approaches, the cost assessment approach and the bicriterion decision making approach, are presented for the implementation of this inspection procedure.

1. INTRODUCTION

In many production lines or distribution centers, items are transported in lots (all items of the same type) or kits (different types of items). Hereafter the word "lot" will represent a lot or a kit. Verifying that a lot contains the correct number of items is necessary to prevent incurring costs of over-shipments and under-shipments. An over-shipment is a lot containing more units than the target or order quantity. An under-shipment is a lot containing fewer units than specified. A 100% counting inspection is commonly used for this verification and errors found are rectified. Such counting is often lengthy and costly, particularly when lots consist of items of varying size and shape.

This paper presents an alternative inspection procedure, called *weighing inspection*, to the 100% manual counting inspection of each lot. The weighing inspection weighs each lot and recommends a 100% inspection only when the actual wieght of the lot deviates from the target weight by more than a predetermined tolerance level. In doing so, it reduces the number of 100% inspections.

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This inspection strategy, for example, can be implemented at distribution centers where multiple parts are retrived from storage according to an order and delivered to a packaging area. A quick lot quantity verification is often required here before its final packaging for shipment to customers.

There have been some studies in the literature that discuss controlling product quality by monitoring product weight. Duling(1976) presents a study concerning a checkweigher to control fill on a packaging line. The use of the checkweigher eliminates the need of any sampling plan or control charts because it is essentially 100% testing, with rectification made as necessary. A similar study has been addressed by Mizenko and Martin (1977). The study investigates various types of weight control systems that used to provide proper operation of filling machines. Overfilling increases costs, while underfilling conflicts with legal requirements. Computer assisted weight control systems have also been proposed by Parobeck, et al. (1981) and Fisher (1983). However, procedures that use the weight of a lot to estimate the number of items in the lot have not been presented in the literature.

Here we discuss the design and implementation of a weighing inspection procedure. In section 2, inspection errors caused by this procedure are defined and illustrated using a numerical example. The impact of the variability of item weight and the tolerance level on the inspection procedure are investigated in section 3, and two approaches to determine the best tolerance level are introduced in section 4.

Assumptions :

a. The unit weight of each item is normally distributed with known mean and variance and is independent between items.

b. The actual number of units of item i in the lot has a probability distribution over a finite range and it is independent between items.

c. There are no manual inspection errors.

2. WEIGHING INSPECTION PROCEDURE AND INSPECTION ERRORS

The weighing inspection procedure is similar to a single inspection to decide whether or not an item is conforming. In this inspection, the decision whether or not each lot contains the exact target quantities is made by weighing the lot and comparing the weight with the target weight. A decision of "good lot" is made if the actual weight deviates from the target weight by less than a predetermined tolerance level, ε. If the lot is not classified as a good lot, a 100% manual inspection is performed The basis of this inspection procedure is that pre-screening of bad lots will make the overall inspection process more effective. Similar approaches can be found in studies dealing with two-inspector problems (Drury et al. (1986)).

Because of the nature of the weighing inspection, the inspection procedure can result in either a type Ⅰ or type Ⅱ error. A type Ⅰ error occurs if the lot is declared bad but found to be good after a 100% manual inspection. A type Ⅱ error occurs if a bad lot is classified as good. The cost of a type Ⅰ error consists of the inspection labor cost and delay cost caused by the manual inspection operation. The type Ⅱ error cost consists of customer dissatisfaction and expediting costs associated with the faulty lot. Let

α and β represent the probabilities of making type I and type II errors, respectively. Then α and β can be mathematically represented as,

$$\alpha = P[\,|W_A - \mu_T| \geq \varepsilon \mid X = N] \quad (1)$$

$$\beta = P[\,|W_A - \mu_T| < \varepsilon \mid X \neq N] \quad (2)$$

where m = number of different types of items in the lot,

n_i = target number of units of item i, i = 1, ⋯, m, in the lot

x_i = actual number of units of item i in the lot,

μ_i = mean weight of item i,

σ_i^2 = variance of weight of item i,

w_{ij} = actual weight of the j^{th} unit of item i,

W_A = the actual weight of the lot

$$(W_A = \sum_{i=1}^{m} \sum_{j=1}^{x_i} w_{ij}),$$

μ_T = the target weight of the lot

$$(\mu_T = \sum_{i=1}^{m} \sum_{j=1}^{m} n_i \mu_i),$$

ε = maximum allowable weight deviation (the tolerance),

$N = (n_1, n_2, \cdots, n_m)$ and

$X = (x_1, x_2, \cdots, x_m)$.

w_{ij}'s and x_i's are *random variables* representing the weights and the number of units of items, respectively.

Samller α and β values would result in better weighing inspection. It will be an ideal inspection if $\alpha = 0$ and $\beta = 0$, which is obtainable only by 100% counting inspection of the lot. Assuming that w_i is normally distributed and independent between the items, the *conditional distribution* of $W_A = \sum_{i=1}^{m} \sum_{j=1}^{x_i} w_{ij}$ for *fixed* X is then normal with $E(W_A \mid X) = \sum_{i=1}^{m} x_i \mu_i$ and $V(W_A \mid X) = \sum_{i=1}^{m} x_i \sigma_i^2$ (Larsen and Marx (1981)). In this case, α can be easily calculated using the standard normal distribution. The value of β, however, depends entirely on the types of faulty lots. Since there are an infinite number of ways a lot can be faulty, it is not possible to list all possible β values. Instead, we use the *unconditional* β value, denoted as $\bar{\beta}$, in order to represent the probability of committing a type II error in the inspection process. Mathematically, $\bar{\beta}$ is defined as

$$\bar{\beta} = \sum_{X \neq N} \{P(X)\, P[\,|W_A - \mu_T| < \varepsilon \mid X]\} \quad (3)$$

where $\sum_{X \neq N}$ represents the summation over the possible $X \neq N$ cases. Note that X may be equal to N in which case a type II error cannot occur. This approach using the unconditional β value is also found in the literature where a human inspector is assumed to have a constant type II inspection error, regardless of the fatality of the defect (Collins, et al. (1973) and Maghsoodloo (1987)).

As mentioned earlier, there are theoretically an infinite number of ways a lot can be faulty (i.e., $X \neq N$ cases). This is, however, not true in practice, and we assume that the actual number of units of item i in the lot, x_i, has a probability distribution *over a finite range*. Using the assumed probability distribution, one can calculate the probabilities of misshipments and $\bar{\beta}$ according to equation (3).

In this paper, we illustrate the calculation procedure of $\bar{\beta}$ using the *normal approxima-*

tion to the distribution. This normal approximation is later compared with three binomial distributions to investigate the impact of the distribution on the weighing inspection. *It is noted that one can use any other discrete distribution or approximation by a continuous distribution that well represents the actual case.* The ideas behind the normal approximation are that

○ It is likely that the actual number of units of item i in the lot has a probability distribution that is symmetric and centered at the target number of units, n_i.

○ The probability that $x_i=n_i$ (correct shipment of item i) is relatively very large as compared to other x_i values

○ The probability rapidly decreases as x_i deviates from the target number of units, and the range of x_i is proportional to n_i (i.e., when a large number of units are picked, the range of misshipments will increase)

The normal approximation with the mean of n_i and the standard deviation of n_ik, where k is a constant coefficient of variation, incorporates these properties. The coefficient of variation is expected to be very small, i.e., the standard deviation is expected to be small compared to the mean. Although we use a single k over all items in the numerical example below, any value of k for each item can be applied to calculate $\bar{\beta}$.

Using the normal approximation, the ranges of x_i's are determined by $n_i \pm [3(n_ik)]^+$, for all i†. One rule of thumb to determine the value of k is to fit this range to the practical rangeof possible misshipment for each item. In this case, the ranges of x_i is directly proportional to n_i. Given the ranges of x_i's and their distributions, $\bar{\beta}$ is now obtainable by enumerating all misshipments within the ranges of all x_i's. Example 1 is used to illustrate the calculations of α and $\bar{\beta}$.

Example 1 : Suppose that the target number of units of 5 different items in a lot are $n_1=5$, $n_2=12$, $n_3=3$, $n_4=3$, and $n_5=7$. Let $\mu_1=10$ (unit=1bs), $\mu_2=7$, $\mu_3=5$, $\mu_4=3$ and $\mu_5=6$, and $\delta_1^2=0.10$, $\delta_2^2=0.07$, $\delta_3^2=0.05$, $\delta_4^2=0.03$ and $\delta_5^2=0.06$. Also assumed is that the standard dev the actual number units, x_i, is $n_i/6$ (i.e., $k=1/6$) for all i

To calculate the probability of misshipment, the ranges of x_i's are first determined by $n_i \pm [3(n_ik)]^+$ for all i. For example, the range of x_i is calculated by $5 \pm [3(5/6)]^+$, or $2 \le x_1 \le 8$. Similarly, the ranges for other x_i's are determined : $6 \le x_2 \le 18$, $1 \le x_3 \le 5$, $1 \le x_4 \le 5$, and $3 \le x_5 \le 11$. Assuming that the tolerance of $\varepsilon=3$(1bs) is used for the weighing inspection, calculation procedures of α and $\bar{\beta}$ are illustrated.

Calculation of α :

α =probability of rejecting a good lot

$=P[\,|W_A-\mu_T|\ge\varepsilon\mid X=N]$

$=P[\,|W_A-\mu_T|\ge 3 \text{ lbs}\mid X=(5,\ 12,\ 3,\ 3,\ 7)]$

$=P[\,|Y|\ge 3\mid X=(5,\ 12,\ 3,\ 3,\ 7)]$

where $Y=W_A-\mu_T$.

Since $Y=(W_A-\mu_T)$ is $N(0, \sigma^2_{A|X})$, where $\sigma^2_{A|X}=V(W_A\mid X)=\sum_{i=1}^{5} x_i\sigma_i^2=2.0$, α can be easily calculated and the resulting $\alpha=0.03390$.

Calculation of $\bar{\beta}$:

$\bar{\beta}$=the unconditional probability to accept a bad lot

† $c=[r]^+$ is the smallest integer such that $c \ge r$. Due to this notation and the normal approximation to the discrete case, even if we used $\pm 3\sigma$ for the range, the probality that x_i is in the range will be sufficiently close to 1.

$$= \sum_{X \neq N} \{P(X)\ P[\ |\ W_A - \mu_T\ |\ < \varepsilon\ |\ X]\}.$$

The calculation of $\bar{\beta}$ requires two probabilities for each misshipment – the probability of the misshipment P(X) and the probability of type Ⅱ error β for a specified $X \fallingdotseq N$. P(X) is calculated by the probability distribution of x_i's. For example, when the normal approximtion is used, P[X=(4, 12, 3, 3, 7)] is computed by the product of $P[3.5 \leq x_1 \leq 4.5]$, $P[11.5 \leq x_2 \leq 12.5]$, $P[2.5 \leq x_3 \leq 3.5]$, $P[2.5 \leq x_4 \leq 3.5]$, and $P[6.5 \leq x_5 \leq 7.5]$, where x_i is $N(n_i, n_i^2/36)$ for all i. The probability of type Ⅱ error, β, for the misship-ment is then calculated using the assumption that the *conditional distribution* of $W_A = \sum_{i=1}^{m} \sum_{j=1}^{ki} w_{ij}$ for fixed X is normal with $E(W_A \mid X) = (W_A \sum_{i=1}^{m} x_i \mu_i$ and $V(W_A \mid X) = \sum_{i=1}^{m} x_i \sigma_i^2$. By calculating two probabilities and summing up their product over all misshipments (i.e., all possible $X \fallingdotseq N$ cases), $\bar{\beta} = 0.11561$ is obtained.

It was noted that any probability distribution, if it is appropriate for the situation, can be used as the distribution of the actual number of units of item i, x_i. In order to investigate the impact of the distribution of x_i on the weighing inspection, a comparative study was performed using Example 1. Three binomial distributions (with p=0.25, 0.5, and 0.75, where p is the probability that a unit is correctly picked) have been compared with the normal approximation for various ε values. The binomial distributions were shifted to the right so that the distributions could be defined over the range obtained by $n_i \pm [(3n_i k)]^+$. For example, when x_1 is assumed to be binomial with p=0.25, the probability distribution is defined as

$$P[x_1 = k] = \binom{6}{k-2} (0.25)^{(k-2)} (0.75)^{6-(k-2)},$$
for $2 \leq k \leq 8$.

The distributions were then used to calculate the probabilities of misshipments for determining $\bar{\beta}$. The results of the study are listed in Table 1.

It is seen from the table that the normal approximation resulted in the worst $\bar{\beta}$ values in all cases. This is because the normal approximation has higher probabilities of making minor (1 or 2 units) misshipments, as

Table 1. The distributions of x_i's and inspection errorss.

ε	α	$\bar{\beta}$			
		Normal	Binomial distribution with		
			p=0.25	p=0.5	p=0.75
0	1.00000	0.00000	0.00000	0.00000	0.00000
1	0.47950	0.03653	0.00046	0.03783	0.00046
2	0.15730	0.07503	0.00093	0.07611	0.00094
3	0.03390	0.11561	0.00143	0.11474	0.00143
4	0.00468	0.15705	0.00196	0.15335	0.00196
5	0.00041	0.19842	0.00252	0.19166	0.00253
6	0.00002	0.23969	0.00314	0.22953	0.00315
7	0.00000	0.28079	0.00382	0.26686	0.00383

the probabilities are higher near the mean as compared to other cases. This implies that the normal approximation would result conservative estimate of $\bar{\beta}$, because an over-estimation of $\bar{\beta}$ is better than an under-estimation.

Note that α is identical regardless of the distribution (see equation (1)). The binomial distributions with p=0.25 and p=0.75 imply tendencies in misshipments, more under-shipments or more over-shipments, respectively. Table 1 indicates, in this case, that the weighing inspection will perform well (i. e., $\bar{\beta}$ is small). This is because the probability of making major (a large number of units) misshipments is high and the probability that such major misshipments pass the weighing inspection is small for a fixed ε.

3. IMPACT OF σ^2 AND ε ON INSPECTION ERRORS

The values of α and $\bar{\beta}$ are important factors for the actual implementation of this inspection procedure. α is dependent on σ^2 and ε, while $\bar{\beta}$ relies on these two factors and k, the coefficient of variation. From equations (1) and (3), it can be easily proven that (a) α monotonically decreases on ε, and (b) $\bar{\beta}$ monotonically increases on ε. This is illustrated in Table 1. This characteristic states that α and $\bar{\beta}$ are in conflict and ε determines α and $\bar{\beta}$ simultaneously. It is noted here that ε can be chosen by the manager in such a way that it results in the most preferred values of α and $\bar{\beta}$. This will be discussed in section 4.

It is also obvious that for a fixed ε, α monotonically increases as the σ_i^2's increase, because a higher variability of item weights increases the possibility of judging good lots as bad. This is illustrated in Figure 1. In the figure, the variances are represented as fractions of the mean weights.

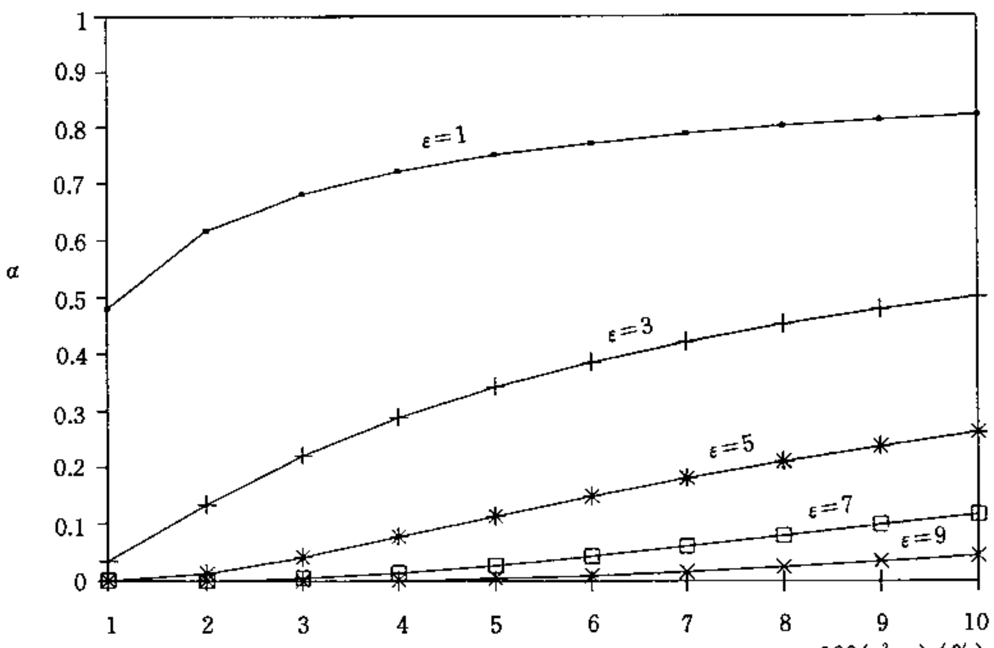

Figure1. The impact of increases in σ_i^2 on α

In Figure 2, $\bar{\beta}$ is plotted against the increases in σ_i^2's. It is shown that $\bar{\beta}$ is decreasing on σ_i^2's. It seems that $\bar{\beta}$ will generally decrease. However, this is not necessarily the case. The reason for this situation can be easily understood by rewriting $P[\,|W_A - \mu_T|<\varepsilon]$ standard normal distribution form and evaluating the probability for various values of μ_T and ε. From the fact that $\bar{\beta}$ is generally decreasing on σ_i^2, it is noted that under-estimation of the variances of the weights will not much affect the inspection process, as it decreases α only. Knowing that a type I error is followed by a 100% manual inspection, $\bar{\beta}$ seems more critical than α. $\bar{\beta}$ is also a function of P(X); i.e., k when the normal approximation is used. Computational time is directly proportional to k.

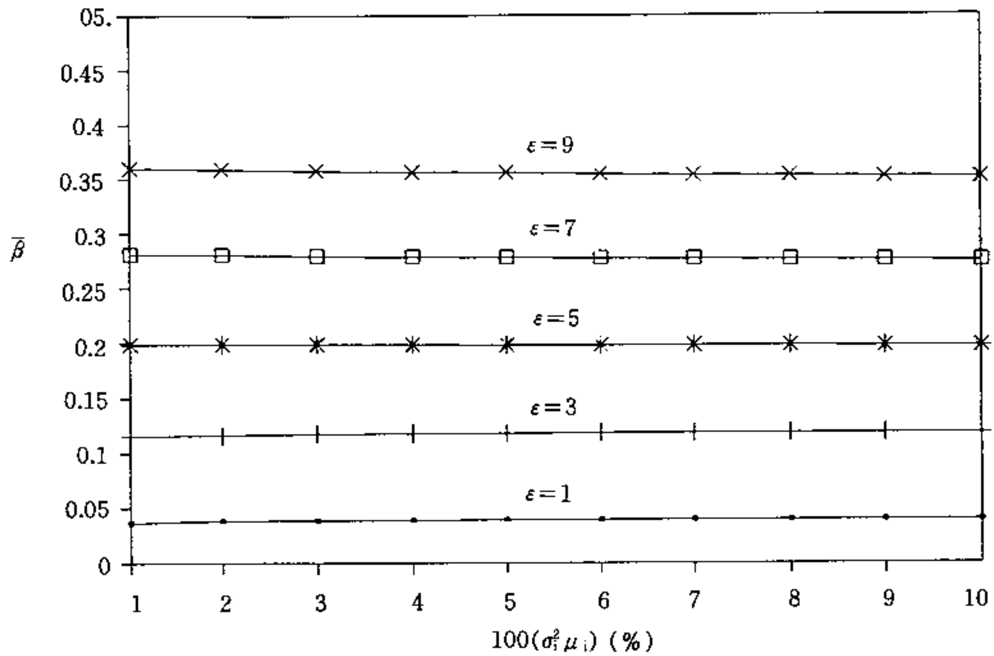

Figure 2. The impact of increases in σ_i^2 on $\bar{\beta}$

4. DETERMINATION OF TOLERANCE LEVEL

One of the most important advantages of this inspection procedure is that the manager can determine α and $\bar{\beta}$ by controlling the tolerance level, ε. The management will prefer a small α if the manual inspection is a costly procedure as compared to the cost of a faulty lot. On the contrary, if the loss caused by a faulty lot is critical, a smaller $\bar{\beta}$ will be preferred. Therefore, a proper selection of ε is very important in the implementation of the weighing inspection. We propose two approaches to determine the tolerance level, ε. They are the cost assessment approach and the bicriterion decision making approach.

The Cost Assessment Aproach

This approach estimates the expected total cost of the inspection and determines ε that minimizes the cost. The expected cost consists of manual inspection cost, the cost of inspecting a good lot(type Ⅰ error), and the cost resulted from a bad lot(type Ⅱ error). The total expected loss is obtainable if the management can provide the monetary loss caused by an under-shipment. In general, the loss of an over-shipment is equivalent to the number of excessive units times unit production cost for each item. Assuming the manual inspection cost is $\sum_{i=1}^{m} x_i c_i$ and there are no manual inspection errors, the total inspection cost (TIC) becomes,

$$\begin{aligned} TIC(\varepsilon) = & \Big[\sum_{X \neq N}\{P(X)\bar{\beta}[\sum_{i \in I_o}(x_i - n_i)c_i^o \\ & + \sum_{i \in I_u}(n_i - x_i)c_i^u\}\Big] \\ & \alpha\Big[P(X=N)\sum_{i=1}^{m} n_i c_i\Big] \\ & + \Big[\sum_{X \neq N}\{P(X)\ (1-\bar{\beta})\ \sum_{i=1}^{m} x_i c_i\}\Big] \end{aligned}$$

where $I_o \equiv \{i \mid x_i > n_i\}$, $I_u \equiv \{i \mid x_i < n_i\}$, c_i^o is the unit over-shipment cost of item i, c_i^u is the unit under-shipment cost of item i, and c_i is the unit inspection cost of item i. $TIC(\varepsilon)$ is a monlinear function and, hence, one can determine the optimal ε by minimizing TIC (ε) using any single variable optimization method such as Golden section search (see Reklaitis et al. (1983)). Note that as $\bar{\beta}$ approaches zero, $TIC(\varepsilon)$ approaches the cost of 100% manual inspection. This is possible when $\varepsilon = 0$, although it is still possible that $\bar{\beta} > 0$.

The Bicriterion Decision Making Approach

When values of the cost factors are not available or difficult to obtain, the cost assessment approach is not practical, in which case we consider the problem as a bicriterion decision making problem. Since smaller α and $\bar{\beta}$ values are preferred, the problem can be formulated as,

$$\begin{aligned} &\text{Minimize} && \alpha \\ &\text{Minimize} && \bar{\beta} \\ &\text{Subject to :} && 0 \leq \varepsilon \leq \varepsilon_{max} \end{aligned}$$

where ε_{max} is an upper bound of ε. ε_{max} is an input by the manager or can be set to a certain fraction of the target weight of the lot. The solution of the above problem seems trivial when a graph of α and $\bar{\beta}$ over the entire ε range can be constructed. However, the construction of the graph requires an excessive computational effort. The manager can determine a proper ε value by using the conflicting nature between α and $\bar{\beta}$. One simple decision procedure is to (1) generate a pair of two error values corresponding to two ε values within the range, (2) interact with the manager for his preference between the pair of two error values, and (3), depending on his preference, eliminate a certain range of ε from further consideration. The procedure is continued until either the manager is satisfied with a solution or the remaining region is very small (Sadagopan and Ravindran (1982)). When a preferred α or $\bar{\beta}$ value is prespecified by the manager, the corresponding ε can be easily determined by the bisection search (see Reklaitis et al. (1983)).

When each lot consists of a large number of items and the manager suspects high possibility in committing various misshipments (i. e., k is large), computing $\bar{\beta}$ is very time consuming. Because of this one might criticize the weighing inspection procedure as being inapplicable to large systems. In this case, one can divide the original lot into several sub-lots. This can be easily (or sometimes automatically) done because a pallet has a limited capacity and the available place for the manual inspection is limited as well. Then the weighing inspection procedure is applied to each of the sub-lots.

Another difficulty may arise when items of two or more types weigh the same amount. In this case, even if the total weight of these items are close to the target weight, there is still a possibility of misshipment (equal number of over and under-shipments of these items). If this situation arises, the efficiency of the weighing inspection process can be increased by palletizing such items appropriately.

5. CONCLUSION

In this paper, we proposed an alternative inspection procedure for counting inspections at production lines or distribution centers. The idea is rather simple and the implementation of the weighing inspection is expected to be inexpensive. Also, management can control the two types of errors of this inspection by properly determining the tolerance level. This research presented two approaches to help the manager determine the tolerance level.

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