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New Approach for Surf Zone Dynamics
碎破帶 動力學에 대한 새로운 接近

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Abstract□A simple surf zone model is presented. The present model takes a quite different approach by showing that wave action is conserved in the surf zone. This condition together with the conservation of energy enables us to develop a surf zone model that requires fewer empirical coefficients. The model is capable of predicting surf zone properties and is presented in analytical forms for the two-dimensional gradually-sloped bottoms. The analytical results were compared favorably with available laboratory data. This surf zone model provides the surface current pattern of the vertical circulation model, and consequently, significantly contributes to solving the three-dimensional current pattern.

要 旨: 간단한 쇄파대 모델이 제시된다. 제시된 모델은 파랑작용(wave action)이 쇄파대내에서 보존된다는 것을 입증함으로써 기존의 방법과는 아주 다른 접근방법을 택하고 있다. 파랑작용의 이러한 보존법칙은 파랑 에너지의 보존법칙과 함께 비교적 적은 실험상수를 필요로하는 쇄파대 모델의 개발을 가능토록하였다. 모델은 쇄파대 특성을 예상 가능케하며 2차원적인 점진적 경사에 대해서 분석적인 형태로 제시된다. 그 분석된 결과는 가능한 실험실자료와 비교하여 만족할만하였다. 이 쇄파대 모델은 수직적 흐름 모델의 수면경계에서의 흐름패턴을 제공하고 따라서 3차원적인 흐름패턴의 해를 구하는데 상당히 기여할 것이다.

1. INTRODUCTION

A common property of waves is their ability to transport energy without the need of any net material transport. In gravity waves, energy is propagated through the fluid media via the oscillations of the potential and the kinetic energies. When waves propagate through a region with currents, their energy is also transported by the moving fluid. The general appearance of the waves including wave height, length and period will also be altered. It is commonly observed that when the currents and waves are in the same direction, waves are lengthened but with reduced wave heights. Opposing currents, on the other hand, shorten the waves but with increased wave heights. This latter situation is particularly hazardous for navigation. Moreover, our recent field and laboratory wave measurements near an inlet entrance seemed to indicate that waves could become unsteady, or modulated, in a nonuniform current field. This unsteadiness is more pronounced if waves counter the current. Lee (1993) showed that this phenomena could be described with help of two new equations termed 'wave action equation' and 'the kinematic conservation of intrinsic angular frequency.' In this paper, the application to surf zone is given.

The flow properties in surf zone are utmost complex owing to the strong interactions among motions induced by waves, currents, and turbulence. The present knowledge on surf zone dynmaics is still inadequate and most of the models are rather rudimentary. Most numerous developments in the study of wave breaking have been made by approximation of the wave energy dissipation. These models can be classified into two groups: one is based

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on the similarity of the wave breaking process with other hydraulic phenomena such as a hydraulic jump (Dally *et al.*, 1984), a tidal bore (Battjes and Janssen, 1978), etc., and the other is based on estimation of the eddy viscosity (Mizuguchi, 1980) or turbulence (Izumiya and Horikawa, 1984).

In this paper a simple surf zone model is presented. This model is based on the consideration of wave energy balance and wave action conservation so that the wave-current interaction is fully taken into account. The model is capable of predicting wave decay and yields mean surface currents both in the across-shore and longshore directions. The model is presented in analytical form for the case of two dimensional gradually-sloped bottoms.

Section 2 describes the wave energy equation and wave action equation for the surf zone. It will be shown that the wave energy equation originates from the conservation of energy whereas the new wave action equation is derived from the free surface boundary condition. The wave energy equation with the addition of an energy dissipation term is derived and the validity of wave action equation in surf zone is established. Subsequent sections are devoted to solve decays of wave heights, mean surface currents, and mean set-ups in surf zone. Whenever possible, the results are compared with available experiemental data.

2. CONSERVATIONS OF WAVE ENERGY AND WAVE ACTION IN SURF ZONE

2.1 Linear Waves on Slowly Varying Water Medium

A velocity potential ϕ is assumed to exist such that the water particle velocities are given by $\nabla\phi$ where ∇ is the 3-dimensional differential operator

$$\nabla\equiv\frac{\partial}{\partial x}\mathbf{i}+\frac{\partial}{\partial y}\mathbf{j}+\frac{\partial}{\partial z}\mathbf{k}$$

It is assumed here that the dynamic free surface boundary condition is still valid with the inclusion of a head loss term. The kinematic and dynamic boundary conditions to be satisfied at the free surface, $z=\eta$, are, respectively,

$$\eta_t+\nabla_h\phi\cdot\nabla_h\eta-\phi_z=0 \tag{1}$$

$$\phi_t+\frac{1}{2}(\nabla\phi)^2+gz+gl=C(t) \tag{2}$$

where $C(t)$ may depend on t, but not on the space variables. We may take $C(t)\equiv0$ without any essential loss of generality. The subscripts t and z indicate the differentiations with respect to time and z-axis, respectively. ∇_h is the horizontal differential operator defined as

$$\nabla_h\equiv\frac{\partial}{\partial x}\mathbf{i}+\frac{\partial}{\partial y}\mathbf{j}$$

The cartesian coordinate system is used with origin at the still water level, $\mathbf{x}(x, y)$ in the horizontal plane and z directed vertically upwards. The velocity vector, $\mathbf{U}(u, v, w)$, is related to ϕ by

$$u=\frac{\partial\phi}{\partial x},\ v=\frac{\partial\phi}{\partial y},\ \text{and}\ w=\frac{\partial\phi}{\partial z}$$

The velocity potential, the free surface displacement and the head loss are assumed to be composed of current and wave components.

$$\phi(\mathbf{x}, z, t)=\phi_c(\mathbf{x};\ t, z)+\varepsilon\phi_w(\mathbf{x}, z, t) \tag{3}$$

$$\eta(\mathbf{x}, t)=\eta_c(\mathbf{x};\ t)+\varepsilon\eta_w(\mathbf{x}, t) \tag{4}$$

$$l(\mathbf{x}, t)=l_c(\mathbf{x};\ t)+\varepsilon l_w(\mathbf{x}, t) \tag{5}$$

where ε is an undefined factor used to separate the current (such as tidal current, wave-induced current, etc.) from the wave parts of the velocity potential. The '; t, z' in the current part is used to recognize that ϕ_c, η_c and l_c may vary slowly over time much longer than the wave period and it could also accommodate small vertical variations in currents. Eqs. (1) and (2) are then expanded in Taylor series to relate the boundary conditions at the mean water level $z=\eta_c$,

$$[\eta_t+\nabla_h\phi\cdot\nabla_h\eta-\phi_z]_{z=\eta_c}+\varepsilon\eta_w\left[\frac{\partial}{\partial z}(\eta_t+\nabla_h\phi\cdot\nabla_h\eta)+\nabla_h^2\phi\right]_{z=\eta_c}+\cdots=0$$

$$[\phi_t+\frac{1}{2}(\nabla\phi)^2+gz+gl]_{z=\eta_c}+\varepsilon\eta_w\frac{\partial}{\partial z}\left[\phi_t+\frac{1}{2}(\nabla\phi)^2+gz+gl\right]_{z=\eta_c}+\cdots=0$$

Substituting Eqs. (3)-(5) into the above equations and separating for current and wave parts, we obtain the followings retaining only the significant terms:

$$(\phi_c)_z = \frac{\partial \eta_c}{\partial t} + \nabla_h \phi_c \cdot \nabla_h \eta_c \tag{6}$$

$$\eta_c = -\frac{1}{g}\left[(\phi_c)_t + \frac{(\nabla_h \phi_c)^2}{2}\right] \tag{7}$$

$$(\phi_w)_z = \frac{D\eta_w}{Dt} + (\nabla_h^2 \phi_c)\eta_w + \nabla_h \phi_w \cdot \nabla_h \eta_c \tag{8}$$

$$\eta_w = -\frac{1}{g}\frac{D\phi_w}{Dt} - l_w \tag{9}$$

where $D/Dt \equiv \partial/\partial t + \nabla_h \phi_c \cdot \nabla_h$.

It should be noted here that the terms retained in the above set of equations are not necessarily of the same order of magnitude for all general conditions. For instance, the last term in Eq. (8) is, in general, a higher order term than the first two and only becomes significant when wave diffraction occurs. The $(\phi_c)_t$ term in Eq. (7) will become zero in steady state. For slowly varying water depth, the wave part of the velocity potential may be written as

$$\phi_w(\mathbf{x}, z, t) = f(z)\hat{\phi}_w(\mathbf{x}, t) + \Sigma(\text{non-propagating modes}) \tag{10}$$

where $f(z) = \cosh k(h+z)/\cosh k(h+\eta_c)$ is a slowly varying function of $\mathbf{x}$, k is a real value wave number and $\hat{\phi}_w$ denotes the velocity potential at the mean water level, termed as 'surface potential.' For progressive waves, the velocity potential and the free surface displacement can be written in terms of the wave-averaged, slowly varying quantities as

$$\phi_w(\mathbf{x}, z, t) = f(z)\mathbf{A}(\mathbf{x};\ t)ie^{i\psi} \tag{11}$$

$$\eta_w(\mathbf{x},\ t) = a(\mathbf{x};\ t)e^{i\psi} \tag{12}$$

where a is commonly defined as wave amplitude. The phase function is defined as $\psi = (\mathbf{K} \cdot \mathbf{x} - \omega t)$, where $\mathbf{K}$ is a wave number vector including the diffraction effects owing to the retention of the 3rd term in Eq. (8), and ω is an absolute frequency. All slowly varying quantities are given here as real numbers. Following the approach by Kirby (1983), a virtual work term proportional to $W(D\phi_w/Dt)$ is introduced to represent the head loss, where W is a positive undefined coefficient indicating the strength of the dissipation. We borrow this approach. The relation between a and A can be established by the dynamic free surface boundary condition specified in Eq. (9), which, after substituting Eqs. (11) and (12) into it, yields

$$-g\eta_w = (1+W)\frac{D\hat{\phi}_w}{Dt}$$

$$-gae^{i\psi} = (1+W)\left\{\frac{\partial}{\partial t} + \bar{\mathbf{U}}_s \cdot \nabla\right\}\{Aie^{i\psi}\}$$

$$= (1+W)\left[\sigma_d A e^{i\psi} + ie^{i\psi}\left\{\frac{\partial A}{\partial t} + \bar{\mathbf{U}}_s \cdot \nabla A\right\}\right] \tag{13}$$

where σ_d is the intrinsic frequency including the diffraction effects, defined as $\sigma_d = \omega - \bar{\mathbf{U}}_s \cdot \mathbf{K}$, and $\bar{\mathbf{U}}_s$ defined as $\nabla_h \phi_c$ at $z = \eta_c$.

This equation states that a and A should have a phase difference unless we impose the condition

$$\frac{\partial A}{\partial t} + \bar{\mathbf{U}}_s \cdot \nabla A = 0 \tag{14}$$

Then, the relation between A and a can be given by the following familiar form

$$A = -g\frac{a}{\sigma_D} \tag{15}$$

where σ_D is defined as $(1+W)\sigma_d$. Similarly, substituting Eqs. (11) and (12) into the kinematic free surface boundary condition given by Eq. (8) yields

$$\sigma_D^2 = (1+W)\left\{gk \tanh k(h+\eta_c) - \frac{g}{A}\nabla_h A \cdot \nabla_h \eta_c\right\} \tag{16}$$

$$\frac{\partial a}{\partial t} + \nabla_h \cdot (\bar{\mathbf{U}}_s a) - A\mathbf{K} \cdot \nabla_h \eta_c = 0 \tag{17}$$

Again, the last term in the above equations reflects the wave diffraction effect and, under normal circumstanaces, is of a higher order.

2.2 Wave Energy Conservation

It is assummed here that the surf zone is coherent in that the essential wave-like periodic motion is retained and is quasi-stationary when time-averaged over wave period. Furthermore, the turbulent motions are of much smaller time scale that the effect on the flow of interest can be simply treated as a dissipative force menifested by an eddy viscosity term. Thus, the familar Navier Stokes' equation of the following form can be applied:

$$\frac{\partial \rho \mathbf{U}}{\partial t}+\nabla\left(\frac{\rho q^2}{2}+p+\rho g z\right)=\rho(\nabla\times\mathbf{U})\times\mathbf{U}+\rho\nu\nabla^2\mathbf{U} \tag{18}$$

where $q^2=u^2+v^2+w^2$ and ν is the kinematic viscosity. Let the surface displacement and the velocity vector, $\mathbf{U}(u, v, w)$, be decomposed into mean value, wave and turbulent fluctuations, which are distinguished by subscripts c and w and prime, respectively; thus,

$$\eta=\bar{\eta}+\eta'=\eta_c+\eta_w+\eta' \tag{19}$$

$$\mathbf{U}=\bar{\mathbf{U}}+\mathbf{U}'=\mathbf{U}_c+\mathbf{U}_w+\mathbf{U}' \tag{20}$$

where the superscript $\bar{}$ is used to denote turbulent averaging. After turbulent-averaging Eq. (18) becomes

$$\frac{\partial \rho \tilde{\mathbf{U}}}{\partial t}+\nabla\left(\frac{\rho \tilde{q}^2}{2}+p+\rho g\tilde{\eta}\right)=\rho(\nabla\times\tilde{\mathbf{U}})\times\tilde{\mathbf{U}}+\rho\nu_t\nabla^2\tilde{\mathbf{U}} \tag{21}$$

where ν_t is the total viscosity including the eddy viscosity due to turbulence. The superscript $\sim$ is omitted hereafter.

Taking the scalar product of $\mathbf{U}(u, v, w)$ with the respective terms in the Navier Stokes equation and summing the products give the mechanical energy conservation equation of depth-integrated form with dissipation:

$$\int_{-h}^{\eta}\left\{\frac{\partial}{\partial t}\left[\frac{\rho q^2}{2}\right]+\nabla\cdot\left[\mathbf{U}\left(\frac{\rho q^2}{2}+p+\rho g z\right)\right]\right\}dz= -\int_{-h}^{\eta}\rho\nu_t\mathbf{U}\cdot\nabla^2\mathbf{U}dz \tag{22}$$

By applying the kinematic boundary conditions at the free surface and the bottom, we obtain

$$\frac{\partial}{\partial t}\int_{-h}^{\eta}\left[\frac{\rho q^2}{2}+\rho g z\right]dz+\nabla_h\cdot\int_{-h}^{\eta}\left[\mathbf{U}\left(\frac{\rho q^2}{2}+p+\rho g z\right)\right]dz +\int_{-h}^{\eta}\rho\nu_t\mathbf{U}\cdot\nabla^2\mathbf{U}dz=0 \tag{23}$$

again, utilizing the Leibnitz' rule of integration. The last dissipative term can be treated as a head loss term in the context of Bernoulli equation, i.e.,

$$\mathbf{D}=\int_{-h}^{\eta}\rho\nu_t\mathbf{U}\cdot\nabla^2\mathbf{U}dz=\nabla_h\cdot\int_{-h}^{\eta}\rho g l\mathbf{U}dz$$

where l is defined as the head loss due to turbulence. Equation (23) can then be expressed as

$$\frac{\partial}{\partial t}\int_{-h}^{\eta}\left[\frac{\rho q^2}{2}+\rho g z\right]dz+ \nabla_h\cdot\int_{-h}^{\eta}\left[\mathbf{U}\left(\frac{\rho q^2}{2}+p+\rho g z+\rho g l\right)\right]dz=0 \tag{24}$$

By applying Bernoulli equation with a head loss,

$$\frac{\rho q^2}{2}+p+\rho g z+\rho g l=-\rho\frac{\partial\Phi}{\partial t}$$

Then, Eq. (24) can be expressed in terms of energy, ε, and transport velocity, $\mathbf{U}_h$ as follows:

$$\frac{\partial\varepsilon}{\partial t}+\nabla_h\cdot(\mathbf{U}_h\varepsilon)=0 \tag{25}$$

where

$$\varepsilon=\int_{-h}^{\eta}\left[\frac{\rho q^2}{2}+\rho g z\right]dz \tag{26}$$

$$\mathbf{U}_h\varepsilon=-\int_{-h}^{\eta}\rho\mathbf{U}\frac{\partial\Phi}{\partial t}dz \tag{27}$$

The velocity, $\mathbf{U}$, shown in Eq. (27) implies the horizontal components. Taking time average over wave period, a wave energy conservation equation valid even for surf zone can be obtained,

$$\frac{\partial E}{\partial t}+\nabla_h\cdot(\mathbf{U}_h E)=0 \tag{28}$$

in which the wave energy, E, and the transport velocity $\mathbf{U}_h$ are the counter parts of $(\rho g\omega H^2/8\sigma)$ and $(\mathbf{Cg}+\bar{\mathbf{U}}_s)$ of non-dissipative case, respectively. Clearly, this transport velocity is different from the non-dissipative case and can be represented by

$$E=\rho g\frac{\omega}{\sigma_D}\frac{H^2}{8} \tag{29}$$

$$\mathbf{U}_h=\mathbf{Cg}+\bar{\mathbf{U}}_s+\mathbf{Cg}_D \tag{30}$$

where the first two terms constitute the transport velocity due to non-dissipative forces whereas the last term manifests the effect due to dissipative force. This term, in general, should be negative indicating a reduced energy flux due to dissipation. In theory, it can be estimated from the time-averaged energy dissipation term as follows:

$$\nabla_h\cdot(\mathbf{C}\mathbf{g}_D E)=D \tag{31}$$

where D is the time-averaged dissipation given by

$$D\equiv\overline{\int_{-h}^{\eta}\rho\nu_t\mathbf{U}\cdot\nabla^2\mathbf{U}dz}=\overline{\nabla_h}\cdot\overline{\int_{-h}^{\eta}\rho g I\mathbf{U}dz}$$

2.3 Wave Action Equation

In this section, the *wave action equation* and the *conservation equation of intrinsic frequency* are derived from the surface boundary conditions using the same definitions given in Eqs. (11), (12) and (16). Substracting Eq. (9)$\times\rho g\eta_w/(1+W)$ from Eq. (8)$\times\rho\hat{\phi}_w$, and ignoring the higher order term due to diffraction effect, we obtain

$$\frac{\partial}{\partial t}(\rho\eta_w\hat{\phi}_w)+\nabla\cdot(\overline{\mathbf{U}}_s\rho\eta_w\hat{\phi}_w)-\rho\left\{\hat{\phi}_w\frac{\partial\phi_w}{\partial z}\Big|_{\eta_c}-\frac{g\eta_w^2}{1+W}\right\}=0 \tag{32}$$

where $\overline{\mathbf{U}}_s$ is current velocity of the mean flow at the water surface level. Substituting Eqs. (11) and (12) into Eq. (38), the following equation is obtained:

$$\frac{\partial}{\partial t}(Bie^{2i\psi})+\nabla\cdot(\overline{\mathbf{U}}_s Bie^{2i\psi})-\sigma_D Be^{2i\psi}\left\{\frac{gk\tanh k(h+\eta_c)}{\sigma_D^2}+\frac{1}{1+W}\right\}=0$$

where B is defined as

$$B=\frac{\rho g}{8}\frac{H^2}{\sigma_D} \tag{33}$$

Expanding and separating the harmonic motions,

$$ie^{2i\psi}\left[\frac{\partial B}{\partial t}+\nabla\cdot(\overline{\mathbf{U}}_s B)\right]-\sigma_D Be^{2i\psi}\left[-\frac{2}{1+W}+\left\{\frac{gk\tanh k(h+\eta_c)}{\sigma_D^2}+\frac{1}{1+W}\right\}\right]=0$$

which yields the dispersion relation, $\sigma_D^2=(1+W)gk\tanh k(h+\eta_c)$, and the following wave action equation:

$$\frac{\partial B}{\partial t}+\nabla_h\cdot[\overline{\mathbf{U}}_s B]=0 \tag{34}$$

The above equation can also be derived directly from Eqs. (14) and (17). Although this wave action equation deals with a quantity identical to that shown in an alternative form of wave energy equation, the meaning of the equation is very different. It was shown that the real quantity conserved by wave energy equation should not be the wave action but the wave energy. Wave action is a surface property of waves governed only by the surface current condition. In steady state, both quanities can be same since the absolute frequency is constant everywhere.

Now if we substitute Eq. (15) into Eq. (17), the following equation is obtained,

$$\frac{\partial\sigma_D A}{\partial t}+\nabla\cdot[\overline{\mathbf{U}}_s\sigma_D A]=0 \tag{35}$$

Eliminating A from Eqs. (35) and (14), we arrive at the final equation that governs the change of the intrinsic wave frequency in a current field:

$$\frac{\partial\sigma_D}{\partial t}+\nabla_h\cdot[\overline{\mathbf{U}}_s\sigma_D]=0 \tag{36}$$

which is termed here as 'the kinematic conservation equation', or simply 'the conservation equation of intrinsic frequency.'

3. WAVE HEIGHT TRANSFORMATION

Waves break when their height reaches a certain limiting value relative to their length or water depth as a result of wave shoaling on a slope. The broken waves normally keep breaking as the water depth decreases, finally reaching the shoreline. Svendsen *et al.* (1978) divided the breaking zone into inner and outer regions: From the breaking point and for some distance shoreward, it is the outer region where a violent transition of the wave slope takes place large scale vortices are formed in this region. After outer region, the inner region begins as the wave becomes very similar to a tidal bore or a hydraulic jump. However, this wave breaking process not yet been fully clarified since the strong currents and turbulence are generated by the broken waves, and interacted with wave breaking. In this section, we suggest the new approach which takes account of the wave-current interaction in the surf zone. Differently from most of the existing wave breaking models, this new model provides the analytical expression of wave height over the wave breaking zone.

The wave energy equation given in Eq. (28), when expressed in terms of wave height, can be written as

$$\frac{\partial}{\partial t}\left(\rho g\frac{\omega}{\sigma_D}\frac{H^2}{8}\right)+\nabla_h\cdot\left[(\mathbf{Cg}+\bar{\mathbf{U}}_s+\mathbf{Cg}_D)\rho g\frac{\omega}{\sigma_D}\frac{H^2}{8}\right]=0 \tag{37}$$

Now, again we assume that the surf zone retains a quasi-steady state when integrated over wave period, then the slowly varying flow properties become time independent, and the absolute frequency becomes a constant. Accordingly, Eq. (37) becomes

$$\nabla_h\cdot\left[(\mathbf{Cg}+\bar{\mathbf{U}}_s+\mathbf{Cg}_D)\frac{\rho g}{8}\frac{H^2}{\sigma_D}\right]=0$$

and applying the wave action equation in the steady state, the wave energy equation in the surf zone is reduced to

$$\nabla_h\cdot\left[(\mathbf{Cg}+\mathbf{Cg}_D)\frac{\rho g}{8}\frac{H^2}{\sigma_D}\right]=0 \tag{38}$$

The quantity corresponding to the dissipative force is assumed here to be proportional to group velocity at the breaking point, Cg_b, since wave height is known to decrease steadily within the surf zone; therefore,

$$\mathbf{Cg}_D=-\beta\mathbf{Cg}_b \tag{39}$$

where β is a positive coefficient. Eq. (38) now becomes

$$\nabla_h\cdot\left(\mathbf{Cg}^*\frac{\rho g}{8}\frac{H^2}{\sigma_D}\right)=0 \tag{40}$$

where the real relative group velocity in the turbulent surf zone is estimated by

$$\mathbf{Cg}^*=\mathbf{Cg}-\beta\mathbf{Cg}_b \tag{41}$$

Eq. (40) is the final form of the proposed energy transformation model. This model has only unknown coefficient, namely, the dissipation coefficient, β and is applicable to the general three-dimensional topography and any arbitrary incident wave angles.

An analytical expression can be obtained for two-dimensional beaches of uniform slopes. Eq. (40) becomes

$$\frac{\partial}{\partial x}\left[(\mathbf{Cg}-\beta\mathbf{Cg}_b)\frac{H^2}{\sigma_D}\right]=0 \tag{42}$$

where x axis is directed onshore. The cross shore component of the above equation gives,

$$H^2\sim\frac{\sigma_D}{\cos\theta(\beta Cg_b-Cg)} \tag{43}$$

Applying the dynamic free surface boundary condition given in Eq. (14) in 2-D steady state condition, we have σ_D proportional to the wave height in surf zone. Eq. (43) can be written as

$$H=\frac{\beta_H}{\cos\theta(\beta Cg_b-Cg)} \tag{44}$$

with β_H determined later. θ can be determined by Snell's law as

$$\theta=\sin^{-1}(C_a\sin\theta_o/C_o)$$

where $C_a=\omega/k$ is an absolute phase speed. Eq. (44) is non-dimesionalized as

$$H'=\frac{1}{H_bCg_b}\frac{\beta_H}{\cos\theta(\beta-Cg')} \tag{45}$$

where $H'=H/H_b$ and $Cg'=Cg/Cg_b$. Now we apply two boundary conditions; $H/H_b=1$ at a breaking point $d/d_b=1$, and $H/H_b=H_s'$ where Cg becomes zero. We then obtain

$$H'=\frac{H_s'}{\cos\theta[1-Cg'(1-H_s'/\cos\theta_b)]} \tag{46}$$

Applying shallow approximation to Cg' gives

$$H'\doteq\frac{1}{\cos\theta}\cdot\frac{H_s'}{1-\sqrt{d'}(1-H_s'/\cos\theta_b)} \tag{47}$$

where $d'=d/d_b$ and d is defined as a total water depth, $(h+\eta_c)$. β is only one independent of the

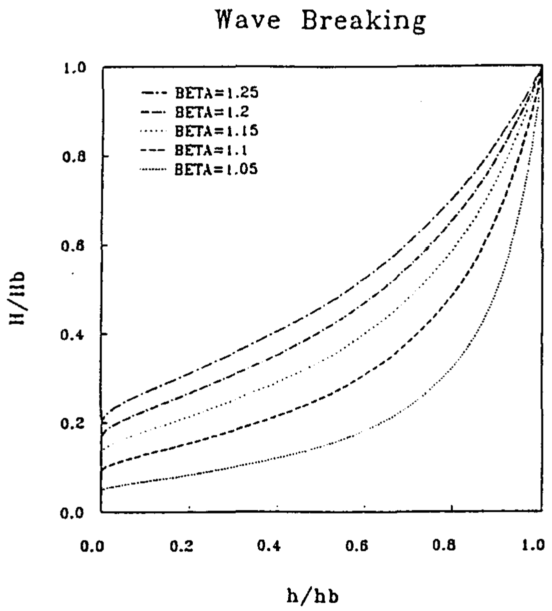

Fig. 1. Effect of a parameter β on the dimensionless wave height in the surf zone.

three parameters, H', β_H and β. If the value of H_s' is determined from experiements, the other two are expressed in terms of breaking properties as

$$\beta_H = \frac{\cos\theta_b H_s'}{\cos\theta_b - H_s'} H_b Cg_b, \quad \beta = \frac{\cos\theta_b}{\cos\theta_b - H_s'}$$

Figure 1 plots the dimensionless wave height in the surf zone for different β values. Figure 2 shows the comparison between the present theory and the laboratory data by Horikawa and Kuo (1966). The dimensionless values of the wave height at the shoreline H_s', are determined from the data; they are 0.22, 0.18, 0.14 and 0.14, respectively, for slopes of 1/20, 1/30, 1/65 and 1/80. It was found that the experimental values of H_s' can be closely approximated by $\sqrt{\tan\alpha}$ with α being the slope of the beach. Extensive laboratory experiments have shown that the pattern of wave height decay across the surf zone is strongly a function of the beach slope.

4. SURFACE CURRENTS

The main mechanism responsible for current generation inside the surf zone is suggested to be due to the excess wave-induced momentum also known as the radiation stresses. Vertical circulations are known to exist as a consequence mass balance to maintain a quasi-steady state. Solution conerning longshore current and its distribution was originally obtained by Longuet-Higgins (1970) and later modified and refined by numerious other investigators. The model was based on the balance between the friction forces and gradients in the radiation stress. Cross-shore current modeling is more recent effort based on such ideas as wave set-up, undertow, etc. In this section, the surface current vectors in the surf zone containing both cross-shore and longshore components are solved by the applications of wave action equation and the steady state wave energy equation:

$$\nabla\cdot\left(\bar{\mathbf{U}}_s \frac{H^2}{\sigma_D}\right)=0, \quad \nabla\cdot\left[(\beta Cg_b - Cg)\frac{\mathbf{K}}{k}\frac{H^2}{\sigma_D}\right]=0 \tag{48}$$

Elimilating H^2/σ_D from the above equations we obtain a pair of simple equations for surface currents,

$$u_s = \beta_O \cos\theta(\beta Cg_b - Cg), \quad v_s = \beta_L \sin\theta(\beta Cg_b - Cg) \tag{49}$$

where u_s and v_s denote, respectively, the cross-shore and longshore components of surface current vector at the mean water level, $\bar{\mathbf{U}}_s$; β_O and β_L are constants of proportionarity.

When expressed in terms of wave height the above pair of equations become,.

$$u_s = \beta_O \beta_H \frac{\sigma_D}{H^2}, \quad v_s = \beta_L \beta_H \frac{\sin\theta}{\cos\theta}\frac{\sigma_D}{H^2} \tag{50}$$

which show that while both onshore and longshore current components are inversely proportional to the wave height sqaured, only longshore component is a function of wave angle. The surface current equations can be more conveniently expressed in non-dimensional forms,

$$u_s' = \beta_O \cos\theta(\beta - Cg'), \quad v_s' = \beta_L \sin\theta(\beta - Cg') \tag{51}$$

where $u_s' = u_s/Cg_b$ and $Cg' = Cg/Cg_b$. Applying the shallow water condition, we have,

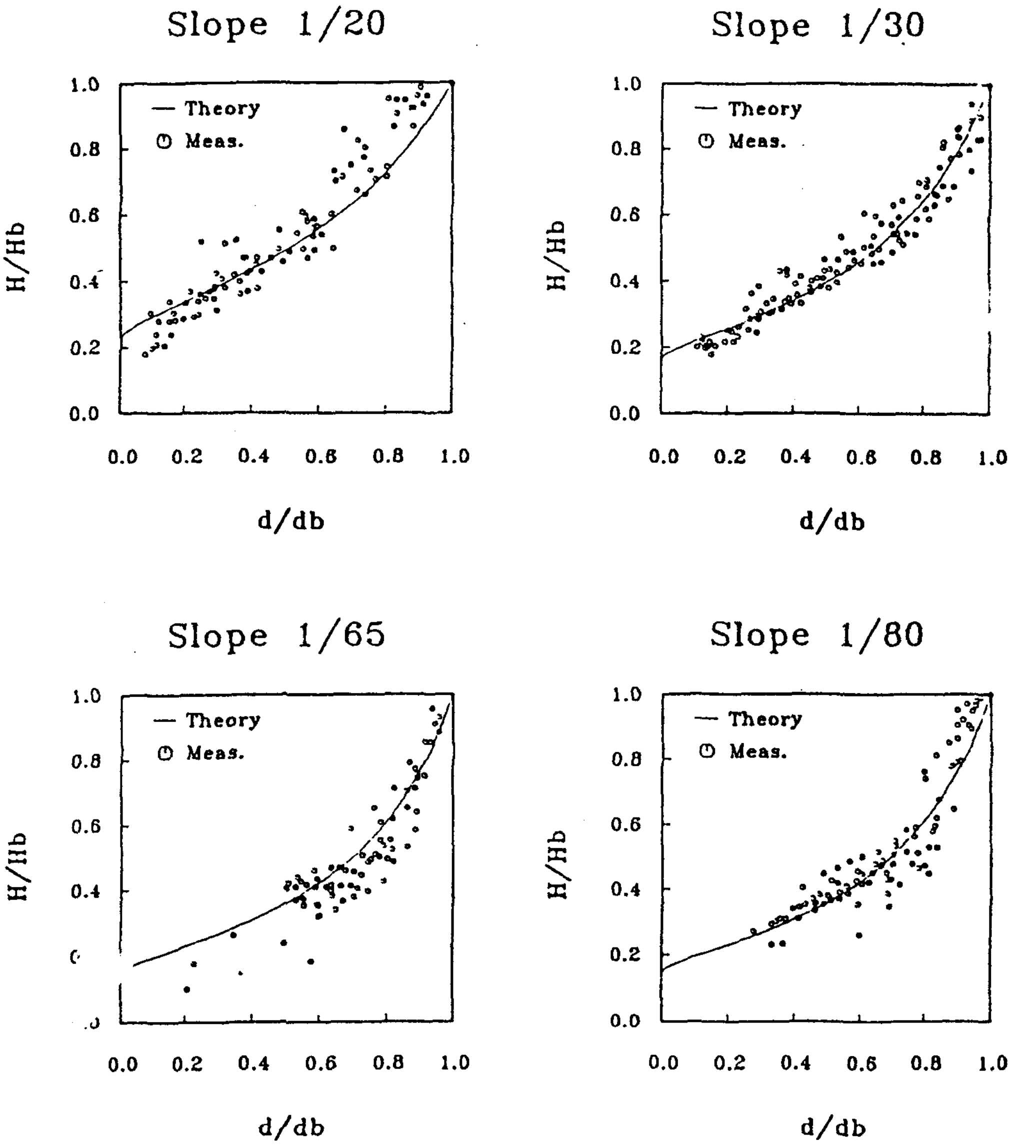

Fig. 2. Comparison with laboratory experiments presented by Horikawa and Kuo (1966).

$$u_s'=\beta_O\cos\theta_o(\beta-\sqrt{h'}),\quad v_s'=\frac{\beta_L\sin\theta_o}{C_o'}\sqrt{h'}(\beta-\sqrt{h'}) \tag{52}$$

Figure 3 illustrates the effect of β on the dimensionless surface onshore current as given by Eq. (52). Unfortunately, no experimental data are available at present. Figure 4 illustrates the effect of β on the dimensionless surface longshore current as given by Eq. (52). Figure 5 compares the theory with the laboratory longshore current data measured by Visser (1991). It should be pointed out here that the Visser's data are depth-averaged and theory is for surface current. In the longshore direction, however, one expects the vertical distribution to be rather uniform. The theory appears to fit the data remarkably well. One of the major advantages of the present model is that it requires only one empi-

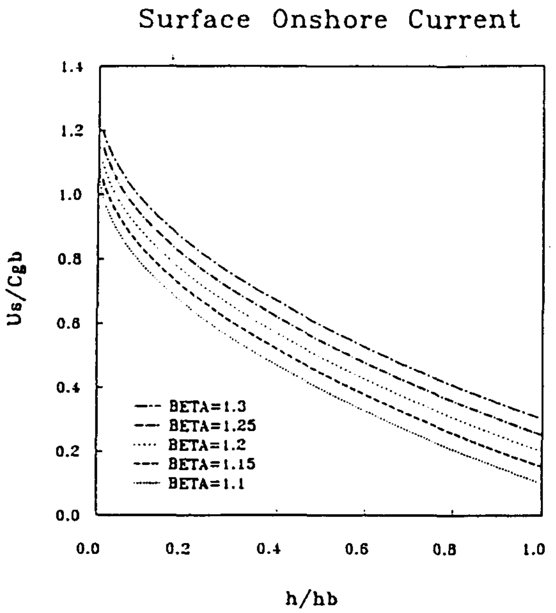

Fig. 3. Effect of a parameter β on the dimensionless surface onshore current in the surf zone.

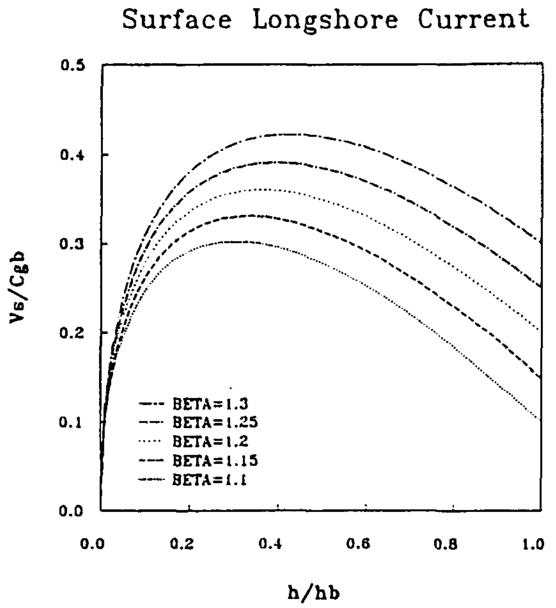

Fig. 4. Effect of a parameter β on the dimensionless surface longshore current in the surf zone.

rical coefficient to control the magnitude and eliminates the troublesome mixing coefficient appeared in most of the existing theories. In order to fit the data one often has to assume large mixing strength without justification.

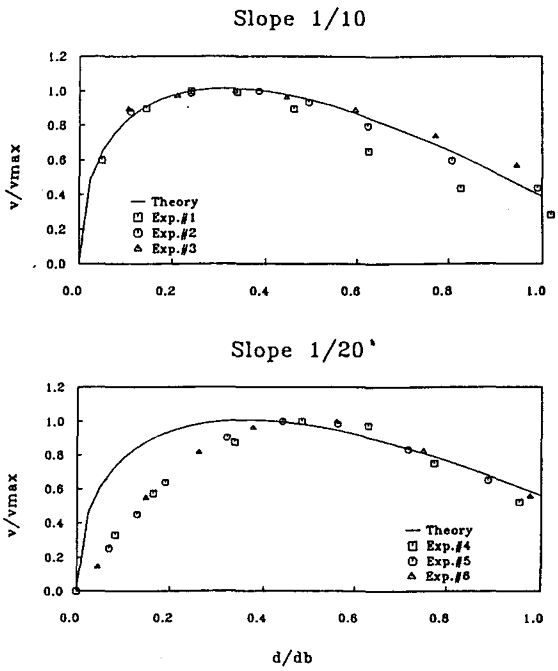

Fig. 5. Comparison of longshore current with laboratory experiments presented by Visser (1991).

5. SET-UP AND SET-DOWN

In steady state, the wave set-up in surf zone is estimated here by the alternative form of 'the kinematic conservation of intrinsic frequency' and the continuity equation,

$$\nabla \cdot (\bar{U}_s H) = 0, \quad \nabla \cdot [\bar{U}(h+\eta_c)] = 0$$

It is assumed here that the magnitude of the depth-averaged return current beneath the mean water level, $\bar{U}$, is proportional to the onshore surface current, $\bar{U}_s$ then

$$H = \kappa(x)(h+\eta_c) \tag{53}$$

where $\kappa(x)$ is, in general, a spatial dependent coefficient. If $\kappa(x)$ is a constant across the surf zone, the above equation is simply the frequently adopted extension of Miche's criterion. From Eq. (53) the set up can be estimated as

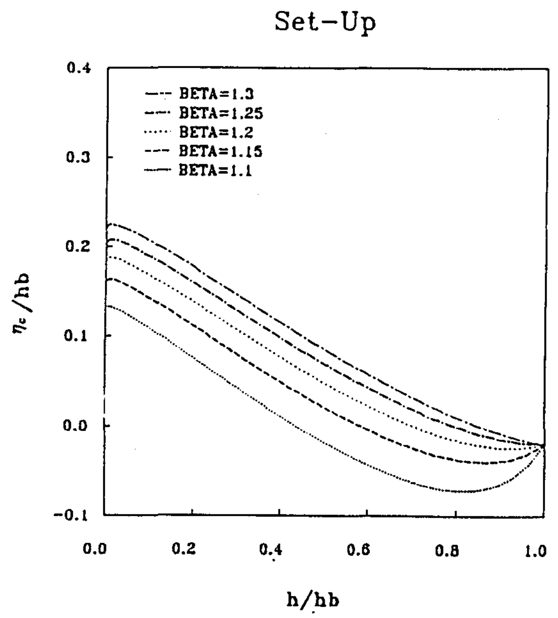

Fig. 6. Effect of a parameter β on the dimensionless set-up in the surf zone.

$$\eta_c = \frac{H}{\kappa(x)} - h \tag{54}$$

or in non-dimensional form assuming $\eta_c(x_b)/h_b \ll 1$,

$$\eta_c' = \frac{\kappa(x_b)}{\kappa(x)} H' - h' \tag{55}$$

where $\eta_c' = \eta_c/h_b$ and x_b indicates the breaking point. Here the definition of the mean water level is limited where the bed is at all times covered by water. The set-up or set-down can be computed at once when the wave height within the surf zone is determined.

Figure 6 illustrates the effect of β on the dimensionless set-up given by Eq. (55) for a constant κ over the surf zone.

There is one notable feature of the model that is generally lacking in most of the existing models. The model predicts rather mild, sometimes near constant, set-down in the transition zone immediately after breaking point where the wave height drops sharply. This phenomenon referred to as transition region 'paradox' has been noted as a significant feature in the transition region (Basco and Yamashita, 1986; Theike, 1988). The conventional theory of balancing the momentum due to radiation stress should produce a jump of set-up in the transition region where wave height reduces sharply. Laboratory data, on the other hand, showed nearly contant set-down across the transition region, where the wave height is reduced nearly proportional to the reduction of water depth, as indicated by Eq. (54). This phenomenon can be further clarified through the momentum balance (Lee, 1993).

6. CONCLUSION

The wave action equation has been derived from the surface boundary conditions. The dispersion equation describes the wave fluctuating motion, whereas the wave action equation describes the slowly varying motion. The wave energy equation and the wave action equation were applied to the surf zone observed in steady state. Both equations in surf zone have been proven to provide practical theories to predict wave height, surface onshore current, and longshore current on the straight coast beach. The variation of wave height is determined by the finite wave height reaching a shoreline. For the nearly normal approaching waves, it is notable that surface onshore current is inversely proportional to the wave height, and the surface longshore current is not only proportional to the wave angle but also inversely proportional to the wave height. The set-up model yields the Miche's type wave breaking model and predicts rather mild, sometimes near constant, set-down in the transition zone immediately after breaking point.

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