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A Hydrodynamic Solution for the Lateral Spreading of a River Plume
河川水 플룸 橫方向 퍼짐의 解釋解

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Abstract□Assuming Gaussian distribution of the density difference between a turbulent jet river plume and its ambient saline water, a hydrodynamic solution for the lateral spreading of a river plume is developed. Two advantages can be expected from the assumption we made. Firstly, we need not consider mixing processes in the plume in dealing with this problem. Secondly, by putting pressure gradients which can be obtained from the density distribution, into the equation of motion, we can solve them easily. We compared the analytic solution with the field data of the Nakdong river plume and found reasonably good correspondence.

要 旨: 하수제트플룸내에서 플룸과 주변해수와의 밀도차가 Gauss 분포를 나타내고 있다는 가정하에 하구만에 흘러 나오는 하수 플룸의 횡방향 퍼짐속도의 해석해를 구하였다. 이 가정을 도입하므로써 얻는 이점은 첫째, 혼합과정의 결과가 밀도분포에 반영되어 있으므로 이 문제를 다루는데 있어서 혼합과정에 대한 고려를 할 필요가 없다는 점이고, 둘째로는 밀도분포가 알려져 있으므로 그로부터 압력경사력을 구하여 운동방정식에 대입, 문제를 쉽게 풀 수 있다는 점이다. 이론해를 낙동강 하수플룸의 관측결과와 비교해 본 결과, 비교적 잘 일치함을 발견하였고 본 연구의 이론식이 하수플룸의 퍼짐을 다루는데 유용하게 쓰일 수 있음을 확인하였다.

1. INTRODUCTION

The dynamic behaviour of the fresh water river plume issuing into a salt water basin has been sought by many investigators since the work of Takano on a river plume (Takano, 1954a; 1954b; 1955). Takano treated the spreading of the river water as it expanded laterally between hyperbolic boundaries and thinned vertically above heavier ambient water. Since then the primary operative forces, such as the inertia of issuing river water and associated turbulent diffusion, friction between the effluent and the bed immediately seaward of the mouth and the buoyancy by the density contrast, and their relative roles have been considered by many workers to solve the dynamic behaviour of the turbulent river water plume (Borichanski and Mikhailov, 1966; Bondar, 1970; Wright and Coleman, 1974; Officer, 1976). There are also some works to deal with the problem by numerical approaches (Gravine, 1987; O'Donnel, 1988, 1990).

For the present work an analytic solution on the lateral expanding velocity of the river plume was investigated by assuming that the density difference between the plume and the ambient saline water has a Gaussian distribution in the lateral cross-section of the plume, i.e.

$$\Delta\rho(x, y, z) = \Delta\rho_s(x)\, e^{-(y/B)^2/2} e^{-(z/H)^2/2}$$

Here x is the distance along the plume axis from the river mouth, y is the lateral distance from the plume axis, and z is the depth from the level surface. $\Delta\rho_s$ is the density difference with the ambient seawater at the surface of the axis. H and B are the depth and half width of the plume at whose boundary $\Delta\rho$ decreases to a small value; i.e. $\Delta\rho$

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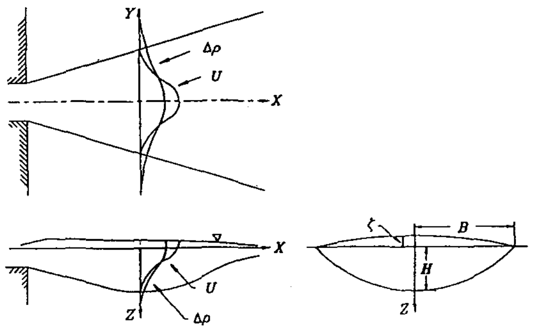

Fig. 1. Definition sketch of the river plume.

$=\Delta\rho_s e^{-1/2} \doteqdot 0.6\ \Delta\rho_s$ which is considered to be negligible (Fig. 1).

The assumption of Gaussian distribution for the density difference was adopted by Bates (1953), and has been supported by other authors (Murota *et al.*, 1983; Fischer *et al.*, 1979; Jirka *et al.*, 1985). Based on this assumption, we can figure out pressure in the plume layer, and the steady state equations of motion for the lateral expanding velocity of river plumes which were established following the example of the Bowden (1983) can be solved easily. This solution has been compared with the field data obtained at the Nakdong river plume in 1981 and 1993.

2. LATERAL SPREADING OF THE PLUME

When we assume a Gaussian distribution for the density difference in a plume cross-section, the mixing processes of the salt water need not to be considered in dealing with the dynamics of the river plume because the effect of the processes is already included in the density distribution. The equations of motion of the plume dynamics can also be solved with the pressure gradient which can be evaluated from the density distribution.

For simplicity, the density distribution in the plume cross-section at a fixed x can be assumed as:

$$\rho_s=\rho_a-\Delta\rho_s e^{-(y/B)^2/2}, \qquad 0\zeta<z\leq 0 \qquad (1)$$

$$\rho_1=\rho_a-\Delta\rho_s e^{-(y/B)^2/2} e^{-(z/H)^2/2}, \qquad 0<z<\infty \qquad (2)$$

where, ρ_s is the surface density of the plume, ρ_a is the density of the ambient water, ζ is the surface elevation above the level surface, and ρ_1 is the density of the water below the level surface.

The equations of motion in the lateral direction of the plume can be established by assuming that the lateral motion is so slow that it can be treated as a steady state. We can calculate the pressure from the density distribution of Eqs. (1) and (2); In the plume layer, we have;

$$\rho_1 N_z \frac{\partial^2 v_1}{\partial z^2}=\frac{\partial P_1}{\partial y} \qquad (3)$$

$$p_1=(\rho_a-\Delta\rho_s e^{-(y/B)^2/2})g\ \zeta+\rho_a g z-\Delta\rho_s e^{-(y/B)^2/2} g \int_0^z e^{-(z/H)^2/2} dz \qquad (4)$$

In the lower layer, we have;

$$\rho_2 N_z \frac{\partial^2 v_2}{\partial z^2}=\frac{\partial P_2}{\partial y} \qquad (5)$$

$$p_2=(\rho_a-\Delta\rho_s e^{-(y/B)^2/2})g\ \zeta+\rho_a g z -\Delta\rho_s e^{-(y/B)^2/2} g \int_0^z e^{-(z/H)^2/2} dz \qquad (6)$$

In the equations the variables with subscripts 1 and 2 are those of the plume layer and the lower layer, respectively, and N_z is the vertical eddy viscosity.

In the lower layer the velocity of the ambient water, v_2, can be assumed as constant, and hence the pressure gradient along the lateral direction is zero. Then Eq. (5) is reduced to;

$$\frac{\partial p_2}{\partial y}=\frac{\partial}{\partial y}\left[(\rho_a-\Delta\rho_s e^{-(y/B)^2/2})g\zeta-\Delta\rho_s e^{-(y/B)^2/2} g \sqrt{\frac{\pi}{2}}H\right]=0 \qquad (7)$$

Here, $\int_0^\infty e^{(z/H)^2/2} dz$ (the error function) was replaced to $\sqrt{\frac{\pi}{2}}H$.

From the boundary condition that the ζ and $e^{-(y/B)^2/2}$ approach zero rapidly for large y, the surface elevation ζ can be solved as;

$$\zeta=\frac{\Delta\rho_s e^{-(y/B)^2/2}}{\rho_a-\Delta\rho_s e^{-(y/B)^2/2}}\sqrt{\frac{\pi}{2}}H \qquad (8)$$

Substituting ζ into Eqs. (3) and (4), we get

$$\rho_1 N_z \frac{\partial^2 v_1}{\partial z^2} = -\Delta\rho_s (y/B^2) g e^{-(y/B)^2/2}\sqrt{\frac{\pi}{2}}H\{1-erf(z)\} \quad (9)$$

and the solution for this equation is

$$v_1 = -\Delta\rho_s (y/B^2) g e^{-(y/B)^2/2}\sqrt{\frac{\pi}{2}}(H^3/\rho_1 N_z)\left(t^2 - 2\int\int_0^t erf(t)\ \text{at}\ dt\right)+C \quad (10)$$

where $t=(z/H)/\sqrt{2}$, and the z-dependency of the upper layer density ρ_1 was temporarily neglected and the eddy viscosity N_z was assumed to be constant.

To determine the integration contant C, no-slip condition was applied at the interface between the plume and the ambient water of the lower layer;

$$v_i = -\Delta\rho_s (y/B^2) g e^{-(y/B)^2/2}\sqrt{\frac{\pi}{2}}(H^3/\rho_a N_z)\{t_i^2 - 2G(t_i)\} + C = v_0 \quad (11)$$

Here the subscript i is used for the interfacial variables, and v_0 is the lateral velocity component of the ambient water and is assumed to be zero. And we put $\int\int_0^t erf(t) dt dt = G(t)$; which can be numerically integrated.

Now we have the solution:

$$v_1 = \Delta\rho_s (y/B^2) g e^{-(y/B)^2/2}\sqrt{\frac{\pi}{2}}(H^3/\rho_1 N_z)[(t_i^2 - t^2) - 2\{G(t_i) - G(t)\}] \quad (12)$$

The surface velocity v_s at $z=\zeta$ can be obtained neglecting ζ;

$$v_s = \Delta\rho_s (y/B^2) g e^{-(y/B)^2/2}\sqrt{\frac{\pi}{2}}(H^3/\rho_1 N_z)[t_i^2 - 2G(t_i)] \quad (13)$$

If we assume that the density difference $\Delta\rho$ at the interface which decreases to $\Delta\rho_s e^{-1/2} = 0.6\Delta\rho_s$ can be regarded negligible (Murota *et al.*, 1984), the interface between the plume and the ambient water can be determined by the equidensity line of density difference 0.6 $\Delta\rho_s$ in the cross-section. That is,

$$\Delta\rho = \Delta\rho_s e^{-(y/B)^2/2} e^{-(z/H)^2/2} = \Delta\rho_s e^{-1/2} \quad (14)$$

Then as the equation of interface we get an ellipse;

$$\frac{y^2}{B^2} + \frac{z^2}{H^2} = 1 \quad (15)$$

From this equation t_i becomes;

$$t_i = \sqrt{\frac{1}{2} - \frac{(y/B)^2}{2}} \quad (16)$$

At last the solution of lateral expanding velocity in the plume cross-section is;

$$v_1 = \Delta\rho_s (y/B^2) g e^{-(y/B)^2/2}\sqrt{\frac{\pi}{2}}(H^3/\rho_1 N_z)[(t_i^2 - t^2) - 2\{G(t_i) - G(t)\}] \quad (17)$$

and its surface value is

$$v_s = \Delta\rho_s (y/B^2) g e^{-(y/B)^2/2}\sqrt{\frac{\pi}{2}}(H^3/\rho_1 N_z)\{t_i^2 - 2G(t_i)\} \quad (18)$$

3. RESULTS AND DISCUSSIONS

In order to verify our theoretical solution, we selected the Nakdong river plume as an example. The data used here are those taken in September 16, 1981 and in August 31, 1993. We have carried out oceanographical observation in this area for a long time since 1981 (Fig. 2). The former are the data of natural river plume and the latter are those of discharged plume from the Nakdong barrage which was constructed in 1987. In Fig. 3, we find that in 1981, the river outflow was in its peak in September. Fig. 4 shows time variation of the vertical density distribution. We can see in the figure that two plume cores passed the observing site during our observation from 10:00 in 16 September until 10:00 next day; one around 18:00 and the other

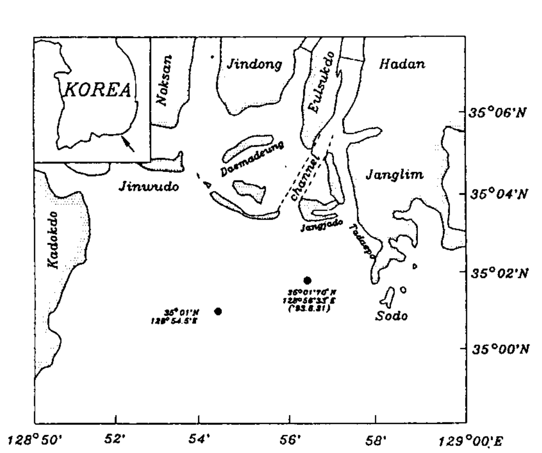

Fig. 2. Location map of the current, salinity, temperature measurement and water sampling.

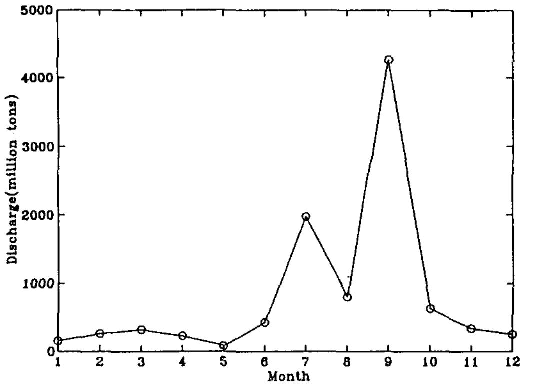

Fig. 3. Amount of river discharge in 1981.

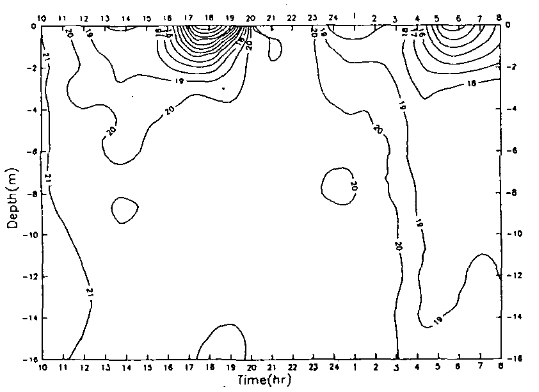

Fig. 4. Variation of vertical density (σ_t) distribution with time (1981. 9. 16~17).

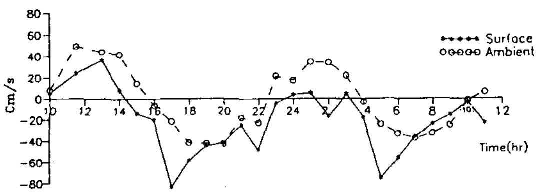

Fig. 5. Time variation of velocity (E-W Comp.) (1981. 9. 16~17).

around 06:00 next day. Fig. 5 illustrates the time variation curves of the surface velocity of the plume (starred) and the depth-mean velocity of the lower layer (circled). We can find bumpy parts around 18:00 and 6:00 next day. They show another indication of the lateral expanding of the plume. We choose the former plume for the analysis, confirming it as the effluent jet plume flowing out of the river mouth of the passage which passed west

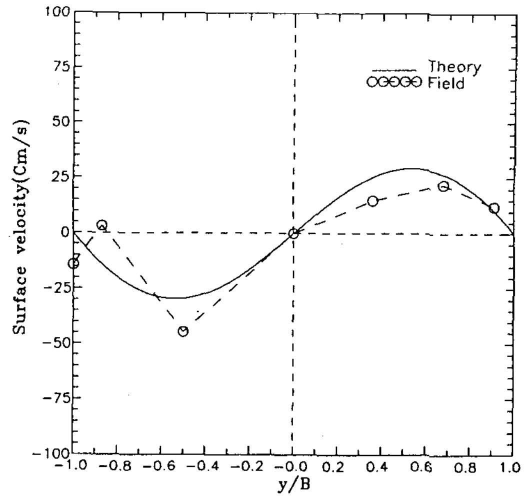

Fig. 6. Surface velocity of lateral spreading of the plume (1981. 9. 16).

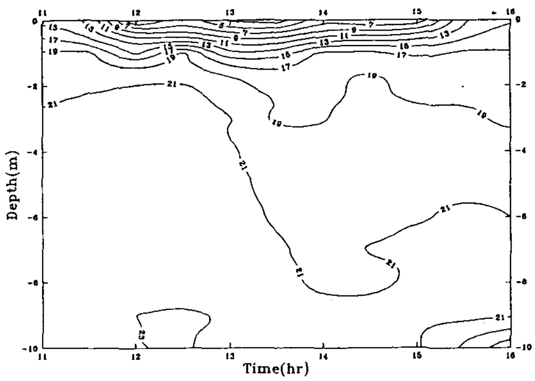

Fig. 7. Variation of vertical density (σ_t) distribution with time (1993. 8. 31).

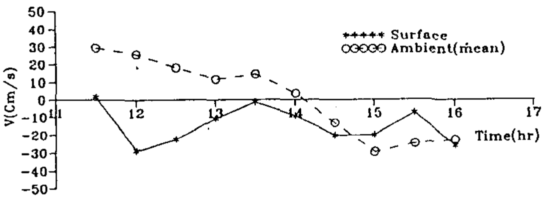

Fig. 8. Time variation f velocity (E-W Comp.) (1993. 8. 31).

side of Ulsukdo, and now blocked by the barrage. We evaluate the lateral expanding velocities of the plume surface by substracting the velocities of the ambient water from the surface velocities, while considering that the plume axis stretched southward

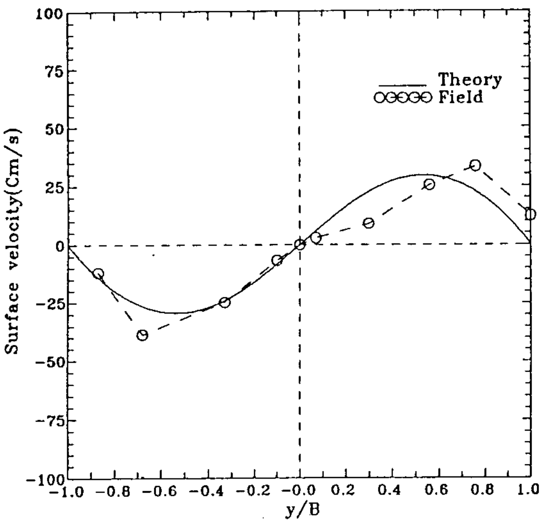

Fig. 9. Surface velocity of lateral spreading of the plume (1993. 8. 31).

and the plume expanded laterally in the east-west direction. And the lateral distances is calculated by numerical integration. Thus we take $B=5$ km, $H=1$ m, and $N_z=3\times10^{-5}$ m^2/sec in this analysis. The result of the analysis is shown in Fig. 6 (dashed line).

The analysis of the data taken in August 1993 (see Fig. 7 and 8) is also carried out in the same way and its result is shown in Fig. 9 (dashed line). It can be found from the above cases that the results of field data correspond well with theoretical curves (solid lines).

4. CONCLUSION

We developed an analytical solution for the lateral spreading velocity of a river plume, assuming the Gaussian distribution for the density difference between plume layer and ambient saline water. Applying the theoretical solution to the real plume of the Nakdong river, we found they correspond well each other. We conclude that our solution can be used satisfactorily for the problems dealing with spreading of river plumes.

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