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A Mathematical Model for Nonlinear Waves due to Moving Disturbances in a Basin of Variable Depth
부등 수심지역의 이동 교란에 의한 비선형파의 수학적 모형

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Abstract□ Mathematical models of nonlinear waves due to disturbances moving with the near critical velocity in a basin of variable depth are discussed. A two-dimensional model for waves of arbitrary amplitude is developed. In the case of small perturbation it is shown that nonlinear ray method can be applied to obtain the generalized forced Korteweg-de Vries equation.

要 旨: 수심이 일정치 않은 지역에서 임계치에 가까운 속도로 이동하는 교란에 의해 발생되는 비선형파의 수학적 모형에 대해 논의하였다. 임의파고를 갖는 파랑의 2차원 모형이 개발되었다. 미소교란의 경우, 비선형 파향선법이 일반화된 Korteweg-de Vries 식을 얻는데 적용될 수 있음을 나타내었다.

1. INTRODUCTION

The forced Korteweg-de Vries equation is now one of canonical models for describing the resonant excitation of nonlinear waves by moving perturbations in a basin of constant depth. It was applied for surface waves due to atmospheric moving perturbations or an obstacle in a flow (Akylas 1984; Cole 1985; Wu 1987; Lee *et al.* 1989; Wu and Wu, 1988), for ship waves in a shallow channel (Mei, 1986; Mei and Choi, 1987), for internal waves in a stratified flow due to an obstacle (Grimshaw and Smyth 1986; Melville and Helfrich 1987; Mitsudera and Grimshaw 1990; 1991), for atmospheric waves due to a local topography (Patoine and Warn 1982; Warn and Brasnett 1983). Resonance between waves and perturbations leads to a complex picture of wave radiation which depends on the ratio of parameters of nonlinearity, dispersion and forcing. For engineering applications it is necessary to take into account the effects of variabillity of basin depth, because the near critical regime of fluid motion occurs in coastal zones.

On the other hand, the "free" Korteweg-de Vries equation, as it is known, is applied for the description of long waves in coastal zones (Mei, 1989; Pelinovsky, 1982; Voltzinger *et al.* 1989). Its generalization for the basin of variable depth was made by Ostrovsky and Pelinovsky (1970) and Johnson (1972). In this context it is interesting to study the case of moving perturbations in a basin of variable depth.

In this paper different models for nonlinear waves in the basin of variable depth due to moving atmospheric perturbations are discussed. In case of small quasi-plane perturbation it is proved that the generalized forced Korteweg-de Vries equation is an appropriate model.

2. NONLINEAR BOUSSINESQ-LIKE MODEL

Let us consider the potential motion of fluid under the action of atmospherical perturbations. The

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governing equation is the Laplace equation for potential Φ:

$$\nabla^2\Phi+\frac{\partial^2\Phi}{\partial z^2}=0 \tag{1}$$

with kinematic boundary conditions on uneven bottom

$$\nabla\Phi\nabla h+\frac{\partial\Phi}{\partial z}=0 \quad (z=-h(x,\ y)) \tag{2}$$

and on free surface

$$\frac{\partial\eta}{\partial t}+\nabla\Phi\nabla\eta=\frac{\partial\Phi}{\partial z} \quad (z=\eta(x,\ y,\ t)) \tag{3}$$

and the dynamic boundary condition on the free surface

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}[(\nabla\Phi)^2)+(\frac{\partial\Phi}{\partial z})^2]+g\eta+\frac{\delta p_{atm}}{\rho}=0 \quad (z=\eta) \tag{4}$$

Here ∇ denotes the horizontal gradient and δp_{atm} is the variation of atmospheric pressure above sea surface which is an arbitrary function of horizontal coordinates and time. This model is exact for arbitrary wave motions (Cauchy-Poisson problem), but for practical realization it is difficult to solve the Laplace equation in a domain with unknown moving boundary and uneven bottom. As a result, only linear theory of waves generated by moving perturbations has been developed (such situation is similar to the generation of ship waves). We will concentrate our attention on the generation of nonlinear waves in shallow water. Nonlinearity in shallow water will be significant only if perturbations move with transcritical speed (the Froude number approaches to 1). Many researchers have investigated the problem for a basin of constant depth. analysis has shown that the generated waves are long so that we may simplify the problem by using the approximations of nonlinear shallow water theory; small bottom slopes and smoothness of wave profile. Such a model has been developed for waves of small but finite amplitude (Boussinesq-like models) and will be here generalized for cases of uneven bottom and arbitrary wave amplitude.

As usual in shallow water theory we shall expand the potential in Taylor series:

$$\Phi=\sum_{0}^{\infty}\Phi_n(x,\ y,\ t)\ [z+h(x,\ y)]^n \tag{5}$$

where all the functions Φ_n are unknown. Substitution into Eq. (1) gives the following recurrence relations

$$(n+2)(n+1)[1+(\nabla h)^2]\Phi_{n+2} +(n+1)[2\nabla\Phi_{n+1}\nabla h+\Phi_{n+1}\nabla^2 h]+\nabla^2\Phi_n=0 \tag{6}$$

Accordingly, we have only two independent functions: Φ_o and Φ_1. From the bottom boundary condition (2), we find Φ_1:

$$\Phi_1=-\frac{\nabla\Phi_0\nabla h}{1+(\nabla h)^2} \tag{7}$$

and thus only Φ_o is independent. It is straightforward to show that

$$\Phi_2=-\frac{1}{2[1+(\nabla h)^2]}\left\{\nabla^2\Phi_o-\frac{\nabla\Phi_o\nabla h\nabla^2 h}{1+(\nabla h)^2} -2\nabla h\nabla\left[\frac{\nabla\Phi_o\nabla h}{1+(\nabla h)^2}\right]\right\} \tag{8}$$

$$\Phi_3=-\frac{1}{6[1+(\nabla h)^2]}\{\nabla^2\Phi_1+2\Phi_2\nabla^2 h+4\nabla\Phi_2\nabla h\} \tag{9}$$

$$\Phi_4=-\frac{1}{12[1+(\nabla h)^2]}\{\nabla^2\Phi_2+3\Phi_3\nabla^2 h+6\nabla\Phi_3\nabla h\} \tag{10}$$

and so on. The expansion, Eq. (5), converges only if the assumptions of shallow water theory are satisfied, but the wave amplitude can be arbirary. After substitution of Eq. (5) in the kinematic (Eq. 3) and dynamic (Eq. (4) boundary conditions, we can obtain two-dimensional equations for Φ_o and η. It is convenient to take gradient of Eq. (4), and then it turns out to be an equation for the horizontal velocity $\nabla\Phi_o$.

$$\frac{\partial}{\partial t}\nabla\Phi+(\nabla\Phi\nabla)\nabla\Phi+\frac{\partial\Phi}{\partial z}\nabla\frac{\partial\Phi}{\partial z}+g\nabla\eta +\frac{\nabla p_{atm}}{\rho}=0 \tag{11}$$

It is recalled that $\nabla\Phi$ and $\partial\Phi/\partial z$ on sea surface are represented by series:

$$\nabla\Phi=\sum_{0}^{\infty}[\nabla\Phi_n+(n+1)\Phi_{n+1}\nabla h](h+\eta)^n \tag{12}$$

$$\frac{\partial\Phi}{\partial z}=\sum_{0}^{\infty}(n+1)\Phi_{n+1}(h+\eta)^n \tag{13}$$

where we may take only the bounded terms no higher than the second order of basin depth. This procedure has been used in many works without taking excitations into account. It is convenitnt to introduce the depth-averaged velocity defined by

$$\vec{u}=\frac{1}{h+\eta}\int_{-h}^{\eta}\nabla\Phi dz \tag{14}$$

By substituting Eq. (12) into the above, we have

$$\vec{u}=\nabla\Phi_o+\frac{1}{2}\nabla\Phi_1(h+\eta)+\Phi_1\nabla h+\frac{1}{3}\nabla\Phi_2(h+\eta)^2$$
$$+\Phi_2\nabla h(h+\eta)+\frac{1}{4}\nabla\Phi_3(h+\eta)^3+\Phi_3\nabla h(h+\eta)^2$$
$$+\frac{1}{5}\nabla\Phi_4(h+\eta)^4+\Phi_4\nabla h(h+\eta)^3+\cdots \tag{15}$$

With the help of the recurrence formula, it can be expressed in terms of $\nabla\Phi_o$. By utilizing the assumptions of shallow water theory, we can obtain the following approximation

$$\nabla\Phi_o\approx\vec{u}+\frac{h+\eta}{2}[\vec{u}\Delta h+2(\nabla h\nabla)\vec{u}]$$
$$+(\nabla h)^2\vec{u}+\frac{(h+\eta)^2}{6}\Delta\vec{u}+\cdots \tag{16}$$

Substituting Eq. (16) into Eqs. (12) and (13), we can transform Eqs. (3) and (11):

$$\frac{\partial\eta}{\partial t}+\nabla\cdot(H\vec{u})=0 \tag{17}$$

$$\frac{\partial\vec{u}}{\partial t}+(\vec{u}\nabla)\vec{u}+g\nabla\eta=\vec{D}-\frac{\nabla p_{atm}}{\rho} \tag{18}$$

where

$$H=h+\eta \tag{19}$$

$$\vec{D}=\frac{1}{H}\nabla(\frac{H^3}{3}R+\frac{H^2}{2}Q)-\nabla h(\frac{H}{2}R+Q) \tag{20}$$

$$R=\frac{\partial}{\partial t}\nabla\cdot\vec{u}+(\vec{u}\nabla)\nabla\cdot\vec{u}-(\nabla\cdot\vec{u})^2 \tag{21}$$

$$Q=\frac{\partial\vec{u}}{\partial t}\nabla h+(\vec{u}\nabla)(\vec{u}\nabla h) \tag{22}$$

We can also derive the pressure with the same accuracy:

$$p(x,y,z,t)=p_{atm}(x,y,t)+\rho g(\eta-z)+\frac{1}{2}\rho[z^2$$
$$+2h(z-\eta)-\eta^2]R-\rho(\eta-z)Q \tag{23}$$

In the absence of atmospheric perturbation in a basin of constant depth, this system reduces to the model by Su and Gardner (1969), was generalized to the case of smooth bottom (Zheleznyak, 1983, see also Voltzinger *et al*, 1989). Our model includes effects of both smooth bottom and atmospheric perturbation. The main advantages of this model are as follows: (1) As a two-dimensional model it is much simpler than the original three-dimensional model. (2) It describes waves of arbitrary (not only small) amplitude which may be very important for the study of resonance effects of wave generation by moving perturbations.

Limitation of this model is related to the smoothness of wave profile (we used small parameter of dispersion) and it is nesessary to check it in numerical computations.

In case of small amplitude waves, the dispersion term in Eq. (18) can be simplified

$$\vec{D}=\frac{h}{2}\frac{\partial}{\partial t}[\nabla\ div\ (h\vec{u})-\frac{h}{3}\nabla\ div\ \vec{u}] \tag{24}$$

and without atmospheric perturbation it was obtained in the paper of Peregrine (1967), see also Mei (1989).

As it is known, the smallness of dispersion in the frame of nonlinear-dispersive models corresponds to the following Taylor series of the exact linear dispersion relation

$$\omega^2=g\,h\,k^2(1-\frac{k^2\,h^2}{3}) \tag{25}$$

and this form is not convenient for numerical calculation because it leads to an instability at large wave numbers. It is known to be more convenient to use the improved dispersion relation in the form

$$\frac{\omega^2 h}{g} = k^2 h^2 \frac{1+\frac{1}{15}k^2 h^2}{1+\frac{6}{15}k^2 h^2} \qquad (26)$$

which is valid for $kh<2$ as Madsen *et al.* (1991) demonstrated. They have also proposed nonlinear equations of improved model. Because the atmospheric factor can be included in nonlinear dispersive models as an additive term, we can follow this line. Final result (in one-dimensional case) takes the form

$$\frac{\partial}{\partial t}(Hu)+\frac{\partial}{\partial x}(Hu^2)+gH\frac{\partial\eta}{\partial x}-\frac{1}{2}h^2\frac{\partial^3 Hu}{\partial x^2\partial t}+\frac{1}{6}h^3\frac{\partial^3}{\partial x^2\partial t}\frac{Hu}{h}=-\frac{H}{\rho}\frac{\partial p_{atm}}{\partial x} \qquad (27)$$

Other variations of approximate dispersion relation for water waves are possible (Hunt, 1979; Kozlov and Pelinovsky, 1989), and they can be used for developing different nonlinear-dispersive wave theories.

For the description of nonlinear waves generated by moving atmospheric perturbations in a basin of variable depth, models of different classes of approximations can be used. For example, Eqs. (17) and (18) can be used for very long waves of arbitrary amplitude; Eqs. (17) and (18) with dispersion term in the form as given by Eq. (24) for long waves of small amplitude; and Eqs. (17) and (27) for moderate length waves of small amplitude.

3. NONLINEAR RAY METHOD

Further simplifications are possible for weakly nonlinear and weakly dispersive waves generated by quasi-plane perturbations of atmospheric pressure when the wave field is practically one-dimensional. In this case the order of equations of nonlinear dispersive model can be reduced by one. It is usual to use ray methods which are well developed for linear approximation and generalized on weakly nonlinear media (Shen, 1975; Ostrovsky and Pelinovsky, 1975; Engelbrecht *et al*, 1988). It is convenient to transform the system of Eqs. (17)-(18) with dispersion term (24) to single wave equation for water level

$$\frac{\partial^2\eta}{\partial t^2}-\nabla\cdot[c^2\nabla\eta]=Q(\eta,\ \vec{u}) \qquad (28)$$

where

$$Q=-\frac{\partial}{\partial t}\nabla\cdot(\eta\vec{u})+\nabla\cdot[h(\vec{u}\nabla)\vec{u}]-\nabla\cdot(h\vec{D})+\nabla\cdot\frac{h\nabla p_{atm}}{\rho} \qquad (29)$$

where $c=\sqrt{q\,h(x,y)}$. We introduce a variable $s=\tau(\vec{r})-t$ instead of time t, where τ is to be determined later in the text. In terms of new variables, Eq. (28) will take the form

$$[1-c^2(\nabla\tau)^2]\frac{\partial^2\eta}{\partial s^2}-\frac{\partial}{\partial s}[2c^2\nabla\tau\nabla\eta+\eta\nabla\cdot(c^2\nabla\tau)]-\nabla\cdot(c^2\nabla\eta)=Q \qquad (30)$$

In order to determine the terms of this equation explicitly, we need some physical assumptions. Assume that the radius of curvature of the wave front is large (a quasi-plane wave approximation) and the depth varies slowly (mild bottom slopes), and then it is natural to accept that the solution depends, primarily, on one coordinate, s, while $\vec{r}$-dependence is weak. Consequently, the terms containing second-order derivatives and the squares of first-order derivatives with respect to the slow coordinate r may be neglected in Eq. (30) to the first approximation. Then Eq. (30) can be written into two independent equations:

$$(\nabla\tau)^2=c^{-2}=(gh)^{-1} \qquad (31)$$

$$\frac{\partial}{\partial s}[2c^2\nabla\tau\nabla\eta+\eta\nabla\cdot(c^2\nabla\tau)]+Q=0 \qquad (32)$$

Eq. (31) is well known in the linear theory of long waves in a basin of slowly varying depth. It is an eikonal equation (see, for example, Mei, 1989). Eq. (31) is a nonlinear partial differential equation for τ of first order and it can be solved by the method of characteristics and rewritten in the form of Hamilton-Jacobi equation. Following Mei (1989), we take a more elementary approach. Let $y(x)$ represent one of the rays, then its slope on plane (x, y) must be given by

$$\frac{dy}{dx}=\frac{\frac{\partial\tau}{\partial y}}{\frac{\partial\tau}{\partial x}} \tag{33}$$

After substituting it Eq. (31) and its derivatives, we obtain

$$\frac{d}{dx}\left[\frac{c^{-1}\frac{dy}{dx}}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}\right]=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\frac{\partial c^{-1}}{\partial y} \tag{34}$$

It is a nonlinear ordinary differential equation of second order for the ray $y(x)$ and we need two initial conditions; initial point $y(x_o)$ and initial direction $dy/dx(x_o)$. The ray path can be solved analytically or numerically. Many specific examples of ray patterns are discussed in the book by Mei (1989). Using rays as a reference coordinate l

$$l=\int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \tag{35}$$

and find the eikonal τ

$$\tau=\int\frac{dl}{c(x,y)} \tag{36}$$

it is another form of the celebrated Fermat's principle and τ is the time of wave motion along a ray. It should be emphasized that in this approximation the ray picture does not depend on weak dispersion and not on weak nonlinearity.

Let us consider now Eq. (32). Flow velocity in function Q may be expressed by means of formulas from a linear theory of long waves:

$$\vec{u}=\vec{i}c(x,y)\frac{\eta}{h(x,y)} \tag{37}$$

where $\vec{i}$ is a unit vector along a ray. Taking only derivatives of coordinate s into account we can write the main terms in Q:

$$Q=\frac{3}{2h}\frac{\partial^2\eta^2}{\partial s^2}+\frac{h}{3g}\frac{\partial^4\eta}{\partial s^4}-\frac{h}{g\rho}\frac{\partial^2 P_{atm}}{\partial s^2} \tag{38}$$

and then the Eq. (32) can be integrated

$$2c^2\nabla\tau\nabla\eta+\eta\,\mathrm{div}(c^2\nabla\tau)+\frac{3}{2h}\frac{\partial\eta^2}{\partial s}+\frac{h}{3g}\frac{\partial^3\eta}{\partial s^3}$$

$$=\frac{h}{g\rho}\frac{\partial P_{atm}}{\partial s} \tag{39}$$

Using the Ostrogradsky-Gauss theorem in curvilinear ray coordinates we can calculate div $\nabla\cdot(\nabla\tau)$.

$$\nabla\cdot(\nabla\tau)=\frac{1}{b}\frac{d}{dl}\frac{b}{c} \tag{40}$$

where b is called the ray separation factor (see book by Mei, 1989) or differential width of ray beams. Eq. (39) may be written in the form

$$\sqrt{gh}\frac{\partial\eta}{\partial l}+\frac{3\eta}{2h}\frac{\partial\eta}{\partial s}+\frac{h}{6g}\frac{\partial^3\eta}{\partial s^3}$$

$$+\frac{\sqrt{gh}}{4hb^2}\frac{d(hb^2)}{dl}\eta=\frac{h}{2g\rho}\frac{\partial p_{atm}}{\partial s} \tag{41}$$

Eq. (41) is a modification of the known forced Korteweg-de Vries equation. In fact it reduces to the forced Korteweg-de Vries equation for plane waves in a basin of constant depth.

Eq. (41) at $h=const$ is a basic equation for description of wave generation by moving perturbation in many papers. On the other hand, Eq. (41) with $P_{atm}=0$ is often used for long wave evolution in the coastal zone (Pelinovsky, 1982; Voltzinger *et al.*, 1989; Mei, 1989).

Mention should also be made of conservation laws for Eq. (41). It is known that there exist an infinite number of conservation laws for the Korteweg-de Vries equation with constant coefficients, which is indicative of its full integrability (Whitham, 1974). A modified forced Korteweg-de Vries equation (41) possesses only one conservation law:

$$h^{1/4}\nabla^{1/2}\int_{-\infty}^{+\infty}\eta(s,l)ds=\mathrm{const} \tag{42}$$

which can be used to validate numerical calculations.

When analysing the wave propagation in basin of variable depth, one must bear in mind the following significant circumstance. The formulas presented above are valid only when the rays do not intersect (b does not turn out to be zero). Such situations are typical in the coastal zone with simple geometry when waves propagate in the region of decreasing depth. If the waves encounter regions of increasing depth, the rays will intersect due to refraction, thus forming caustics. Linear problems of monochromatic wave transformation on a straight caustic can be solved exactly, see for example, Mei (1989). The solution is expressed in terms of the Airy function. The amplification factor (it is proportional to $\lambda^{-1/6}$) is not large for long waves, therefore, wave can be considered as weak nonlinear everywhere, including the caustic zone. Some nonlinear corrections to the Airy function were made in the book of Engelbrecht *et al.* (1988). It is more important that the wave form must be changed in caustic zone even in linear theory. The reflected (or transformed) monochromatic wave has the same amplitude as the incident wave but differs from the latter in phase by $\pi/2$. Such a phase transformation is equivalent to the Hilbert transformation for wave of arbitrary form

$$\eta_{tr}(s,l)=\frac{1}{\pi}\int\eta_{in}(\tau,l)\frac{d\tau}{s-\tau} \qquad (43)$$

and this equation can be used for describing the nonlinear wave transformation in a caustic zone having the size order of wavelength. Accuracy of this approach was checked by comparison with numerical modeling of cylindrical solitary waves (Pelinovsky and Stepanyants, 1981; Chwang and Wu, 1977).

Eqs. (33), (41) and (43) consitute a complete system of equations of a nonlinear-ray theory of wave generation and transformation in a basin of variable depth.

4. CONCLUSION

A physical-mathematical model for the nonlinear wave generation by moving atmospheric perturbations and its transformation in the coastal zone have been developed. It is based on a modified forced Korteweg-de Vries equation. The rays in this model are determined from the linear theory of long waves and the wave amplitude from the nonlinear evolution equation. The model is supplemented by a special treatment which allows the calculation of wave transformation in the caustic zone.

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