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Effects of Free Surface Fluctuation on the Response of Submerged Structure
波浪에 의한 海水面의 變化가 海洋構造物의 動的擧動에 미치는 影響

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Abstract□In this paper, the effects of free surface fluctuation on the dynamic response of offshore structure is studied. In order to make the mathematical treatment of problem more tractable, only a single degree of freedom system subjected to long crested, stationary, Gaussian, non-breaking random waves of arbitrary bandwidth is considered. Wave force is computed based on the Morison equation in which wave induced fluid particle velocity and acceleration are modified to account for the effect of intermittent submergence of structural members near the free surface. It is shown that the response spectrum is reduced and higher harmonic response component appears when the intermittent submergence of structural member is considered. Furthermore, it is also found that the amount of reduction in the response spectrum is getting smaller as frequency is increased which might be attributed to the higher harmonic component caused by intermittent submergence and these effects are getting profound as water depth is decreased.

要 旨：1次 自由度를 갖는 海洋構造物을 선정하여 long crested, stationary, Gaussian random waves가 構造物의 動的應答 스펙트럼에 미치는 영향에 대해 규명하였다. 해수면 부근에서 주기적으로 수면위로 浮上하는 연직부재를 고려하기 위해 hybrid Morison equation을 제안했고 이 방정식을 기초로 波力을 계산했다. 주기적으로 수면위로 부상하는 효과를 고려한 결과 동적응답 스펙트럼은 감소하였으며 고차 응답성분이 발생했다. 또한 동적응답 스펙트럼은 주파수가 증가할수록 감소하는 경향을 보였으며 이는 해수면 위로 주기적으로 부상하는 현상을 고려할 때 발생하는 고차응답성분 때문인 것으로 사료된다.

1. INTRODUCTION

In studying the dynamic response of slender offshore structures to wave actions, it is often assumed that wave forces by calculated on the basis of Morison equation, in which the force at any section of a member is expressed in terms of wave induced fluid particle velocities and accelerations. Sections of a vertical member near the water surface are intermittently submerged, so that wave forces should be modified from those acting on continuously submerged sections. For the case of random waves, the corresponding statistical properties of the water particle kinematics for intermittently submerged locations near the free surface were examined theoretically by Tung (1975a, b) and Pajouhi and Tung (1975). The statistical properties of the corresponding intermittent forces on such sections were investigated theoretically by Tung (1975a) and more recently by Issacson and Baldwin (1990a). Subsequently, Isaacson and Baldwin (1990b) and Isaacson and Subbiah (1990) compared these theoretical predictions with experiments and with numerical simulations, respectively. The effects of these intermittent forces on the response of structures have received less attention save for the work of Kanegonkar and Haldar (1987) and Tung and Yang (1991) the scope of which, however, is somewhat limited and only

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deterministic waves are considered. It is the purpose of this study to rexamine this subject. For convenience, only a single degree of freedom system is considered. Furthermore, waves are assumed to be long crested, stationary, Gaussian, non-breaking and of arbitrary bandwidth.

2. PROBLEM STATEMENT

Denoting the displacement of the cylinder as y, the differential equation of motion is

$$y(\mathrm{s,t}) = -M\ddot{y}f + \int_0^L F_i(s,t)\,g(s)ds + \int_0^L F_d(s,t)\,g(s)\,ds \quad (1)$$

where

$$f = \frac{L^3}{3EI} \quad (2)$$

and

$$g(s) = \frac{s^2(3L-s)}{6EI} \quad (3)$$

are respectively static displacements of the mass due to unit horizontal forces applied at $s=L$ and $s=s$. Furthermore, F_i(s,t) and $F_d(s,t)$ are respectively the distributed inertia and drag force intensities given by (Sarpkaya and Isaacson, 1981)

$$F_i(s,t) = K_m\dot{u}(s,t) - M_a\ddot{y}(s,t) \quad (4)$$

and

$$F_d(s,t) = K_d[u(s,t) - \dot{y}(s,t)]\,|u(s,t) - \dot{y}(s,t)| \quad (5)$$

where

$$K_m = C_m\rho\frac{\pi D^2}{4} \quad (6)$$

$$M_a = C_a\rho\frac{\pi D^2}{4} \quad (7)$$

and

$$K_d = C_d\rho\frac{D}{2} \quad (8)$$

In equation (6), (7) and (8), C_m, $C_a = C_m - 1$ and C_d are respectively the inertia, added mass and drag coefficients, D is the diameter of the cylinder, u is the horizontal particle velocity and overdot denotes the differentiation with respect to time. Because of quadratic drag forces, the response of the structure is governed by nonlinear differential equation. In a random wave field, this equation is stochastic and the solution has been achieved by statistical equivalent linearization method (Dao and Penzien, 1982). Following Dao *et al.* (1982), the drag force, F_d, is

$$F_d(s,t) \cong K_d u(s,t)|u(s,t)| - 2K_d E[|u(s,t)|]\dot{y}(s,t) \quad (9)$$

where $E[\cdot]$ is used to denote the expected value of the quantity enclosed in the brackets. If the waves are such that they remain mostly within the limits of the boundary of the member, then equation (4) and (9) gave a good approximation. If, however, the wave amplitude is larger compared with the cross-sectional dimension of the cylinder, then the events of complete submergence or emergence of the cyliner are associated with finite values of probability. Taking the possibility of intermittent submergence of the cylinder into consideration, the equation (4) and (9) are modified to

$$F_i(s,t) = K_m\dot{u}(s,t)H[\eta + L - s] - M_a\ddot{y}(s,t) \quad (10)$$

$$F_d(s,t) \cong K_d u(s,t)|u(s,t)|H[\eta + L - s] - 2K_d E[|u(s,t)|H[\eta + L - s]]\dot{y}(s,t) \quad (11)$$

where $H[\cdot]$ is the Heaviside unit step function and η is water surface displacement. Equation (10) and (11) simply state that, for any point fixed in space, as long as it is immersed in water, u(s,t)$=u(s,t)$ and $\dot{u}(s,t)=\dot{u}(s,t)$; otherwise, they are set equal to zero. For further simlification of equation (1), we assume the deflected shape of the cylinder to be

$$y(\mathrm{s,t}) = \varphi(s)Y(t) \quad (12)$$

where $Y(t)$ is the generalized coordinate and $\varphi(s)$ is a shape function which is given by

$$\varphi(s) = \frac{s}{L} \quad (13)$$

Upon substituting equation (10) and (11) into (1), the equation of motion becomes

$$\tilde{M}\ddot{Y} + \tilde{C}\dot{Y} + \tilde{K}Y = \tilde{F} \quad (14)$$

where

$$\tilde{M}=M+M_w \quad (15)$$

is total mass,

$$M_w=\frac{11}{40}M_aL \quad (16)$$

is added mass,

$$\tilde{C}=2\frac{Kd}{f}\int_0^L E[|u(s,t)|H[\eta+L-s]]\varphi(s)g(s)\,ds \quad (17)$$

is hydrodynamic damping coefficient, $\tilde{K}=1/f$ is structural stiffness and

$$\tilde{F}=\frac{K_m}{f}\int_0^L \dot{u}(s,t)H[\eta+L-s]\,g(s)\,ds + \frac{K_d}{f}\int_0^L u(s,t)|u(s,t)|H[\eta+L-s]\,g(s)\,ds. \quad (18)$$

Even though the integral in equation (17) and (18) may be evaluated straight forwardly, in a interest to identify the key parameters of wave-structure interaction, the random variables involved are nondimensionalized. Following Lipset (1986a), the quantities Y, η, s, t, u and $\dot{u}$ are nondimensionalized to represent Y/D, η/σ_η, s/L, $\omega_n t$, u/σ_u and $\dot{u}/\sigma_{\dot{u}}$, respectively, where $\omega_n=(1/\tilde{M}f)^{1/2}$ is the natural frequency of vibration of the structure in water and σ is used to denote the standard deviation of the quantity in the subscript. In terms of nondimensional variables, the equation of motion is now written as

$$\ddot{Y}+C\dot{Y}+Y=F \quad (19)$$

where

$$C=c_1\int_0^1 \sigma_u E[|u(s,t)|H[\eta-a]](3s^3-s^4)\,ds \quad (20)$$

and

$$F=c_2\int_0^1 \sigma_{\dot{u}}\dot{u}(s,t)H[\eta-a](3s^2-s^3)\,ds + c_3\int_0^1 \sigma_u^2(s,t)|u(s,t)|H[\eta-a](3s^2-s^3)\,ds \quad (21)$$

In equation (20) and (21),

$$a=-\frac{L(1-s)}{\sigma_\eta} \quad (22)$$

$$c_1=\frac{K_d}{\tilde{M}\omega_n} \quad (23)$$

$$c_2=\frac{K_mL}{2\tilde{M}D\omega_n^2} \quad (24)$$

and

$$c_3=\frac{K_dL}{2\tilde{M}D\omega_n^2} \quad (25)$$

Rewriting equation (19) gives

$$Y(t)=\int_{\infty}^{\infty} h(t-t)F(\tau)d\tau \quad (26)$$

where $h(\cdot)$ is the impulse response function of equation (19) and τ is time lag. The auto-correlation function $r_{YY}(\tau)$ is the expected value of the product of $Y(t)$ and $Y(t+\tau)$. That is,

$$r_{YY}(\tau)=E[Y(t)Y(t+\tau)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(t-\tau_1)h(t+\tau-\tau_2)E[F(\tau_1)F(\tau_2)]d\tau_1 d\tau_2 \quad (27)$$

Denoting θ_1, θ_2 as $t-\tau_1$ and $t+\tau-\tau_2$, respectively, the equation (20) is reduced to

$$r_{YY}(\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\theta_1)h(\theta_2)r_{FF}(\tau+\theta_1-\theta_2)d\theta_1 d\theta_2 \quad (28)$$

The spectrum of Y is given by the Fourier transform of the covariance function $C_{YY}(\tau)=r_{YY}(\tau)-E^2[Y]$. That is,

$$S_{YY}(\omega)=H^*(\omega)H(\omega)S_{FF}(\omega) \quad (29)$$

where ω is frequency, $S(\omega)$ denotes the spectrum (auto or cross spectrum) of the quantities indicated in the subscript, the symbol* denotes the complex conjugation operation,

$$H(\omega)=\frac{1}{1-\omega^2+iC\omega} \quad (30)$$

is unit frequency response function and $i=\sqrt{-1}$ is the imaginary unit.

3. AUTO-CORRELATION FUNCTION AND SPECTRUM

There are basically two ways in determining $r_{YY}(\tau)$ (Tung and Laurence, 1992). The first method is due to Borgman (1965) in connection with the study of auto-correlation function and spectrum of wave force on vertical members. Subsequently, the same

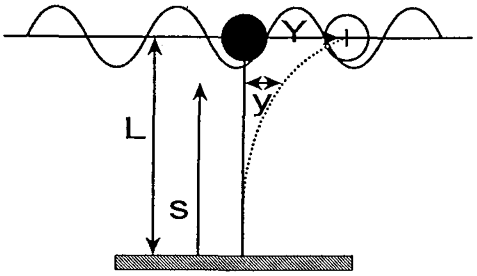

Fig. 1. Definition sketch.

method was used to study the effect of free surface fluctuation on auto-correlation function and spectrum of a random wave field (Pajouhi and Tung, 1975a, Tung and Pajouhi, 1976, Tung, 1975) and of wave force on vertical cylinders (Pajouhi and Tung, 1975b). Based on the definition of expectation, the method takes the joint probability density function of the random variables involved and carries out the necessary integrations. If the method is applied to the present study, the auto-correlation function of F would be a nonlinear function of the correlaton coefficient function $r_{\dot{u}\dot{u}}(\tau)=E=\dot{u}(t(\dot{u}(t+\tau)]$, $r_{\eta\eta}(\tau)=E[\eta(t)\eta(t+\tau)]$, $r_{u\eta}(\tau)=E[u(t)\eta(t+\tau)]$ and $r_{uu}(\tau)=E[u(t)u(t+\tau)]$ (hereafter referred to collectively as r). The spectrum of Y must be performed numerically. Following Borgman (1985), we may expand the nonlinear function of correlation functions r into Taylor's series and retain only the constant and linear terms based on the facts that the correlation coefficient is less than unity. In this way, the Fourier transform of $C_{YY}(\tau)$ may be carried out in closed form. A parallel way of determining the auto-correlation function $r_{YY}(\tau)$ is to express the joint probability density function of two Gaussian random variables as series in terms of the Hermite polynomials (Erdely *et al.*, 1953)

$$h_n(z)=\frac{(-1)^n}{(n\,!\,)^{1/2}}\exp\left(\frac{z^2}{2}\right)\frac{d^n}{dz^n}\left[\exp\left(-\frac{z^2}{2}\right)\right] \qquad (31)$$

This is,

$$\frac{1}{(1-r^2)^{1/2}}\exp\left[-\frac{1}{2(1-r^2)}(x^2+y^2-2rxy)\right]$$
$$=\sum_{n=0}^{\infty}h_n(x)h_n(y)r^n\exp\left[-\frac{1}{2}(x^2+y^2)\right]. \qquad (32)$$

When the expected value under consideration involves more than two random variables, it is convenient to resort to the use of conditional expectation in conjunction with the Hermite polynomial series to perform the task. In line with linear approximation suggested by Borgman (1965) mentioned above, the higher order terms in the Hermite polynomial representation may be truncated. It is the approach that we shall follow in this study. To demonstrate the various techniques involved in the estimation of the expected value, the estimating procedure of the expected value appeared in equation (27) will be shown in detail.

- $c_1\int_0^1\sigma_u E[\,|u(s,t)|H[\eta+a]](3s^3-s^4)\,ds$

Using the fact that u and η is jointly Gaussian due to stationerity assumption,

$$E[\,|u(s,t)|H[\eta+a]]=\int_a^{\infty}\int_{-\infty}^{\infty}|u(s,t)|f_{u\eta}(u,\eta)\,du\,d\eta \qquad (33)$$

where $f_{u\eta}(\,\cdot\,,\cdot\,)$ is the probability density function of the jointly Gaussian random variables u and η. By expanding the joint probability density function into series as shown in equation (32) and taking only the zeroth and first two terms, we have

$$E[\,|u(s,t)|H[\eta+a]]=\frac{1}{2\pi}\int_a^{\infty}\int_{-\infty}^{\infty}|u(s,t)|$$
$$(1+u\eta r_{u\eta}(0))e^{-1/2(u^2+\eta^2)}\,du\,d\eta. \qquad (34)$$

The integration in equation (34) may be easily carried out to give

$$E[\,|u(s,t)|H[\eta+a]]=\sqrt{\frac{2}{\pi}}Q(a) \qquad (35)$$

where

$$Q(a)=\int_a^{\infty}Z(z)\,dz \qquad (36)$$

and

$$Z(z)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}z^2\right) \qquad (37)$$

is the probability density function of the zero-mean Gaussian random variable of unit standard deviation. Hereafter, for brevity, let the subscripts 1 and 2, respectively, refer to quantities evaluated at the time $t_1=t$ and $t_2=t+\tau$..

- $c_2^2\int_0^1\int_0^1\sigma_{\dot{u}}^2 E[\dot{u}_1\dot{u}_2H_1H_2](3s_1^2-s_1^3)(3s_2^2-s_2^3)\,ds_1ds_2$

Exploiting the statistical independence of $\dot{u}$ and η because of the stationerity assumption, we can rewrite the expected value as

$$\begin{aligned}E[\dot{u}_1\dot{u}_2H_1H_2]&=E[\dot{u}_1\dot{u}_2]E[H_1H_2]\\&=r_{\dot{u}\dot{u}}(\tau)\frac{1}{2\pi}\int_a^\infty\int_a^\infty\\&\quad(1+\eta_1\eta_2r_{\eta\eta}(\tau))\,e^{-1/2(\eta_1^2+\eta_2^2)}d\eta_1d\eta_2\\&=r_{\dot{u}\dot{u}}(\tau)[Q^2(a)+r_{\dot{\eta}\dot{\eta}}(\tau)]\end{aligned}\tag{38}$$

- $c_3^2\int_0^1\int_0^1\sigma_u^4 E[u_1|u_1|u_1|u_2|H_1H_2](3s_3^2-s_1^3)(3s_2^2-s_2^3)\,ds_1ds_2$

Since the expected value involves four random variables u_1, u_2, η_1 and η_2, it would be desirable to make use of conditional expectation (Papoulis, 1965). Thus,

$$E[u_1|u_1|u_2|u_2|H_1H_2]=E[H_1H_2E[u_1|u_1|u_2|u_2|\ |\eta_1,\eta_2]\tag{39}$$

where $E[A|B]$ is the conditional expectation of event A given the occurrence of event B. In equation (39), $Eu_1|u_1|u_2|u_2|\ |\eta_1,\eta_2]$ is the conditional correlation function of $u_1|u_1|$ and $u_2|u_2|$ and is equal to the sum of the conditional covariance function C_{12} and the product $\mu_1\mu_2$ of the conditional mean values, μ_1, of $u_1|u_1|$ and μ_2, of $u_2|u_2|$. The linear mean square estimation technique requires that μ^1 and μ^2 be a linear function of η_1 and η_2. That is,

$$\mu_1=a_1\eta_1+b_1\eta_2\tag{40}$$

$$\mu_2=a_2\eta_1+b_2\eta_2\tag{41}$$

where the coefficients a_1, b_1, a_2 and b_2 are selected in such a way to make $u_2|u_1|-\mu_1$ and $u_2|u_2|-\mu_2$ orthogonl to η_1 and η_2

$$E[(u_1|u_1|-\mu_1)\eta_1]=0,\ E[(u_1|u_1|-\mu_1)\eta_2]=0,\tag{42}$$

$$E[(u_2|u_2|-\mu_2)\eta_1]=0,\ E[(u_2|u_2|-\mu_2)\eta_2]=0,\tag{43}$$

from which we get, to the order of r,

$$a_1=\frac{4}{\sqrt{2\pi}}\,r_{\eta u}(0)\tag{44}$$

$$b_1=\frac{4}{\sqrt{2\pi}}r_{\eta u}(\tau)-\frac{4}{\sqrt{2\pi}}r_{\eta u}(0)r_{\eta\eta}(\tau)\tag{45}$$

and

$$a_2=b_1,\ b_2=a_1.\tag{46}$$

The conditional covariance function C_{12} of $u_1|u_1|$ and $u_2|u_2|$ is

$$C_{12}=E[(u_1|u_1|-a_1\eta_1-b_1\eta_2)(u_2|u_2|-a_2\eta_1-b_2\eta_2)]\tag{47}$$

and by the virtue of the orthogonality properties and exploiting the facts that u and η is jointly Gaussian

$$\begin{aligned}C_{12}&=E[(u_1|u_1|-a_1\eta_1-b_1\eta_2)u_2|u_2|]\\&=\frac{8}{\pi}r_{uu}(\tau)-\frac{4a_1}{\sqrt{2\pi}}r_{\eta u}(\tau)-\frac{4b_1}{\sqrt{2\pi}}r_{\eta u}(0).\end{aligned}\tag{48}$$

From equation (39),

$$E[u_1|u_1|u_2|u_2|H_1H_2]=C_{12}E[H_1H_2]+E[\mu_1\mu_2H_1H_2]\tag{49}$$

Substituing μ_1, μ_2 and C_{12} into equation (49) and performing the expectation operation,

$$\begin{aligned}E[u_1|u_1|u_2u_2|H_1H_2]&=\frac{8}{\pi}Z^2(a)r_{\eta u}^2(0)+\frac{8}{\pi}Q^2(a)r_{uu}(\tau)\\&\quad+\frac{16}{\pi}aZ(a)Q(a)r_{\eta u}(0)r_{\eta u}(\tau)\\&\quad\mp\frac{8}{p}a^2Z^2(a)r_{\eta u}^2(0)r_{\eta\eta}(\tau).\end{aligned}\tag{50}$$

After all the expected values in equation (27) being derived, the $\tau_{FF}(\tau)$ takes the form, to the order of r,

$$r_{FF}(\tau)=G_1^2r_{\dot{u}\dot{u}}(\tau)+G_2^2r_{uu}(\tau)+G_3^2r_{\eta u}(\tau)+G_4^2r_{\eta\eta}(\tau)+G_5^2\tag{51}$$

where

$$G_1=c_2\int_0^1\sigma_{\dot{u}}Q(a)(3s^2-s^3)\,ds\tag{52}$$

$$G_2=\sqrt{\frac{8}{\pi}}c_3\int_0^1\sigma_u^2Q(a)(3s^2-s^3)\,ds\tag{53}$$

$$G_3=\sqrt{\frac{16}{\pi}}c_3\int_0^1\sigma_u^2\sqrt{aZ(a)Q(a)}\,\sqrt{r_{\eta u}(0)}(3s^2-s^3)\,ds\tag{54}$$

$$G_4=\sqrt{\frac{8}{\pi}}c_3\int_0^1\sigma_u^2aZ(a)r_{\eta u}(0)(3s^2-s^3)\,ds\tag{55}$$

$$G_5=\sqrt{\frac{8}{\pi}}c_3\int_0^1\sigma_u^2 Z(a) r_{\eta u}(0)(3s^2-s^3)\,ds. \tag{56}$$

It may be verified that G_5^2 is nothing but the square of the mean value of F. With the auto correlation function of F determined, the spectrum of force F is, to the order of r,

$$S_{FF}(\omega)=G_1^2 S_{\dot{u}\dot{u}}(\omega)+G_2^2 S_{uu}(\omega)+G_3^2 S_{\eta u}(\omega)+G_4^2 S_{\eta\eta}(\omega). \tag{57}$$

If we represent the stationary random process $\eta(t)$ and $u(t)$ by its Fourier-Stieltjes integral

$$\eta(t)=\frac{1}{\sigma_\eta}\int_\omega dA(\omega)\,e^{-\omega t} \tag{58}$$

$$u(t)=\frac{1}{\sigma_u}\int_\omega \omega dA(\nu)\frac{\cosh kLs}{\sinh kL}e^{-i\omega t} \tag{59}$$

where $dA(\omega)$ is the complex, random, zero-mean Gaussian Fourier-Stieltjes coefficient, then, it is known (Phillips, 1980) that

$$E[dA(\omega)\,dA^*(\omega')] \quad \text{if } \omega\neq\omega'$$

$$=\frac{1}{\sigma_\eta^2}S_{\eta\eta}(\omega) \quad \text{if } \omega=\omega'. \tag{60}$$

Based on equations (58), (59) and (60), it is easy to see that

$$r_{\eta\eta}(\tau)=\frac{1}{\sigma_\eta^2}\int S_{\eta\eta}(\omega)\,e^{i\omega\tau}d\omega \tag{61}$$

$$r_{uu}(\tau)=\frac{1}{\sigma_u^2}\int \omega^2 S_{\eta\eta}(\omega)\frac{\cosh^2 kLs}{\sinh^2 kL}e^{i\omega\tau}d\omega \tag{62}$$

$$r_{\eta u}(\tau)=r_{u\eta}(\tau)=\frac{1}{\sigma_u\sigma_\eta}\int \omega S_{\eta\eta}(\omega)\frac{\cosh kLs}{\sinh kL}e^{i\omega\tau}d\omega \tag{63}$$

and

$$r_{\dot{u}\dot{u}}(\tau)=\frac{1}{\sigma_{\dot{u}}^2}\int \omega^4 S_{\eta\eta}(\omega)\frac{\cosh^2 kLs}{\sinh^2 kL}e^{i\omega\tau}d\omega \tag{64}$$

so that

$$S_{uu}(\omega)=\frac{1}{\sigma_u^2}\omega^2\frac{\cosh^2 kLs}{\sinh^2 kL}S_{\eta\eta}(\omega) \tag{65}$$

$$S_{\eta u}(\omega)=\frac{1}{\sigma_\eta\sigma_u}\omega\frac{\cosh kLs}{\sinh kL}S_{\eta\eta}(\omega) \tag{66}$$

and

$$S_{\dot{u}\dot{u}}(\omega)=\frac{1}{\sigma_{\dot{u}}^2}\omega^4\frac{\cosh^2 kLs}{\sinh^2 kL}S_{\eta\eta}(\omega) \tag{67}$$

4. NUMERICAL RESULTS

To quantify the above results, we must specify the wave spectrum from which the quantities σ_η, σ_u, $\sigma_{\dot{u}}$ and $r_{\eta u}(0)$ can be calculated. In this study, we elect to use the Wallops spectrum (Huang *et al.*, 1981) which takes the form

$$S_{\eta\eta}(\omega)=\frac{\alpha g^2}{\omega^m\omega_0^{5-m}}\exp\left[-\frac{m}{4}\left(\frac{\omega_0}{\omega}\right)^4\right] \tag{68}$$

where

$$m=\left|\frac{\log(2\pi^2\xi^2)}{\log 2}\right| \tag{69}$$

is the absolute value of the slope of the spectrum (on the log-log scale) in the high frequency range and

$$\xi=\sigma_\eta/L_0 \tag{70}$$

is the significant wave slope, L_0 being the wave length whose frequency ω_0 corresponds to that of peak of the single peak Wallops spectrum. In equation (68), the coefficient α is given by

$$\alpha=\frac{(2\pi\xi)^2 m^{(m-1)/4}}{4^{(m-5)/4}}\frac{1}{\Gamma[(m-1)/4]} \tag{71}$$

where $\Gamma[\cdot]$ is the gamma function (Abramowitz and Stegun, 1968). Closer examination of the above shows that the Wallops spectrum is specified by the two parameters ξ and ω_0; the former is a measure of the severity of the sea and rarely exceeds the value of 0.02 in the field although can reach as high as 0.04 in wind-wave tank experiments in the laboratory. From equation (68), it may be verified that

$$\sigma_\eta=E^{1/2}[\eta(t)\eta(t)]=\frac{2\pi\xi g}{\omega_0^2} \tag{72}$$

$$\sigma_u=\frac{2\pi\xi g}{\omega_0}\left(\frac{m}{4}\right)^{1/4}\left\{\frac{\Gamma((m-3)/4)}{\Gamma((m-1)/4)}\right\}^{1/2}\frac{\cosh kLs}{\sinh kL} \tag{73}$$

$$\sigma_{\dot{u}}=2\pi\xi g\left(\frac{m}{4}\right)^{1/2}\left\{\frac{\Gamma((m-5)/4)}{\Gamma((m-1)/4)}\right\}^{1/2}\frac{\cosh kLs}{\sinh kL} \tag{74}$$

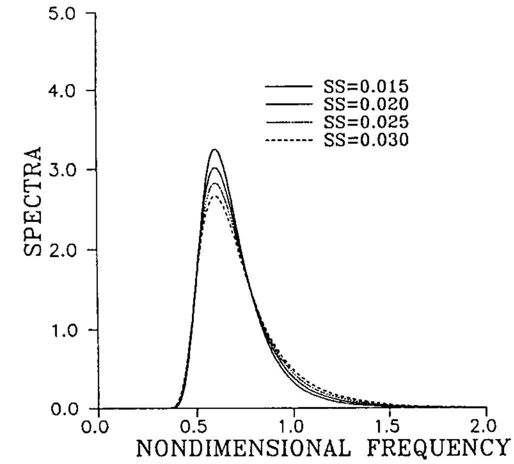

Fig. 2. Wallops wave spectrum (ss is ξ).

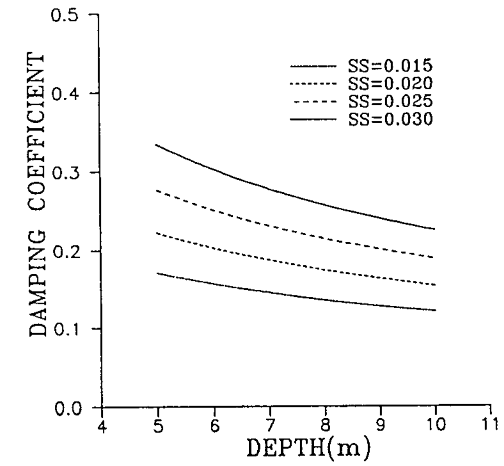

Fig. 3. Hydrodynamic damping coefficient for varying ξ and depth.

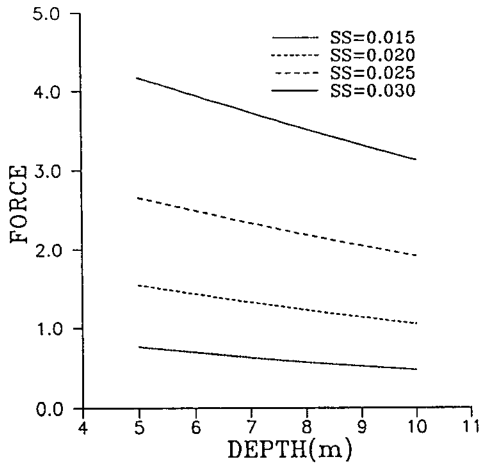

Fig. 4. Expected value of wave force for varying ξ and depth.

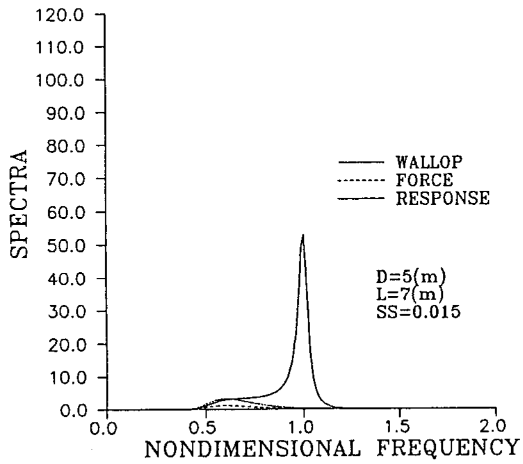

Fig. 5. Response spectrum (ss is ξ).

and

$$r_{\eta u}(0)=\frac{\Gamma((m-2)/4)}{\{\Gamma((m-1)/4)\Gamma((m-3)/4)\}^{1/2}} \qquad (75)$$

The parameters involved in determining $E[F]$, $E[F^2]$, $S_{FF}(\omega)$ and $S_{YY}(\omega)$ are the force coefficients C_m, C_a and C_d, the sea state parameters ξ and ω_0, water depth L, and the diameter of cylinder D. Following Lipset (1986a), we select $C_m=1.25$, $C_d=1.45$ and $\rho D^2/M=0.785$. For the sea state, we use $\xi=0.015$, 0.02, 0.025 and 0.03 and $\omega_0=0.6$ rad/sec. We allow $\dot{D}$ to take values of 1.0m and 5.0m. In Fig. 2, the Wallops wave spectrum is plotted for $\xi=0.015$, 0.02, 0.025 and 0.03. Nondimensionalized hydrodynamic damping coefficient, C, and the expected value of F, $E[F]$, plotted in Fig. 3 and Fig. 4, respectively, for varying ξ and depth which are expected to be a major factors affecting the effects of free surface fluctuation on the dynamic response of the structure. It is shown that as the nonlinearity of random wave field is getting increased, the wave energy is spreading toward the relatively low and high frequency range. It is also shown that the damping coefficient and $E[F]$ are getting larger with increased ξ, whereas this trend is reversed when the water depth is increased. With respect to the response

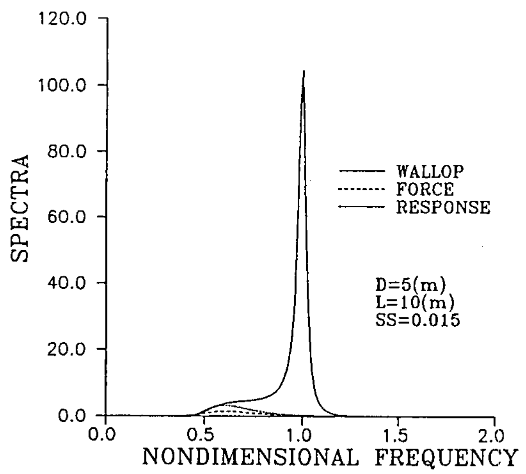

Fig. 6. Response spectrum (ss is ξ).

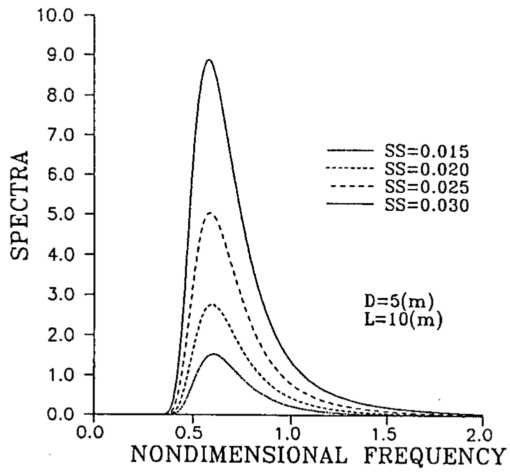

Fig. 8. Wave force spectrum (ss is ξ).

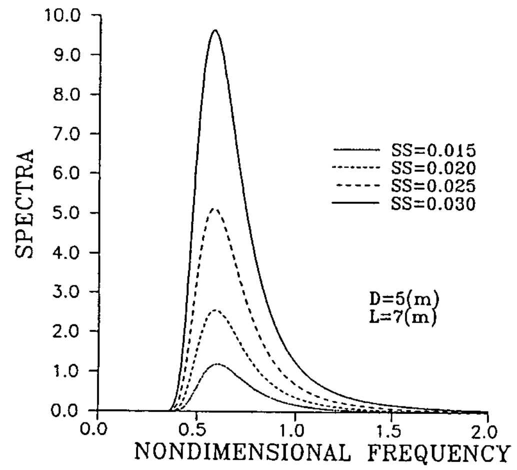

Fig. 7. Wave force spectrum (ss is ξ).

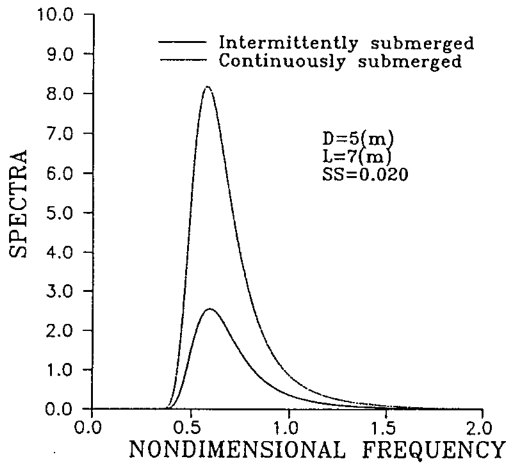

Fig. 9. Wave force spectrum (ss is ξ).

spectrum, Fig. 5 shows the spectra for $\xi=0.015$ and $L=7$m and Fig. 6 for $\xi=0.015$ and $L=10$m where the wave and force spectra are also included for the comparison. Note that in this case the response spectrum is broadened due to resonance like condition with an associated large response occurring around $\omega/\omega_n=1.0$ whereas the force and wave height spectra have a peak around $\omega/\omega_n=0.6$. Fig. 7 and Fig. 8 show the wave force spectra for $\xi=0.015$, 0.02, 0.025 and 0.03 at $L=7$m and, at $L=10$m respectively. It is shown that with increased ξ, the force spectra are getting larger. It is also shown that when the depth is getting larger, for a case in which ξ is less than 0.03, the force spectra are increased whereas in a case where ξ is larger than 0.03, the force spectra is getting decreased. For the comparison, the wave force spectra considering the effect of the intermittent submergence are also plotted with a continuously submerged counterpart in Fig. 9 for $L=7$m and in Fig. 10 for $L=10$m respectively. Response spectra for varying ξ and depth are plotted in Fig. 11 and Fig. 12, Fig. 13 and 14 show the response spectra with a continuously submerged counterpart. It is shown that the response spectra are reduced when the intermittent submergence is considered and the amount of reduction in the response spectrum is getting smaller as frequency is increased, which might be attributed to the higher

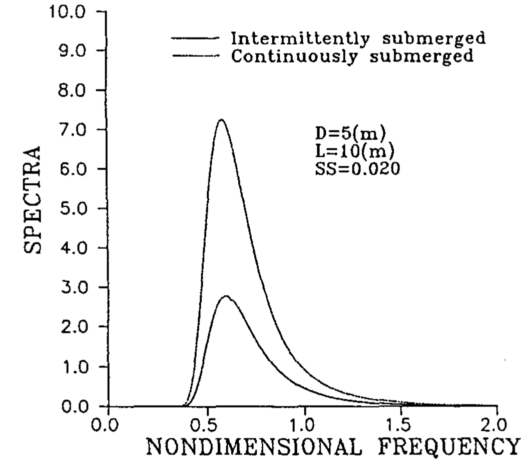

Fig. 10. Wave force spectrum (ss is ξ).

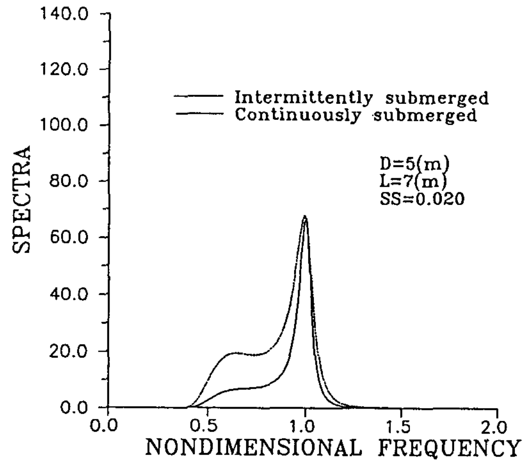

Fig. 13. Response spectrum (ss is ξ).

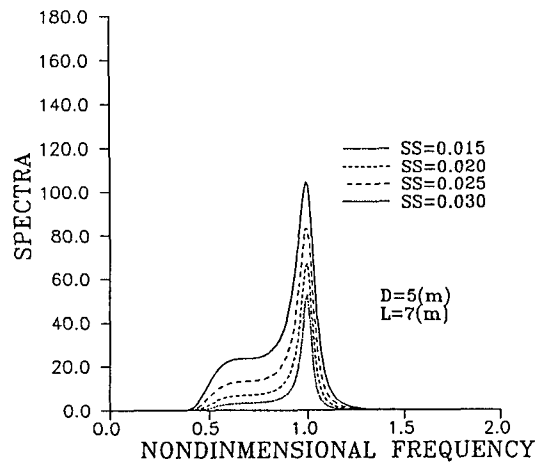

Fig. 11. Response spectrum (ss is ξ).

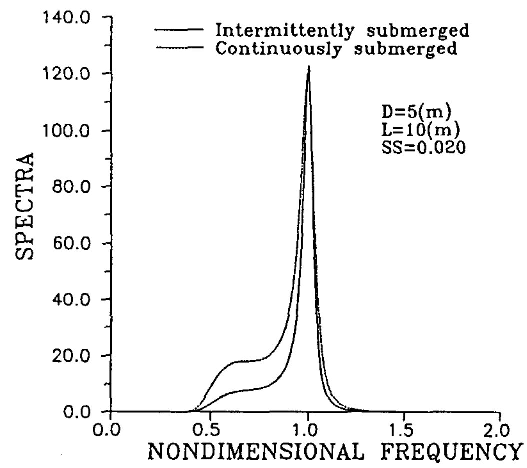

Fig. 14. Response spectrum (ss is ξ).

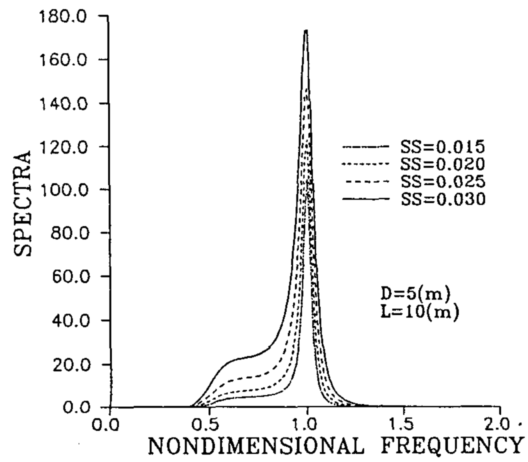

Fig. 12. Response spectrum (ss is ξ).

harmonic response component caused by the intermittent submergence.

5. CLOSURE

In this study, we have demonstrated the manner in which the response spectrum of offshore structure may be obtained. Knowledge of wave force spectrum which is prerequisite for the determination of the dynamic response of flexible offshore structure susceptible to dynamic action is also derived considering the effect of intermittent submergence of offshore structure. Although it has been shown that free surface fluctuation has profound effects on wave field kinematics, dynamics and wave forces especially in the vicinity of still water level, it has not been clear how free surface fluctuation affects structural response. This study shows that the response spectrum is reduced as expected and higher harmonic response component is appeared when the intermittent submergence of structural member is considered.

REFERENCES

Abramowitz, M. and Stegun, I.A., 1968. Handbook of mathematical functions, Dover, Mineola, NY.

Borgman, L.E., 1965. A statistical theory for hydrodynamic force on objects, Technical Report HEL-9-6, Hydraulics Engineering Laboratory, University of California, Berkeley, California.

Dao, B.V. and Penzien, J., 1982. Comparison of treatment of nonlinear drag force acting on fixed offshore platform, Applied Ocean Research, Vol. 4, No. 2, 77-72.

Erdely, A., Magnum, W., Oberhettinger, F. and Tricomi, F.G., 1953. Higher transcendental functions, McGraw-Hill Book Co. Inc., New York, NY, Vol. 2.

Huang, N.E., Long, S.R., Tung, C.C., Yuan, Y. and Bliven, K.F., 1981. A unified two-parameter wave spectral model for a general sea state, *J. Fluid Mech.*, **112**: 203-224.

Huang, N.E., Long, S.R., Tung, C.C., Yuan, Y. and Bliven, L.F., 1983. A non-Gaussian statistical model for surface elevation of nonlinear random wave fields, *J. Geophys. Res.*, **88**: 7597-7606.

Isaacson, M. and Baldwin, J., 1990a. Random wave forces near the free surface, *Journal of Waterways, Port, Coastal and Ocean Engineering Division, ASCE*, Vol. 116, No. 2, 232-251.

Isaacson, M. and Baidwin, J., 1990. Measured and predicted random wave forces near the free surface, *Journal of Waterways, Port, Coastal and Ocean Engineering Division, ASCE*, Vol. 116, No. 6, 742-763. Appl. Oc. Res.

Isaacson, M. and Subbiah, K., 1990. Random wave slamming on cylinders, *Journal of Waterways, Port, Coastal and Occan Engineering Division, ASCE*, Vol. 116, No. 6, 742-763.

Kagegaokar, H.B. and Halda, A., 1987. Non-gaussian response of offshore platforms: dynamics, *J. Struct. Div., ASCE*, Vol. 113, No. 9, 1882-1898.

Lipsett, A.W., 1986a. A perturbation solution for nonlinear structural response to oscillatory flow, *Applied Ocean Research*, Vol. 8, No. 4, 183-189.

Lipsett, A.W., 1986b. Nonlinear structural response in random waves, *Journal of Structural Division, ASCE*, Vol. 113, 1882-1898.

Pajouhi, K. and Tung, C.C., 1975a. Statistics of random wave field, *Journal of Waterways, Port, Coastal and Ocean Engineering Division, ASCE*, Vol. 101, No. 4, 435-449.

Pajouhi, K. and Tung, C.C., 1975b. Statistical properties of fluid motion and fluid force in a random wave field, The Center for Marine and Coastal Studies Report No. 75-1, North Carolina State University, Raleigh, NC.

Papoulis, A., 1965. Probability, random variables and stochastic processes, McGraw-Hill book Co., New York, NY.

Phillips, O.M., 1980. The dynamics of the upper ocean, 2nd edition. Cambridge University Press, New York, NY.

Sarpkaya, T. and Isaacson, M., 1981. Mechanics of wave forces on offshore structures, Van Nostrand Reinhold, New York, 31, 159.

Tung, C.C., 1975a. Statistical properties of kinematics and dynamics of a random gravity wave field, *Journal of Fluid mechanics*, Vol. 70, No. 2, 251-255.

Tung, C.C., 1975b. Statistical properties of wave forces, *Journal of Engineering Mechanics, ASCE*, Vol. 100, No. 1, 1-11.

Tung, C.C., 1976. Statistical properties of pressure in a random wave field, *Journal of Physical Oceanography*, Vol. 6, No. 4, 589-591.

Tung, C.C. and Pajouhi, K., 1976. Covariance functions and spectra of a random wave field, *Journal of Physical Oceanography*, Vol. 5, No. 1, 104-107.

Tung, C.C., 1979. On response of structures to random waves, *Applied Ocean Research*. Vol. 1, No. 4, 209-212.

Tung, C.C., Huang, N.E., Yuan, Y. and Long, S.R., 1989. Probability function of breaking limited surface elevation, *J. Geophys. Res.*, 94, No. C1, 967-972.

Tung, C.C. and Yang, C.H., 1992. Effects of free surface fluctuation on response of offshore structures, Ph.D. Thesis. North Carolina State University.

Tung, C.C. and Laurence, Z.H., 1992. Random wave force on horizontal members, Ph.D. Thesis. North Carolina State University.