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Vibration Analysis of Frame Structural Systems by the Receptance Method

S.Y. Han* and K.C. Kim**

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Abstract

There exist many frame structural systems having some attachments reducible to damped spring-mass systems, concentracted masses and spring supports. For free and forced vibration analyses of such a system an analytical method based on the receptance method is presented. A framed structure having attachments is considered as a combined system composed of various Timoshenko beam and bar elements and the attachments. So, the vibration characteristics of the system are calculated by synthesizing receptances and Support Displacement Transfer Ratio (SDTR) of beam and bar elements in spectral and/or closed forms, and receptances of the attachments. In forced vibration analysis, arbitray excitation forces at a point on the structure and displacement excitations at boundaries are considered. Numerical investigations are carried out for verification of the presented method, and the results show good accuracy and very high computational efficiency.

1. Introduction

There exist various kinds of framed structures composed of beam/bar elements and subsystems reducible to damped spring-mass systems. It is needed to develope an efficient method for vibration analysis of such a combined structural system. In this study, an analytical method based on the receptance method is developed to calculate efficiently free vibration characteristics and dynamic responses to arbitrary excitations.

As to applications of the receptance method, Bishop et al.[1, 2](1960,1965) showed a formulation for vibration analysis of an Euler beam having a concentrated mass Azimi et. al.[3, 4](1984, 1986) carried out natural vibration analysis of a combined system composed of rectangular plate elements. Kelkel[5](1987) obtained natural vibration characteristics of a rectangular stiffened plate by treating it as a combined system composed of a rectangular plate and Euler beam elements. Han et. al. [6, 7](1989) derived an efficient formulation for free and forced vibration analysis of a rectangular plate having damped

* Member, Maritime Research Institute, Samsung Heavy Industries
** Member, Seoul National University

spring-mass systems by considering it as a combined system. In their works Support Displacement Transfer Ratio(SDTR) which was conceptually similar to receptance was introduced for efficient calculations of dynamic responses to displacement excitation applied to boundaries of the combined system.

In this study, an analytical method based on the receptance method is developed for free and forced vibration analysis of a framed structure having damped spring-mass systems, concentrated masses and /or spring supports. In the formulation of the problem, a framed structure having the attachments is treated as a combined system composed of various Timoshenko beam and bar elelements. Free vibration characteristics and dynamic responses of the combined system to arbitrary excitations are calculated by synthesizing receptances and SDTR's of the elements which are derived in both spectral and closed forms. As to dynamics responses, arbitrary excitation forces applied at a point on the structure and displacement excitations applied to boundaries are considered.

Some numerical investigations are made to verify accuracy and computational efficiency of the presented method in comparisons with the finite elements method.

2. Receptance and SDTR of a Uniform Timoshenko Beam

2.1 Receptance

Receptance is one of dynamic influence coefficients, and defined as a ratio of a steady state response to a harmonic excitation force. In the coordinate system of a uniform Timoshenko beam element as shown in Fig.1, four kinds of receptances can be defined according to combinations of components of responses and forces.

In case of a translational harmonic excitation force $Fe^{i\omega t}$ applied at $x=h$, two kinds of receptances are defined as

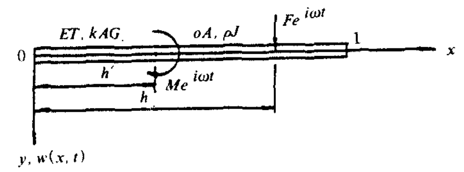


Fig. 1 Coordinate system of uniform beam subjected to the point excitation force and moment.

$$\alpha_{xh}(\omega)=\frac{W(x)}{F},\ \alpha_{xh}(\omega)=\frac{\Psi(x)}{F} \tag{1}$$

Similarly, in case of a rotational harmonic excitation moment $Me^{i\omega t}$ applied at $x=h'$, two kinds of receptances are defined as

$$\alpha_{xh'}(\omega)=\frac{W(x)}{M},\ \alpha_{xh'}(\omega)=\frac{\psi(x)}{M} \tag{2}$$

In (1) and (2), $W(x)$ and $\psi(x)$ are translational and rotational amplitudes of the steady state responses at a point x, respectively. The relation between $W(x)$ and $\psi(x)$ is,

$$\psi(x)=\frac{dW(x)}{dx}-\Gamma(x) \tag{3}$$

where $\Gamma(x)$ is a shear deformation.

When a translational harmonic excitation force $Fe^{i\omega t}$ is applied at $x=h$ and a rotational harmonic excitation mement $Me^{i\omega t}$ is applied at $x=h'$ as shown in Fig.1, the governing equations of forced vibration of a uniform Timoshenko beam are given as follows[8, 9] ;

$$\rho A\frac{\partial^2 w}{\partial t^2}=kAG(1+\beta\frac{\partial}{\partial t})(\frac{1}{l^2}\frac{\partial^2 w}{\partial \xi^2}-\frac{1}{l}\frac{\partial \psi}{\partial \xi}) + \frac{F}{l}\delta(\xi-c_1)e^{iet}$$

$$\rho I\frac{\partial^2 \psi}{\partial t^2}=EI(1+\alpha\frac{\partial}{\partial t})\frac{1}{l^2}\frac{\partial^2 \psi}{\partial \xi^2}-kAG(1+\beta\frac{\partial}{\partial t}) \cdot (\frac{1}{l}\frac{\partial w}{\partial \xi}-\psi)+\frac{M}{l}\delta(\xi-c_2)e^{iet} \tag{4}$$

where $\omega(\xi, t)$ and $\psi(\xi, t)$ are lateral and angular displacement, ρA mass per unit length, ρI rotary inertia per unit length, EI and kAG bending and shear rigidity per unit length, $\delta(x)$ Dirac delta function, αE and βG viscoelastic coefficient due to internal friction for bending and shear detormations, and ξ, c_1 and c_2 are nondimensional parameters defined as

$$\xi = \frac{x}{l},\ c_1 = \frac{h}{l},\ c_2 = \frac{h'}{l} \tag{5}$$

From (1) and (2), the receptance of a uniform Timoshenko beam can be derived by obtaining the steady state responses of (4) which are described as

$$w(\xi, t) = W(\xi)e^{iet},\ \psi(\xi, t) = \psi(\xi)e^{iet} \tag{6}$$

Assuming

$$w(\xi, t) = \sum_{r=1}^{n} W_r(\xi)q_r(t)$$
$$\psi(\xi, t) = \sum_{r=1}^{n} \psi_r(\xi)q_r(t)$$

with Timoshenko beam functions[10] $W_r(\xi)$ and $\psi_r(\xi)$, and time dependent generalized coordinates. $q_r(t)$, and neglecting cross mode effects of internal frictions, the steady state responses of (4) can be obtained in a spectral form. Hence, the receptances can be neglected, $\alpha E = 0 = \beta G$, the exact solutions of the steady state responses of (4) can be obtained with the aids of Laplace transformation technique, and receptances can be expressed in closed forms[6].

2.2 SDTR

As shown in Fig.2, from the steady state responses at a point x to a translational harmonic displacement excitations $\omega_o(t) = W_o e^{i\omega t}$ and a rotational harmonic angular displacement excitations $\theta(t) = \Theta_o e^{i\omega t}$ at supporting ends of the beam elements, four kinds of SDTR conceptually similar to the receptance are defined as follows ;

$$\tau_x(\omega) = \frac{w_T(x,t)}{w_o(t)},\ \tau_x'(\omega) = \frac{\psi_T(x,t)}{w_o(t)}$$
$$\tau'_x(\omega) = \frac{w_T(x,t)}{\theta_o(t)},\ \tau'{}_x{}'(\omega) = \frac{\psi_T(x,t)}{\theta_o(t)} \tag{8}$$

where $\omega_T(x, t)$ and $\psi_T(x, t)$ are the steady state responses at a point x in the support motion. They are the sum of a rigid body motion and relative elastic motion and can be expressed as follows under the assumption of small θ_o ;

$$w_T(x, t) = w_o(t) + x\theta_o(t) + w_e(x, t)$$
$$\psi_T(x, t) + \psi_e(x, t) \tag{9}$$

where $w_e(x, t)$ and $\psi_e(x, t)$ are translational and rotational components of relative elastic motions.

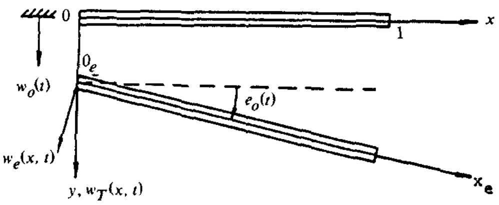


Fig. 2 Coordinate system of uniform beam subjected to the translational and rotational displacement excitations along boundary

The governing equations of such a support motion can be described as

$$\rho A\frac{\partial^2 w_e}{\partial t^2} - kAG(1+\beta\frac{\partial}{\partial t})(\frac{1}{l^2}\frac{\partial^2 w_e}{\partial \xi^2} - \frac{1}{l}\frac{\partial \psi_e}{\partial \xi})$$
$$= -\rho A(l\xi\frac{d^2\theta_o}{dt^2} + \frac{d^2 w_o}{dt^2})$$
$$\rho I\frac{\partial^2 \psi_e}{\partial t^2} - EI(1+\alpha\frac{\partial}{\partial t})(\frac{1}{l^2}\frac{\partial^2 \psi_e}{\partial \xi^2} - kAG(1+\beta\frac{\partial}{\partial t})$$
$$\cdot(\frac{1}{l}\frac{\partial w_e}{\partial \xi} - \psi_e) = -\rho I\frac{d^2\theta_o}{dt^2}) \tag{10}$$

The SDTR's defined in (8) can be derived in spectral forms by using steady state responses of (10) which can be obtained by the classical modal analysis in a similar way to derive receptance in section 2.1. In this case, the relative elastic motions are expressed as

$$w_e(\xi, t) = \sum_{r=1}^{n} W_r(\xi) q_r(t)$$
$$\psi_e(\xi, t) = \sum_{r=1}^{n} \psi_r(\xi) q_r(t) \qquad (11)$$

In case the internal damping can be neglected, SDTR can be derived in closed forms[6] using the exact solutions of (10) and Laplace transformation technique.

3. Receptance and SDTR of a Uniform Bar

3.1 Receptance

As shown in Fig.3, the receptance of a uniform bar is defined as the ratio of the steady state response $\omega(x, t) = W(x)e^{i\omega t}$ to a harmonic excitation force $Fe^{i\omega t}$ applied at $x = h$ i.e.,

$$\alpha_{xh}(\omega) = \frac{W(x)}{F} \qquad (12)$$

Fig. 3 Coordinate system of uniform bar subjected to the point excitation force and the translational displacement excitation along boundary

The equation of motion for longitudinal vibration of a uniform bar may be expressed as follows including viscoelastic coefficient, αE ;

$$\rho A \frac{\partial^2 w}{\partial t^2} - EA(1 + \alpha \frac{\partial}{\partial t}) \frac{\partial^2 w}{\partial x^2} = F\delta(x - h)e^{iet} \qquad (13)$$

To derive the receptance in spectral forms[6], the steady state response should be obtained by the classical modal analysis neglecting coupled mode effects of internal damping. And it turns out to be in the form of

$$w(x, t) = \sum_{r=1}^{n} \phi_r(x)\, q_r(t) \qquad (14)$$

where $\phi_r(x)$ and $q_r(t)$ are eigenfunctions and time dependent generalized coordinates.

In case the internal damping is neglegible, the receptance can be derived in closed forms[6], using the exact solution of (13).

3.2 SDTR

The SDTR of a uniform bar is defined as a ratio of the absolute steady state response $\omega_T(x, t) = W_T(x)e^{i\omega t}$ to a harmonic displacement excitation $\omega_o(t) = W_o e^{i\omega t}$ applied at supporting part of the bar, and it can be expressed as

$$\tau_x(\omega) = \frac{W_T(x)}{W_o} \qquad (15)$$

The absolute motion is the sum of the rigid body motion ω_0(t) and the relative elastic motion $\omega_e(x, t)$ i.e.

$$w_T(x, t) = w_o(t) + w_e(x, t) \qquad (16)$$

The governing equation for the support motion of a uniform bar may be expressed as follows including viscoelastic coefficient, αE ;

$$\rho A \frac{\partial^2 w_e}{\partial t^2} - EA(1 + \alpha \frac{\partial}{\partial t}) \frac{\partial^2 w_e}{\partial x^2} = -\rho A \frac{\partial^2 w_e}{\partial t^2}) \qquad (17)$$

The steady state response of (17) is obtained by the classical modal analysis neglecting coupled mode effects of internal damping and turns out to be in the form of

$$w_e(x, t) = \sum_{r=1}^{n} \phi_r(x) q_r(t) \qquad (18)$$

where $\phi_r(x)$ and $q_r(x)$ are eigenfunctions and time dependent generalized coordinates. Then, the SDTR can be derived in spectral forms[6].

In case the internal damping is neglegible, the SDTR in cosed forms[6] can be derived using the exact solution of (17).

4. Vibration Characteristics of Framed Structures with Various Attachments

A two-dimensional framed structure having a spring-mass system is shown in Fig.4 as an example. An efficient formulation for obtaining vibration characteristics of the 2-dimensional combined system by synthesizing receptances and /or SDTR's of various beam/bar elements and receptances of the attachments will be described in this section.

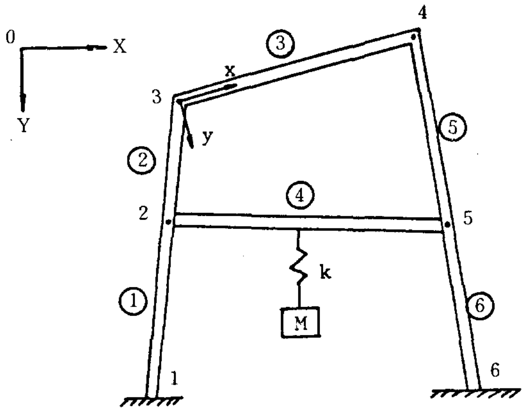


Fig. 4 2-D frame structure having spring-mass system

For convenience of the formulation, identification numbers are designated to each beam/bar element and connection point, and a local coordinate system(x, y) and a gloabal corrdinate system (X, Y) will be introduced as shown in Fig.4.

4.1 Receptance and SDTR matrices of a Beam/ Bar Element in a Local Coordinnate System

In a local coordinate system as shown in Fig.5, the steady state responses $U_j e^{i\omega t}, j=1, 2, 3$, at a point x to the harmonic excitation forces $F_j e^{i\omega t}$, $j=1, 2, 3$, applied at $x=h$ can be described as follows ;

$$\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix}_x = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & \alpha_{23} \\ 0 & \alpha_{32} & \alpha_{33} \end{bmatrix}_{xh} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}_h \qquad (19)$$

or $\{U\}_x = [\alpha]_{xh}\{F\}_h$,

where the coupled effects of axial and bending deformation are neglected, and

$$\alpha_{ij} = \frac{U_i(x)}{F_j},\ i, j=1, 2, 3$$

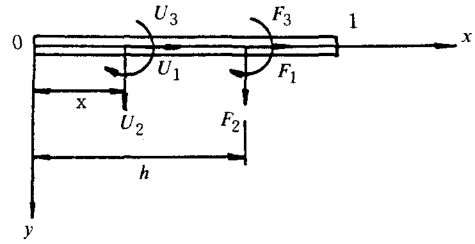


Fig. 5 Beam element in local coordinate

In (19), $[\alpha]_{xh}$ is a receptance matrix of a beam/bar element in a local coordinate system.

In case of harmonic displacement excitations $D_j e^{i\omega t}, j=1, 2, 3$, applied to boundaries of the framed structure, amplitudes of steady state responses at an arbitrary point are given as

$$\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix}_x = \begin{bmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & \tau_{23} \\ 0 & \tau_{32} & \tau_{33} \end{bmatrix}_{xh} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix}_h$$

or $\{U\}_x = [\tau]_x\{D\}$,

where $\tau_{ij} = \frac{U_i(x)}{D_j},\ i, j=1,2,3$

In (20), $[\tau]_x$ is a SDTR matrix of a beam/bar element in a local coordinate system.

In (19) and (20), α_{11} and τ_{11} are a receptance and a SDTR, respectively, of a uniform bar derived in chapter 3 and remained ones are of a uniform Timoshenko beam derived in chapter 2.

4.2 Receptance and SDTR matrices of an Beam/ Bar Element in the Global Coordinate System

The coordinate transformation matrix between a local and the global coordinate systems shown in Fig.6 becomes

$$[T]=\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{21}$$

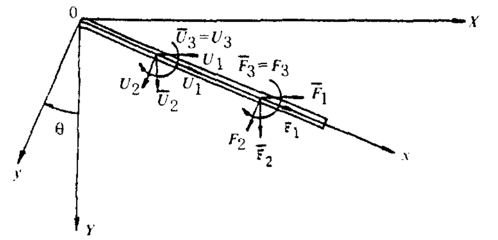


Fig. 6 Beam element in global coordinate

Hence the relations between U_i, F_i and D_i in a local coordinate system and $\bar{U}_i$, $\bar{F}_i$ and $\bar{D}_i$ in the global coordinate system are as follows ;

$$\begin{aligned} \{U\}&=[T]\{\bar{U}\} \\ \{F\}&=[T]\{\bar{F}\} \\ \{D\}&=[T]\{\bar{D}\} \end{aligned} \tag{22}$$

Therefore, the receptance and the SDTR matrices in the global coordinate system can be obtained as

$$\begin{aligned} [\bar{\alpha}]_{xh}&=[T]^T[\alpha]_{xh}[T] \\ [\bar{\tau}]_x&=[T]^T[\tau]_x[T] \end{aligned} \tag{23}$$

4.3 Free Vibration Characteristics of a Framed Structure

It is assumed that n_k beam elements are connected at a common node k each other and the opposite node of each element is an isolated node except the one of an arbitrary element d supported rigidly as shwon in Fig.7.

When harmonic excitation forces $\bar{F}_j e^{i\omega t}$, $j=1$, 2, 3, are applied at $x=h_f$ in an arbitrary i-th element, and harmonic displacement excitations $\bar{D}_j e_{i\omega t}$, $j=1$, 2, 3, are applied at a supported node of the d-th element, steady state responses at any point can be obtained by synthesizing receptances and SDTR's of the elements. Using the steady state responses, the receptance and SDTR matrices of the framed structure can be obtained.

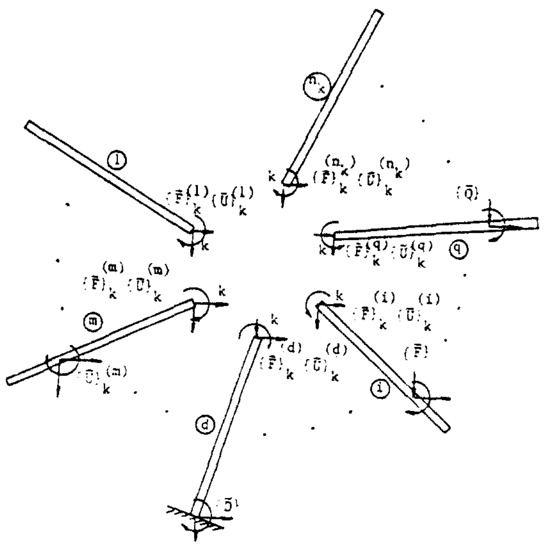


Fig. 7 Free body diagram of nK beam elements connected at the common node k

In the free body diagram shown in Fig.7, the displacement $\{\bar{U}\}_k^{(i)}$, and the reaction force $\{\bar{F}\}_k^{(i)}$ at the node k of the element i can be introduced. Now, the relation between displacements and forces at any point in an arbitrary element m can be expressed using (23) as follows ;

$$\begin{aligned} \begin{Bmatrix} \{\bar{U}\}_k^{(m)} \\ \{\bar{U}\}_x^{(m)} \end{Bmatrix} &= \begin{bmatrix} [\bar{\alpha}]_{kk}^{(m)} \\ [\bar{\alpha}]_{xk}^{(m)} \end{bmatrix} \{\bar{F}\}_x^{(m)} \quad (m\neq d,\ i) \\ \begin{Bmatrix} \{\bar{U}\}_k^{(d)} \\ \{\bar{U}\}_x^{(d)} \end{Bmatrix} &= \begin{bmatrix} [\bar{\alpha}]_{kk}^{(d)} & \{\bar{\tau}\}_k^{(d)} \\ [\bar{\alpha}]_{xk}^{(d)} & \{\bar{\tau}\}_x^{(d)} \end{bmatrix} \begin{Bmatrix} \{\bar{F}\}_k^{(d)} \\ \bar{D} \end{Bmatrix} \\ \begin{Bmatrix} \{\bar{U}\}_k^{(i)} \\ \{\bar{U}\}_x^{(i)} \end{Bmatrix} &= \begin{bmatrix} [\bar{\alpha}]_{kk}^{(i)} & \{\bar{\alpha}\}_{kf_f}^{(i)} \\ [\bar{\alpha}]_{xk}^{(i)} & \{\bar{\alpha}\}_{x,hf}^{(i)} \end{bmatrix} \begin{Bmatrix} \{\bar{F}\}_k^{(i)} \\ \bar{F} \end{Bmatrix} \end{aligned} \tag{24}$$

From (24), the displacements at the common node k can be described as the sum of displacements to reaction forces and external excitation forces as follows ;

$$\{\bar{U}\} = [\beta]_{kk}\,\{\bar{F}\}_k + [\beta]_{kf}\,\{\bar{R}\} \tag{25}$$

where

$$\{\bar{U}\} = [\{\bar{U}\}_k^{(1)T}\ \{\bar{U}\}_k^{(2)T}\ \cdots\ \{\bar{U}\}_k^{(n_k)T}\ ]^T$$

$$\{\bar{F}\}_k = [\{\bar{F}\}_k^{(1)T}\ \{\bar{F}\}_k^{(2)T}\ \cdots\ \{\bar{F}\}_k^{(n_k)T}\ ]^T$$

$$\{\bar{R}\}_k = [\{\bar{D}\}^T\ \{\bar{F}\}^T\ ]^T$$

The reaction forces and the displacements in (25) should satisfy the equilibrium and compatibility conditions as follows ;

$$\sum_{i=1}^{n_k} \{\bar{F}\}_k^{(i)} = \{0\} \tag{26}$$

$$\{\bar{U}\}_k^{(1)} = \{\bar{U}\}_k^{(2)} = \cdots\ \{\bar{U}\}_k^{(n_k)} \tag{27}$$

To describe the equilibrium condition (26) in a matrix form, we introduce a constraint matrix shown below ;

$$[C]_k = \begin{bmatrix} [I]_3 & & & & \\ & [I]_3 & & \bigcirc & \\ & & \ddots & & \\ & \bigcirc & & \ddots & \\ & & & & [I]_3 \\ -[I]_3 & -[I]_3 & \cdot & \cdots & -[I]_3 \end{bmatrix} \tag{28}$$

where

$$[I]_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, the eqilibrium condition (26) can be expressed as

$$\{\bar{F}\} = [C]_k\,\{\bar{F}\}_k^R \tag{29}$$

where

$$\{\bar{F}\}_k^R = [\{\bar{F}\}_k^{(1)T}\ \{\bar{F}\}_k^{(2)T}\ \cdots\ \{\bar{F}\}_k^{(n_k-1)T}\ ]^T$$

The compatibility condition (27) can be also expressed as

$$[C\}_k^T\,\{\bar{U}\} = \{0\} \tag{30}$$

Substituting (29) and (30) into (25), gives the following equation ;

$$\{0\} = [C\}_k^T\,[\beta]_{kk}\,[C\}_k\,\{\bar{F}\}_k^R + [C\}_k^T\,[\beta]_{kf}\,[\bar{R}\} \tag{31}$$

Hence, by substituting (29) into (31), the reaction forces at the node k of each element in the free body diagram can be obtained ;

$$[\bar{F}\} = -[C]_k([C]_k^T\,[\beta]_{kk}\,[C]_k)^{-1}\,[C]_k^T\,[\beta]_{kf}\,\{\bar{R}\} \tag{32}$$

$[\beta]_{kf}\,\{R\}$ can be devided into two parts according to the excitation types ;

$$[\beta]_{kf}\,\{\bar{R}\} = [\beta]_{kd}\,\{\bar{D}\} + [\beta]_{ki}\,\{\bar{F}\} \tag{33}$$

Then (32) becomes

$$\{\bar{F}\} = -[\bar{C}]_k\,([C]_k^T\,[\beta]_{kk}[C]_k)^{-1}\,[C]_k^T\,([\beta]_{kd}\{\bar{D}\} + [\beta]_{ki}\,\{\bar{F}\}) \tag{34}$$

Therefore, from (24) and (34), the receptance and the SDTR matrices of the combined system shown in Fig.7 can be derived. That is, the amplitudes of steady state responses at a point x in the element m to the harmonic excitation forces applied to $x=h_f$ in the element i are obtained by substitution of (34) into (24), and the receptance matrix $[\bar{\gamma}]_{xf}^{(mi)}$ can be calculated by

$$\{\bar{\gamma}\}_{xf}^{(mi)} = \begin{cases} [\bar{\alpha}]_{xh}^{(m)}\,[A]_f^{(m)} & for\quad m \neq i \\ [\bar{\alpha}]_{xh}^{(m)}\,[A]_f^{(m)} + [\bar{\alpha}]_{xh}^{(m)} & for\quad m = i \end{cases} \tag{35}$$

where $[A]_f^{(m)}$ is a matrix composed of the elements corresponding to the element m in $(-[C]_k\,([C]_k^T\,[\beta]_{kk}\,[C]_k)^{-1}\,[C]_k^T\,[\beta]_{ki})$. Also, the amplitudes of steady state responses at any point x of the element m to the hamonic displacement excitations applied to the supported node of the element d are obtained by substitution of (34) into (24), and a SDTR matrix $[\bar{\eta}]_x^{(md)}$ can be calculated by

$$\{\bar{\eta}\}_{xj}^{(md)} = \begin{cases} [\bar{\alpha}]_{xh}^{(m)} \, [A]_d^{(m)} & for \ m \neq d \\ [\bar{\alpha}]_{xh}^{(m)} \, [A]_d^{(m)} + [\bar{\tau}]_x^{(m)} & for \ m = i \end{cases} \quad (36)$$

where $[A]_d^{(m)}$ is a matrix composed of the elements corresponding to the element m in $-[C]_k$ $([C]_k^T \, [\beta]_{kk} \, [C]_k)^{-1} \, [C]_k^T \, [\beta]_{kd}.)$.

From now on, let us consider a generalized case which has the multiple common nodes. It is supposed that there are totally N nodes in the structure which consist of N_c common nodes and N_o isolated nodes, that is, $N = N_c + N_o$. When we imagine the free body diagram of the framed structure as shown in Fig.7, the reaction forces and displacements at the common nodes of each element can be defined. Constructing the relation between the displacements and forces in a similar way of (24), using receptances and SDTR's of the elements and rearranging the ones for the common nodes (25), the following equations can be obtained ;

$$\{\bar{U}\}_c = [\beta]_{cc} \, \{\bar{F}\}_c + [\beta]_{cf} \{\bar{R}\}_G \quad (37)$$

where

$$\{\bar{U}\}_c = \begin{Bmatrix} \{\bar{U}\}_{L_1} \\ \cdot \\ \cdot \\ \cdot \\ \{\bar{U}\}_{L_{N_c}} \end{Bmatrix}, \ \{\bar{U}\}_{L_k} = \begin{Bmatrix} \{\bar{U}\}_{L_1}^{K_{k1}} \\ \cdot \\ \cdot \\ \cdot \\ \{\bar{U}\}_{L_{N_c}}^{K_{kE_k}} \end{Bmatrix},$$

$$\{\bar{F}\}_c = \begin{Bmatrix} \{\bar{F}\}_{L_1} \\ \cdot \\ \cdot \\ \cdot \\ \{\bar{F}\}_{L_{N_c}} \end{Bmatrix}, \ \{\bar{F}\}_{L_k} = \begin{Bmatrix} \{\bar{F}\}_{L_1}^{K_{k1}} \\ \cdot \\ \cdot \\ \cdot \\ \{\bar{F}\}_{L_{N_c}}^{K_{kE_k}} \end{Bmatrix},$$

and L_i, $i = 1, 2, \cdots, N_c$, are nodal numbers of the common nodes, E_i the number of beam elements connected to an arbitrary common node L_i, K_{ij}, $j = 1, 2, \cdots, E_i$, the element numbers of the above elements.

The reaction forces and displacements defined at each common node should satisfy equilibrium (26) and compatibility conditions (27). If the conditions for all common nodes are considered, the constraint matrix $[C]_G$ for the global system can be written as

$$[C]_G = \begin{bmatrix} [C]_1 & & 0 \\ & \ddots & \\ 0 & & [C]_{N_c} \end{bmatrix} \quad (38)$$

where $[C]_k$ is a constraint matrix for a common node k as given in (28).

Then the equilibriun and compatiblity conditions can be decribed as

$$\begin{aligned} \{\bar{F}\}_c &= [C]_G \, [\bar{F}]_c^R \\ \{\bar{0}\}_c &= [C]_c^T \, [\bar{U}]_c^R \end{aligned} \quad (39)$$

Substituting (39) into (37), the equation

$$\{0\} = [C]_G^T \, [\beta]_{cc} \, [C]_G \, \{\bar{F}\}_c^R + [C]_G^T \, [\beta]_{cf} \{\bar{R}\}_G \quad (40)$$

can be obtained, and the unknown reaction forces can be calculated by

$$\{\bar{F}\}_c = -[C]_G (C)_G^T [\beta]_{cc} [C]_G)^{-1} [C]_G^T [\beta]_{cf} \{\bar{R}\}_G \quad (41)$$

Therefore, the receptance and SDTR of the framed structure can be derived in a similar way of (35) and (36). Also, a latent root problem for the free vibration is derived from (40).

That is, omitting the excitation term,

$$\{0\} = [C]_G^T \, [\beta]_{cc} \, [C]_G \, \{\bar{F}\}_c^R \quad (42)$$

is obtainable and from which a characteristic equation can be obtained as

$$det\left([C]_G^T\,[\beta]_{cc}\,[C]_G\right)=0 \qquad (43)$$

When the r-th latent vector $\{\bar{F}\}_c^{R(r)}$ of (42) without damping is substituted into (39), the r-th mode shape of the framed structure can be calculated using the relation shown in (24) for each element.

4.4 Framed Structure having Attachments

In the formulation for calculating vibration characteristics of a framed structure having attachments, only damped spring-mass systems are considered as attachments because a spring-mass system, a concentrated mass or a support spring can be reduced by numerical control of parameters.

If an attached subsystem is composed of three components of damped spring-mass systems as shown in Fig.9, a direct receptance matrix of a subsystem at the attached point may be described as

$$[\beta]=\begin{bmatrix}\beta_{11} & 0 & 0\\ 0 & \beta_{22} & 0\\ 0 & 0 & \beta_{33}\end{bmatrix} \qquad (44)$$

where a receptance β_{ii} of a damped spring-mass system as shown in Fig.8 is given as[5]

$$\beta_{ii}=\frac{k_i+j\omega c_i-m\omega^2}{m\,i\omega^2(k_i+j\omega c_i)},\; j=\sqrt{-1}, \quad i=1,2,3, \qquad (45)$$

Also, a direct receptance matrix of the attached subsystem at mass points can be given as

$$[\beta_o]=\begin{bmatrix}\beta_{o1} & 0 & 0\\ 0 & \beta_{o2} & 0\\ 0 & 0 & \beta_{o3}\end{bmatrix} \qquad (46)$$

where

$$\beta_{oi}=-\frac{1}{m\,i\omega^2} \qquad i=1,2,3$$

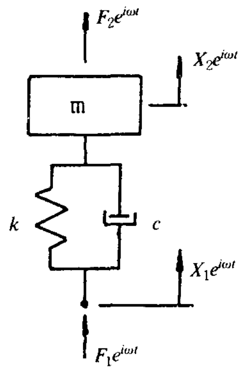
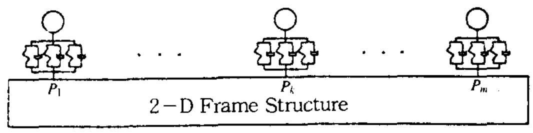


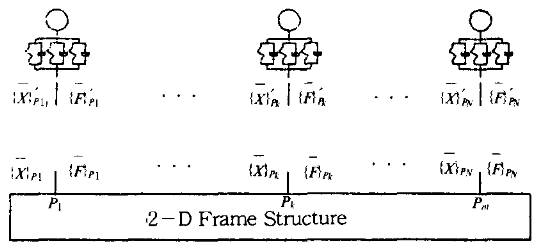
Fig. 8 Coordinate system of damped spring-mass system

It is supposed that N subsystems be attached at points P_k, $k=1, 2, \cdots, N$, on the framed structure as shown in Fig.9(a), and excitation forces $\{\bar{F}_2\}e^{i\omega t}$ and displacement excitations $\{\bar{R}\}e^{i\omega t}$ be applied at an arbitrary point Q of the structure and supporting boundaries, respectively. When the total system is depicted as a free body diagram, the reaction forces and displacements can be defined as shown in Fig.9(b). In the description of displacements and forces in Fig.9(b), a time factor $e^{i\omega t}$ is omitted as a matter of convenience.



(a)

(a) Block diagram of the composite system



(b)

(b) Free body diagram of the composite system

Fig. 9 Block diagram of 2-D frame structure having damped spring-mass systems

The amplitudes of steady state responses at any point can be described as follows using receptances and SDTR's of the framed structure and attached subsystems ;

at an arbitrary point P of the framed structure

$$\{\bar{X}\}_P = [\gamma]_{PQ}\{\bar{F}\}_e + [\eta]_P\{\bar{R}\}_e + \sum_{j=1}^{N}[\gamma]_{PP_j}\{\bar{F}\}_{P_j} \tag{48}$$

at an attached point $P_k (k=1, 2, \cdots, N)$

$$\begin{aligned} \{\bar{X}\}_{P_k} &= [\gamma]_{P_kQ}\{\bar{F}\}_e + [\eta]_{P_k}\{\bar{R}\}_e + \sum_{j=1}^{N}[\gamma]_{P_kQ}\{\bar{F}\}_{P_j} \\ \{\bar{X}\}_{P_k} &= [\beta]_{kk}\{\bar{F}\}_{P_k}' \quad , \end{aligned} \tag{49}$$

at a mass point of the k-th subsystem

$$\{\bar{X}_o\}_{P_k} = [\beta_o]_k \{\bar{F}\}_{P_k}' \ (k=1, 2, \cdots, N) \tag{50}$$

The reaction forces and displacements should satisfy the following equilibrium and compatibility conditions at attached points ;

$$\{\bar{F}\}_{P_k} = \{\bar{F}\}_{P_k}' = \{0\} \qquad (k=1, 2, \cdots, N) \tag{51}$$

$$\{\bar{X}\}_{P_k} = \{\bar{X}\}_{P_k}' \tag{52}$$

From (49), (51) and (52), the simultaneous equations from which the unknown reaction forces $\{\bar{F}\}_{P_k}$, $k=1, 2, \cdots, N$, can be obtained are given in a matrix form as follows ;

$$[A]\{\bar{F}\}_c = -[B]\{\bar{F}\}_e - [C]\{\bar{R}\}_e \tag{53}$$

where

$$\{\bar{F}\}_c = [\{\bar{F}\}_{P_1}^T \cdot \cdot \cdot \{\bar{F}\}_{P_k}^T \cdot \cdot \cdot \{\bar{F}\}_{P_N}^T]^T$$

Then, the reaction forces can be obtained as

$$\{\bar{F}\}_c = -[A](\ [B]\{\bar{F}\}_e + [C]\{\bar{R}\}_e) \tag{54}$$

Hence, the amplitudes of steady state responses can be calculated by substituting (54) into (48), (49) and (50) as follows ;

$$\begin{aligned} \{\bar{X}\}_P &= ([\gamma]_{PQ} - [D][A]^{-1}[B]\{\bar{F}\}_e \\ &\quad + ([\eta]_P - [D][A]^{-1}[C]\{\bar{R}\}_e \\ \{\bar{X}_o\} &= [E][A]^{-1}[B]\{\bar{F}\}_e + [E][A]^{-1}[C]\{\bar{R}\}_e \end{aligned} \tag{55}$$

where

$$[D] = [[\bar{\gamma}]_{PP_1} \cdot \cdot \cdot [\bar{\gamma}]_{PP_k} \cdot \cdot \cdot [\bar{\gamma}]_{PP_N}]$$

$$[E] = \begin{bmatrix} [\beta_o]_1 & & 0 \\ & \ddots & \\ 0 & & [\beta_o]_N \end{bmatrix},$$

$$\{\bar{X}_o\} = [\{\bar{X}_o\}_1^T \cdot \cdot \cdot \{\bar{X}\}_k^T \cdot \cdot \cdot \{\bar{X}\}_N]^T$$

Therefore, the receptance and SDTR of the total system can be derived from (55) according to their definitions. And a latent root problem for the free vibration analysis of the total system can be obtained from (53). That is, omitting the excitation term,

$$[A]\{\bar{F}\}_c = \{0\} \tag{56}$$

is obtainable. And from the characteristic equation $\det([A])=0$ of (56), the r-th latent root is obtainable and can be expressed as

$$\lambda_r = -\sigma_r + j\omega_d^{(r)}, \quad j=\sqrt{-1}, \quad r=1,2, \cdot \cdot \cdot \tag{57}$$

where σ_r and $\omega_d^{(r)}$ are positive real values. Using (57) under the assumption of neglecting cross mode effects of damping, the r-th modal damping ratio ζ_r of the framed structure can be estimated by

$$\zeta_r \approx \frac{\sigma_r}{\sqrt{\sigma_r^2 + \omega_d^{(r)2}}} \tag{58}$$

And, the r-th mode shapes of the total system can be obtained by substituting the r-th latent vector $\{\bar{F}\}_c^{(r)}$ of (56) (excluding damping) into

(48), (49) and (50) (excluding external excitations). That is,

$$\{\bar{X}\}_P^{(r)} = [D]\{\bar{F}\}_c^{(r)}$$
$$\{\bar{X}_o\}_P^{(r)} = -[E]\{\bar{F}\}_c^{(r)} \tag{59}$$

where $[D]$, $[E]$ and $\{\bar{X}_o\}_c^{(r)}$ are same as those in(55).

4.5 Dynamic Responses of Frame Structure having Attachments

Dynamic responses of the combined system can be efficiently calculated by using the global receptance or SDTR matrices obtained by synthesizing the receptances and /or SDTR's of beam /bar elements and attachments.

When excitation forces $\{\bar{F}\}_Q e^{i\omega t}$ are applied at a point Q of the framed structure, the steady state responses at an arbitrary point are directly calculated by using the global receptance matrix $[\bar{\gamma}]_{PQ}$;

$$\{\bar{X}\}_P = [\bar{\gamma}]_{PQ}\{\bar{F}\}_Q \tag{60}$$

In case displacement excitations $\{\bar{R}\}e^{i\omega t}$ along supporting boundaries of the system, the steady state responses can be calculated by using SDTR matrix $[\bar{\eta}]_P$;

$$\{\bar{X}\}_P = -[\bar{\eta}]_P\{\bar{R}\} \tag{61}$$

The transient responses at a point P to arbitrary excitations can be obtained from the global receptance or SDTR matrix by application of Fourier transformation method. Then, the transient response to arbitrary excitation forces $\{\bar{f}(t)\}_Q$ applied at a point Q is

$$\{x(t)\}_P = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\bar{\gamma}]_{PQ}\{\bar{F}(\omega)\}_Q e^{i\omega t}\, d\omega \tag{62}$$

and that to arbitrary displacement excitation $\{\bar{r}(t)\}$ applied to suppoting boundaries is

$$\{\bar{x}(t)\}_P = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\bar{\eta}]_P\{\bar{R}(\omega)\}e^{i\omega t}\, d\omega \tag{63}$$

where $\{\bar{F}(\omega)\}_Q$ and $\{\bar{R}(\omega)\}$ are Fourier transformations of $\{\bar{f}(t)\}_Q$ and $\{\bar{r}(t)\}$, respectively.

5. Numerical Examples and Discussions

For the verification of the present method, numerical investigations are carried out for a framed structure having a damped spring-mass system, as shown in Fig.10. The framed structure is made of mild steel whose material properties are $E = 2.1\times10^{11}$ N/m^2, $\rho = 7.85\times10^3\ kg/m^3$ and Poisson's ratio 0.3. The sectional shapes of the element 1 and 2 are a rectangular section of width × depth of 2 cm×3 cm and that of the element 3 is 2 cm×6 cm. The structural damping is neglected. The magnitude of mass of a damped spring-mass system is 1/10 of that of the framed structure, and its spring constant is determined so that the natural frequency of the spring-mass system may coincide to the fundamental natural frequency of the framed structure.

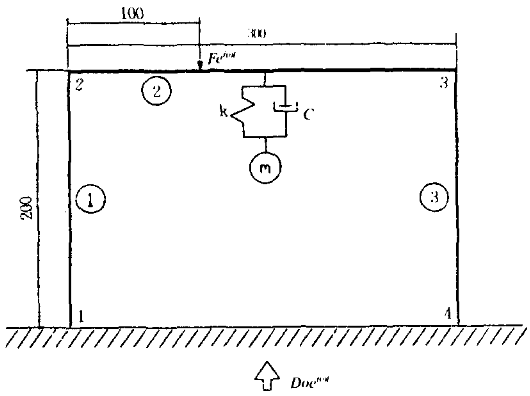


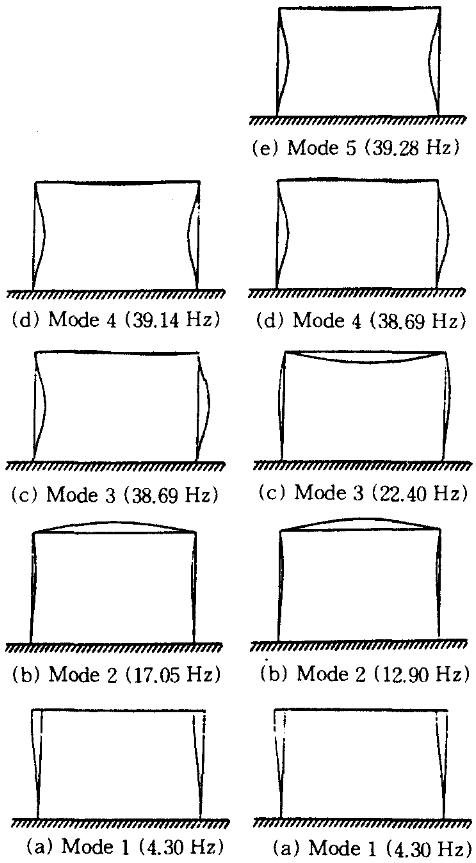
Fig. 10 2-D frame structure having a damped spring-mass system subjected to point excitation or base excitation

Natural frequencies and mode shapes of the framed structure having a spring-mass system are calculated by the formulations described in sections 4.3 and 4.4. The calculated results are compared with those by the finite element method in Table 1 and Fig.11. The both result are in good agreements.

Table 1 Natural frequencies of the frame structure having a spring-mass system : refer to Fig. 10

Model / Method / Order	Frame structure		Frame structure having a spring-mass system	
	R.M*	F.E.M.	R.M.	F.E.M.
1	4.30	4.30	4.30	4.30
2	17.05	17.05	12.90	12.90
3	38.69	38.61	22.40	22.40
4	39.14	39.10	38.69	38.61
5	65.42	65.43	39.28	39.24

* R.M. : the receptance method



(A) Base system (B) With a damped spring-mass system

Fig. 11 Mode shapes of 2-D frame structure having a spring-mass system : receptance method coincides with FEM

Also, calculations of steady state responses to harmonic excitations are carried out for both cases of a framed structure with and without a damped spring-mass system. The damping ratio of the attached subsystem is assumed to be 0.1 and two types of excitations are considered as shown in Fig.10 ; a harmonic excitation force applied to a point on the element 2 and harmonic displacement excitation applied to supporting boundaries. In both types of excitations, the magnitudes of excitations are taken as unity, $F_o = D_o = 1$. The calculated results of steady state responses by the receptance method are plotted in Fig.12 and Fig.13. For the purpose of comparison, calculations by the finite element method are also carried out for the case of the excitation force and its results are also plotted in Fig.12. From the figure, it can be seen that they are in good agreements.

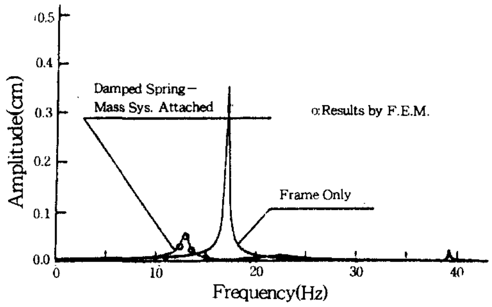


Fig. 12 Steady-state response at the midpoint of beam 2(Excitation : F=1Kgf, Do=0. in Fig. 10)

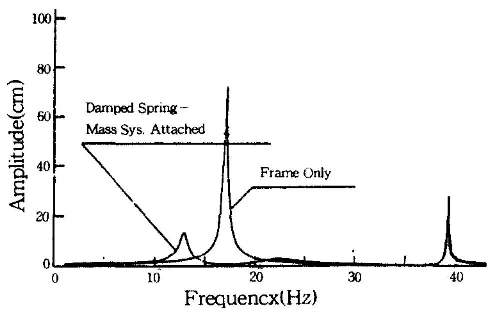


Fig. 13 Steady-state response at the midpoint of beam 2(Excitation : F=0., Do=1cm in Fig. 10)

6. Conclusions

For an efficient vibration analysis of a framed structure having attachments such as damped spring-mass systems, support springs and/or concentrated masses, analytical approaches based on the receptance method are presented. And, to verify accuracy and computational efficiency of the presented method, some numerical investigations are carried out. Major conclusions are as follows :

(1) Receptances of a uniform Timoshenko beam and a bar are derived in spectral and closed forms. Also, the SDTR (support displacement transfer ratio) of the Timoshenko beam/bar is defined with similar concepts to the receptance and derived in spectral and closed forms.

(2) The vibration characteristics of the framed structure having attachments can be calculated by synthesizing receptances of the beam/bar elements and the attachments in good accuracy with higher computational efficiency. In the process of the synthesis, modal damping properties of the global system can be estimated appropriately.

(3) The dynamic responses of the framed structure having attachments to both point excitation forces and displacement exitations along supporting boundaries can be easily calculated in good accuracy by using the receptance or SDTR of the global system obtained by synthesizing receptances and/or SDTR's of beam/bar elements and receptances of the attachments.

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