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A Bayes Linear Estimator for Multi-proportions Randomized Response Model

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Abstract

A Bayesian approach is suggested to the multi-proportions randomized response model. O'Hagan's (1987) Bayes linear estimator is extended to the inference of unrelated question-type randomized response model. Also some numerical comparisons are provided to show the performance of the Bayes linear estimator under the Dirichlet prior.

1. Introduction

It is often difficult to get reliable information in sample surveys of human population. Since some respondents who are not cooperative have a tendency to give incorrect information, it may cause the response error. These difficulties arise more seriously when the respondents are queried about sensitive or highly personal matters which deal with phenomena that are illegal or looked upon as morally condemnable by society. For example, "Did you have abortions during your life time?" or "Have you ever been charged with drunken driving for the last three months?"

Randomized response (RR) method, devised by Warner (1965), was suggested to get more credible estimators for the proportion of sensitive attribute. Many authors have extended the RR method to various cases, such as multi-proportions problem and sensitive quantitative data problem. Furthermore, some authors attempted to present Bayesian approaches to the inference of parameter in the RR model. Initially, Winkler and Franklin (1979) suggested a Bayesian estimator for the Warner's model, Pitz (1980) for the unrelated question model. All of them used a beta prior and calculated the posterior distibutions of π_A, which were written as

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mixtures of beta distributions. But their calculations were too complicated. Recently O'Hagan (1987) introduced a Bayes linear estimator for both model. Two important advantages of the Bayes linear estimator are simplicity and robustness.

While many classical approaches for multi-proportions RR model have been studied, Bayesian approaches have been studied only to the binomial proportion problem. In this study, a Bayes linear estimator for multi-proportions RR model is formulated. And some numerical comparisons between the new estimator and the classical maximum likelihood estimator are also made under the Dirichlet prior.

2. Bayes Linear Estimator for Unrelated Question Multi-proportions Model

2.1 Bayes Linear Estimator

The Bayes linear estimator has been discussed by several statisticians. Especially, O'Hagan (1987) applied the Bayes linear estimator for the inference of the RR model. An important advantage of the Bayes linear estimator is that it is distribution-free. Neither prior distributions nor likelihoods need be specified fully. Only first- and second-order moments are needed. The basic theorem is as follows:

[Theorem] (Brunk, 1980) Let $\underline{X}=(X_1,X_2,\cdots,X_m)'$ and $\underline{Y}=(Y_1,Y_2,\cdots,Y_n)'$ be random vectors whose components belong to a Hilbert space of square-integrable random variables with inner product $<\underline{x},\underline{y}'>=E(\underline{x}\underline{y}')$. Where $<\underline{x},\underline{y}'>$ denotes the $m\times n$ matrix that has the inner product $<x_i,y_j>$ in the i-th row, j-th column, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$. The subspace of constant random variables that corresponds to specified parameters is spanned by the random variable that is identically equal to one and is denoted by $\underline{1}$.

The Bayes linear estimator of $\underline{Y}$ given $\underline{X}$ is called the linear expection of $\underline{Y}$ given $\underline{X}$ and is denoted as $\hat{\underline{Y}}=LE(\underline{Y}|\underline{X})$. The linear covariance of $\underline{Y}$ given $\underline{X}$ is

$$V_{\underline{Y}}=LV(\underline{Y}|\underline{X})=E[(\underline{Y}-\hat{\underline{Y}})(\underline{Y}-\hat{\underline{Y}})'].$$

The vector $\hat{Y}$ is the orthogonal projection of Y on space $\{x, 1\}$, the sapce spanned by x and 1, that minimizes

$$E(Y-t)'(Y-t) \quad \text{for} \quad t \in \text{space}\{x, 1\}.$$

In other words, $\hat{Y}$ has the smallest posterior squared error loss among linear functions of the data, X. Then,

$$\begin{aligned} \hat{Y} &= LE(Y|X) \\ &= E(Y) + Cov(Y,X)[Cov(X)]^{-1}(X - E(Y)). \end{aligned} \tag{2.1}$$

The linear covariance of Y given X is

$$\begin{aligned} V_Y &= LV(Y|X) \\ &= Cov(Y) - Cov(Y,X)[Cov(X)]^{-1}Cov(X,Y). \end{aligned} \tag{2.2}$$

In the context of a Bayes linear estimation, X is a random vector to be observed, and Y is a quantity to be estimated (typically an unobservable parameter). The proof of the above theorem is given in Brunk (1980).

2.2 Data-Gathering Device

In the unrelated question randomized response model, nonsensitive question is used as the alternate question. It can be written schematically as follows.

P : Are you a member of class A?
1-P : Are you a member of class B?

The formulation is much more flexible, and generally more efficient than the Warner's. One extension is to replace the binary responses by quantitative ones. Thus, schematically,

P : State your characteristic A?
1-P : State your characteristic B?

Let A_k and B_k be the values of the two characteristics for individual k ($k = 1, 2, \cdots, N$). In practice, A_k is sensitive but B_k is not, and for the respondent to feel protected, both should have identical ranges of possible values.

Liu and Chow (1976) developed a new multi-proportions model using a new randomizing device. A number of balls of two different colors, e.g. red and white, will be placed in the body of the randomizing device (figure 2.1). A Discrete number, such as 1, 2, ⋯ , t, will be marked on the surface of white balls. The proportion of red to white balls, and of white balls with different figures (1,2, ⋯, t), will be predetermined.

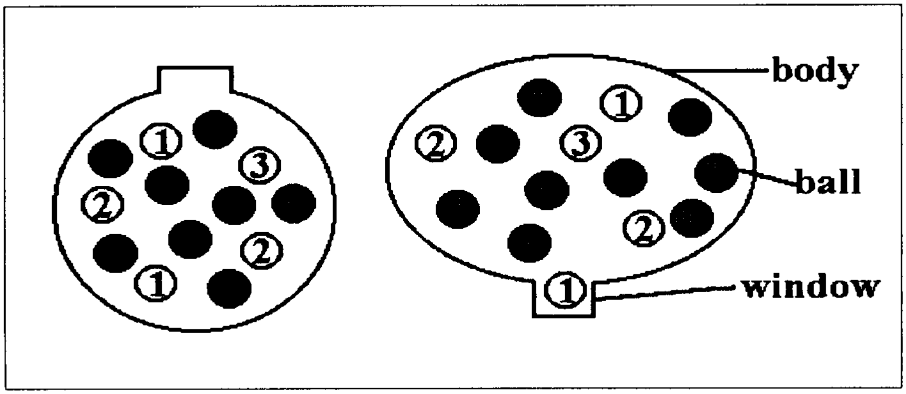

Figure 2.1 The randomizing device in the Liu and Chow's model

The respondent is asked to turn the device upside down, and shake the device. If the ball in the window is a red ball, the respondent will be asked to answer the sensitive question. Otherwise, the respondent simply tells the number marked on the white ball.

The proportions of the red balls and white balls are P and 1-P, respectively. The proportions of white balls only is $p_1, p_2, \cdots, p_t$ where $\sum p_i = 1-P$ and P, $(p_1, p_2, \cdots, p_t)$ are predetermined values. Let π_i= the true proportion of the i-th category in the population for sensitive question A, where $i=1,2,\cdots,t$ and $\sum \pi_i = 1$.

The above model can be written as follows.

Category number	1	2	⋯	i	⋯	t
True proportion	π_1	π_2	⋯	π_i	⋯	π_t
Proportion of white ball	p_1	p_2	⋯	p_i	⋯	p_t

($\sum p_i = 1-P$, P = proportion of red balls)

2.3 Formulation and Estimation

Let A_k's and B_k's be the sensitive characteristics and the nonsensitive characteristics respectively for individual k, $k=1,2,\cdots,N$ where N is the population size. Suppose that the A_k's are square integrable and exchangeable, and so are the B_k's. Also in the unrelated question model, it is regarded that characteristic A_k's and B_k's are independent, that is

$$Cov(A_k, B_{k'})=0 \text{ for all } k \text{ and } k'$$

Regard each multichotomous characteristic A_k and B_k as two sets of s binary characteristic, where s=t-1. That is, let

$$A_{jk}=I(A_k=j) \ ,$$
$$B_{jk}=I(B_k=j) \ ,$$

where $j=1,2,\cdots s$ and $k=1,2,\cdots,N$ and $I(\cdot)$ means the indicator function. Consider prior knowledge of the A_{jk}'s and B_{jk}'s. Let

$$E(A_{jk}) = m_j,$$
$$Var(A_{jk}) = v_j,$$
$$Cov(A_{jk},A_{j'k'}) = \begin{cases} c_j & \text{if } j=j' \\ c_{jj'} & \text{if } j\neq j' \end{cases},$$

$$E(B_{jk}) = m_{Bj},$$
$$Var(B_{jk}) = v_{Bj},$$
$$Cov(B_{jk},B_{j'k'}) = \begin{cases} c_{Bj} & \text{if } j=j' \\ c_{Bjj'} & \text{if } j\neq j' \end{cases},$$

Also let X_k be the response of the k-th respondent, then X_k is a multichotomous random variable. Let

$$X_{jk}=I(X_k=j)$$

then we can show the followings:

$$\begin{aligned} E(X_{jk}) &= P\Pr(A_{jk}=j)+(1-P)\Pr(B_{jk}=j) \\ &= Pm_j+(1-P)m_{Bj} \end{aligned} \tag{2.3}$$

$$\begin{aligned} Var(X_{jk}) &= Pv_j+(1-P)v_{Bj}+P(1-P)(m_j-m_{Bj})^2 \ , \\ Cov(X_{jk},X_{jk'}) &= P^2c_j+(1-P)^2c_{Bj} \ , \\ Cov(X_{jk},X_{j'k'}) &= P^2c_{jj'}+(1-P)^2c_{Bjj'} \ . \end{aligned} \tag{2.4}$$

where $k,k'=1,2,\cdots,N,\ k\neq k',\ j,j'=1,2,\cdots,s$ and $j=j'$.

The parameters of interest are

$$Y_j=\sum A_{jk}/N \ , \ j=1,2,\cdots,s \ , \tag{2.5}$$

where Y_j represents the true proportion of member who belong to the j-th category of characteristic A. Then the properties of Y_j are derived by the equations (2.1) and (2.2) as follows :

$$\begin{aligned} E(Y_j) &= m_j \ , \\ Var(Y_j) &= \{v_j+(N-1)c_j\}/N \ , \\ Cov(Y_j,Y_{j'}) &= \{-m_jm_{j'}+(N-1)c_{jj'}\}/N \ . \end{aligned} \tag{2.6}$$

Let

$$\overline{X_j}=\sum_{k=1}^{n}X_{jk}/n \tag{2.7}$$

which means the proportion of respondents who answered 'j ' in the sample. Since all A_k's and B_k's are exchangeable, the estimator will be a function of $\overline{X_j}$. Then from (2.3) to (2.7) expectations and variances of $\overline{X_j}$'s and covariance of $\overline{X_j}$ and $\overline{X_{j'}}$ for $j\neq j'$ are derived as follows:

$$\begin{aligned} E(\overline{X_j}) &= Pm_j+(1-P)m_{Bj} \\ Var(\overline{X_j}) &= \{Pv_j+(1-P)v_{Bj}+P(1-P)(m_j-m_{Bj})^2 \\ &\quad +(n-1)P^2c_j+(n-1)(1-P)^2c_{Bj}\}/n \ , \end{aligned} \tag{2.8}$$

$$\begin{aligned} Cov(\overline{X_j},\overline{X_{j'}}) &= \{P^2m_jm_{j'}+(1-P)^2m_{Bj}m_{Bj'} \\ &\quad +(n-1)[P^2c_{jj'}+(1-P)^2C_{Bjj'}]\}/n \ , \\ Cov(Y_j,\overline{X_j}) &= \{v_j+(N-1)c_j\}(P/N) \ , \\ Cov(Y_j,\overline{X_{j'}}) &= \{-m_jm_{j'}+(N-1)c_{jj'}\}(P/N) \ . \end{aligned} \tag{2.9}$$

Thus, by equations (2.1) , (2.2) and (2.6) to (2.9), the Bayes linear estimator $\hat{\underline{Y}}$ for $\underline{Y}=(Y_1,Y_2,\cdots,Y_s)'$ and the variance measure of $\hat{\underline{Y}}$ are obtained as follows :

$$\hat{\underline{Y}}=E(\underline{Y})+Cov(\underline{Y},\overline{\underline{X}})[Cov(\overline{\underline{X}})]^{-1}(\overline{\underline{X}}-E(\overline{\underline{X}})) \ ,$$

and

$$\begin{aligned} Var(\hat{Y}(\overline{X})) &= Cov(Y) - Cov(Y,\overline{X})[Cov(\overline{X})]^{-1}Cov(\overline{X},Y))) \ , \\ &= [V(Y_j, Y_{j'})]_{s\times s} \ , \quad j,j' = 1,2,\cdots,s \ , \end{aligned} \tag{2.10}$$

where

$$\begin{aligned} \overline{X} &= (\overline{X_1}, \overline{X_2}, \cdots, \overline{X_s})' \\ E(\overline{X}) &= (E(\overline{X_1}), E(\overline{X_2}), \cdots, E(\overline{X_s}))', \\ E(Y) &= (E(Y_1), E(Y_2), \cdots, E(Y_s))', \\ Cov(Y,\overline{X}) &= [Cov(Y_j, \overline{X_j})]_{s\times s} \ , \\ Cov(\overline{X}) &= [Cov(\overline{X_j}, \overline{X_{j'}})]_{s\times s} \ , \\ Cov(Y) &= [Cov(Y_j, Y_{j'})]_{s\times s} \ , \quad j,j' = 1,2,\cdots,s \ . \end{aligned}$$

So the Bayes linear estimator and the variance measure for $\hat{Y}_t$ is

$$\begin{aligned} \hat{Y}_t &= 1 - \sum \hat{Y}_j \ , \\ V(\hat{Y}_t) &= V(\hat{Y}_1 + \hat{Y}_2 + \cdots + \hat{Y}_s) \\ &= \sum V(\hat{Y}_j) + \sum_{j<}\sum_{j'} 2V(\hat{Y}_j, \hat{Y}_{j'}) \ . \end{aligned}$$

If the population size N is infinite, as assumed in the randomized response literature, then from (2.6) and (2.9) following relations are hold

$$\begin{aligned} Var(Y_j) &= c_j \ , \\ Cov(Y_j, Y_{j'}) &= c_{jj'} \ , \\ Cov(Y_j, \overline{X_j}) &= Pc_j \ , \\ Cov(Y_j, \overline{X}_{jj'}) &= Pc_{jj'} \ . \end{aligned} \tag{2.11}$$

So if informations about m_j's c_j's and $c_{jj'}$'s are available, the Bayes linear estimator can be computed easily.

Several levels of prior information about the B_{jk}'s may be expressed within the general formulation. Abul-Ela et al. (1967), in the framework of a binary response and infinite population, considered the mean of B_{jk}'s to be either known or unknown. Liu and Chow (1976) suggested some models where the mean of the B_{jk}'s are known. In the finite population, however, there is a further distinction to be made.

If the population mean $\overline{B_j}$ is known, then its variance is zero and we find

$$c_{Bj} = -v_{Bj}/(N-1) \ .$$

On the other hand, if the B_{jk}'s are generated by independent randomization, we have $c_{Bj}=0$.

In the general RR model it is assumed that N is infinite, so it is impossible to let $c_{Bj}=0$. Furthermore, the B_{jk}'s are generated by independent randomization in the Liu and Chow's model.

2.4 Prior Specification

In the case of binomial proportion model with an infinite population, Pitz (1980) used a uniform prior which is a special case of beta distribution. To find a Bayes linear estimator in the unrelated question-type multi-proportions RR model, not only m_j, c_j and $c_{jj'}$ but also m_{Bj}, c_{Bj} and $c_{Bjj'}$, $j,j' = 1,2,\cdots,s$ and $j \neq j'$, are needed to be specified. It is simple to specify informations about B_{jk}'s.

Here also if we could obtain a truthful answer to the question of interest without using a randomizing device, the sampling could be multinomial in $\underline{Y}'=(Y_1,Y_2,\cdots,Y_s)$.

Let's assume t=3 and $\underline{Y}'=(Y_1,Y_2,Y_3)$ has a Dirichlet distribution with parameters (a,b,c) then followings can be found (Johnson and Kotz (1972)) :

$$E(Y_1)=a/S, \quad E(Y_2)=b/S, \tag{2.12}$$

$$Var(Y_1)=a(S-a)/[S^2(S+1)], \quad Var(Y_2)=b(S-b)/[S^2(S+1)], \tag{2.13}$$

$$Cov(Y_1,Y_2)=-ab/[S^2(S+1)], \quad Cov(Y_1,Y_3)=-bc/[S^2(S+1)], \tag{2.14}$$

where $S=(a+b+c)$. From(2.11) and (2.12) to (2.14) the values of m_j, c_j and $c_{jj'}$ can be obtained. Since it is assumed that the informatioons about B_{jk}'s are already known, the Bayes linear estimator can be computed easily by substituting above values into equations (2.6) through (2.10). Thus, only Dirichlet parameters (a,b,c) are required to find the Bayes linear estimator.

3. Numerical Comparison

It is very difficult to compare the Bayes linear estimator (BLE) with the classical maximum likelihood estimator (MLE) analytically. In this case it is very useful to compare the performance of BLE with that of MLE through numerical comparisons. For this purpose numerical comparisons are made. In this section, the results of numerical comparisons are presented and a brief comment on the results is given.

3.1 Design of Numerical Comparison

This comparison will be investigated in one set of π_j ($\pi_1 = .05$, $\pi_2 = .15$, $\pi_3 = .80$) The sample size is 500 and the comparisons are based on 500 replications. 0.6, 0.7, 0.8 and 0.9 are considered as randomizing probabilities. The proportion of white balls marked 'i' , i.e. p_i are given to be equal.

The Dirichlet distribution is specified as prior for the BLE. Since Winkler and Franklin (1979) suggested the beta prior distribution for binomial case, the Dirichlet prior distribution, which is a natural conjugate of multinomial distribution, is suggested as a prior for trinomial case. Following values are considered as the prior parameter values :

(1,2,6)	(1,2,9)	(1,2,12)	(1,2,15)	(1,3,8)	(1,3,12)
(1,3,16)	(1,3,20)	(1,4,10)	(1,4,15)	(1,4,20)	(1,4,25)

The population proportion of each category is (.05, .15, .80), the prior parameter value (1, 3, 16) corresponds to it. Since the population proportion is unknown, it is difficult to give exact prior parameter value. But it is possible to give relevant prior information in many sampling survey problems. To show the performances of the BLE under various prior informations, many different parameter values are suggested.

To investigate the efficiencies of the BLE with respect to the MLE, MSEs of both estimators are calculated and the relevant efficiency of two estimates, which is defined as the inverse ratio of the MSEs, that is,

$$\text{Ref(BLE, MLE)} = \text{MSE(BLE)} \,/\, \text{MSE(MLE)}$$

is computed.

3.2 Comparison Result

Table 3.1 - Table 3.4 show the relative efficiencies of the BLE with respect to the MLE for various prior parameter values when the randomizing probability changes from 0.6 to 0.9 with increment 0.1.

Table 3.1 The Ref(BLE, MLE)s under various parameter values

P=0.6, $(\pi_1, \pi_2, \pi_3)=(0.05, 0.15, 0.80)$, $n=500$

prior	category 1	category 2	category 3
Dirichlet(1,2, 6)	1.10	1.06	1.25
Dirichlet(1,2, 9)	0.98	0.93	1.10
Dirichlet(1,2,12)	0.88	0.85	0.97
Dirichlet(1,2,16)	0.74	0.79	0.71
Dirichlet(1,3, 8)	0.99	1.12	1.27
Dirichlet(1,3,12)	0.85	0.97	1.07
Dirichlet(1,3,16)	0.74	0.85	0.92
Dirichlet(1,3,20)	0.66	0.80	0.87
Dirichlet(1,4,10)	0.88	1.18	1.25
Dirichlet(1,4,15)	0.75	0.99	1.01
Dirichlet(1,4,20)	0.66	0.85	0.88
Dirichlet(1,4,25)	0.62	0.78	0.87

Table 3.2 The Ref(BLE, MLE)s under various parameter values

P=0.7, $(\pi_1, \pi_2, \pi_3)=(0.05, 0.15, 0.80)$, $n=500$

prior	category 1	category 2	category 3
Dirichlet(1,2, 6)	1.18	1.08	1.25
Dirichlet(1,2, 9)	1.04	0.95	1.11
Dirichlet(1,2,12)	0.94	0.86	1.00
Dirichlet(1,2,15)	0.81	0.85	0.83
Dirichlet(1,3, 8)	1.05	1.16	1.27
Dirichlet(1,3,12)	0.92	1.00	1.09
Dirichlet(1,3,16)	0.82	0.89	0.97
Dirichlet(1,3,20)	0.76	0.82	0.92
Dirichlet(1,4,10)	0.95	1.23	1.28
Dirichlet(1,4,15)	0.82	1.02	1.05
Dirichlet(1,4,20)	0.75	0.90	0.94
Dirichlet(1,4,25)	0.70	0.81	0.89

Table 3.3 The Ref(BLE, MLE)s under various parameter values

P=0.8, (π_1, π_2, π_3) = (0.05,0.15,0.80), n = 500

prior	category 1	category 2	category 3
Dirichlet(1,2, 6)	1.29	1.12	1.26
Dirichlet(1,2, 9)	1.12	0.97	1.11
Dirichlet(1,2,12)	1.01	0.86	1.00
Dirichlet(1,2,15)	0.82	0.83	0.85
Dirichlet(1,3, 8)	1.13	1.21	1.28
Dirichlet(1,3,12)	0.97	1.03	1.11
Dirichlet(1,3,16)	0.87	0.91	0.98
Dirichlet(1,3,20)	0.78	0.83	0.90
Dirichlet(1,4,10)	1.01	1.29	1.31
Dirichlet(1,4,15)	0.88	1.08	1.10
Dirichlet(1,4,20)	0.79	0.93	0.97
Dirichlet(1,4,25)	0.73	0.83	0.90

Table 3.4 The Ref(BLE, MLE)s under various parameter values

P=0.9, (π_1, π_2, π_3) = (0.05,0.15,0.80), n = 500

prior	category 1	category 2	category 3
Dirichlet(1,2, 6)	1.47	1.16	1.24
Dirichlet(1,2, 9)	1.23	0.98	1.10
Dirichlet(1,2,12)	1.08	0.85	0.98
Dirichlet(1,2,15)	0.89	0.81	0.87
Dirichlet(1,3, 8)	1.23	1.27	1.27
Dirichlet(1,3,12)	1.04	1.06	1.10
Dirichlet(1,3,16)	0.90	0.92	0.97
Dirichlet(1,3,20)	0.82	0.80	0.87
Dirichlet(1,4,10)	1.08	1.37	1.31
Dirichlet(1,4,15)	0.91	1.11	1.10
Dirichlet(1,4,20)	0.79	0.95	0.96
Dirichlet(1,4,25)	0.73	0.85	0.89

From the above Tables, the following can be observed:

1. As long as the prior information is not so different from the population, the BLE has smaller MSE than the MLE except the cases of the Dirichlet parameters (1,2 6), (1,2 9), (1,3, 8), (1,3,12), (1,4,10). When proportions of the prior parameter values are near to (1,3,16), the expected proportions of each

group is the same with population proportion ($\pi_1=0.05$, $\pi_2=0.15$, $\pi_3=0.80$), Tables show that the BLE is more efficient than the MLE. So in case relevant information about the population is available, it it efficient to use the BLE.

2. The gain of efficiency of the BLE for the category 1, which has the smallest proportion, is greater than that of other categories. Because the most sensitive category has the smallest proportion in general, estimation of the smallest category proportion is more important than other categories. So it is regarded that the BLE has good performance.
3. As the randomizing probability P increases, the gain of efficiency decreases. When P is equal to 1.0, that is, only direct questioning method is used, we know that the MLE is the best unbiased estimator. In general randomized response study, 0.7-0.8 is used as randomizing probability. So it is recommendable to use the BLE.

References

[1] Abul-Ela, Abdel-Latif A., Greenberg, B. G. and Horvitz, D. G. (1967), "A Multi-proportions Randomized Response Model," *Journal of the American Statistical Association*, **62**, 990-1008.

[2] Brunk, H. D. (1980), "Bayesian Least Squares Estimates of Univariate Regression Functions," *Communications in Statistics -Theory and Methods.* **A9(11)**, 1101-1136.

[3] Greenberg, B. G., Abul-Ela, Simmons, W. R. and Horvitz, D. G. (1969), "The Unrelated Question Randomized Response Model : Theoretical Framework," *Journal of the American Statistical Association,* **64**, 502-539.

[4] Liu, P. T. and Chow, L. P. (1976), " A New Discrete Quantitative Randomized Response Model," *Journal of the American Statistical Association,* **71**, 72-73.

[5] Johnson, N. L. and Kotz, S. (1972), *Distributions in Statistics : Continuous Multivariate Distributions,* John Wiley & Sons, New York.

[6] O'Hagan, A. (1987), " Bayes Linear Estimator for Randomized Response Models," *Journal of the American Statistical Association,* **82**, 580-585.

[7] Pitz, G. F., (1980), "Bayesian Analysis of Random Response Models," *Psychological Bulletin*, **87**, 209-212.

[8] Warner, S. L. (1965), "Randomized Response : A Survey Technique for Eliminating Evasive Answer Bias," *Journal of the American Statistical Association*, **60**, 63-69.

[9] Winkler, R. L. and Franklin, L. A. (1979), "Warner's Randomized Response Model : A Bayesian Approach," *Journal of the American Statistical Association*, **74**, 207-214.

무관질문형 다지확률응답모형에서의 베이즈 선형추정량에 관한 연구

박 진 우[1]

요 약

다지확률응답모형인 경우에 대한 베이지안 접근방법을 연구하였다. O'Hagan (1987)의 베이즈 선형추정량을 다지확률 응답모형의 경우로 확장하였다. 한편 수치비교방법에 의해 새로이 연구된 베이즈 선형 추정량과 기존의 최대우도추정량과의 효율을 비교해 보았다. 이때 베이지안 방법의 사전분포로는 Dirichlet 분포를 사용하였다.

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