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A Location Competition Model of the Obnoxious Facilities with Spillover Effects*

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1. Introduction

Imagine that the people of a region require new public facilities. We wonder : would a centralized or a decentralized political regime yield a more efficient pattern of spatial development? Would the residents of the region be better off if their territory were divided into competitive jurisdictions that individually selected sites to increase the advantage of their residents or if a regional government centrally assigned facilities to geographic locations? How is the answer to that question affected by the nature of the facilities, the presence of space, and its environmental impacts?

Unfortunately, however, formal economists have dealt with the efficient provision of local public goods usually without regard to the space. To the contrary, space is vital when the level of benefits varies according to the location of the patron's residence.

Furthermore, government expenditures alter the distribution and the aggregate level of public services over space and subsequently affect the locational choice opportunities open to households and other economic agents. All this is mediated through the working of land market. In this way, the welfare of residents in a locality is closely linked to how the space is organized. Obviously, nonspatial models do not consider the relation between the location of the service recipients and the benefit level.

It is relatively recent to incorporate formally the spatial dimension of local public expenditures in the theoretic models. In fact, a series of papers dealing with optimal city size in 1970s were the first which paid serious attention. For example, the authors such as Arnott (1979), Arnott and Stiglitz (1979), Schuler (1974), Helpman, Pines, and Borukhov (1976) and Helpman and Pines (1980) discuss the local public goods provided for the whole city produced at the city center or for a particular neighborhood at each point over the continuous space. However, in this initial stage of development, the locational question of public facilities did not arise. The reason is that the resident's level of benefits depends only on the level of public expenditures. In that sense, the service level was distance-insensitive. Indeed, in the models a la Arnott the main question on the local public goods was the optimal level of public expenditures. In reality,

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however, an important class of local public goods is provided by the facilities whose benefits usually decrease in proportion to the distance from the facility location. The bulkiness and the associated discreteness of the public facilities are captured by the Sakashita's insightful paper under the influence of land market (Sakashita 1987). He shows that the externality-internalizing effect of land market does not invalidate the search for optimal location of public facilities. In fact, the equilibrium utility is maximized when the public facility is located at the center, which is what he calls 'Columbus Egg.' Fujita (1986) extended his model further into the two dimensional case with more general form of utility function by using 'area dominance' approach.

This distance-sensitivity coupled with the locational discreteness gives rise to another essential difference from the distance-insensitive models. Namely, because the benefits vary over distance, there arises the possibility of their spilling-over into adjacent jurisdictions and the associated inefficiency in the provision of local public services by competitive local governments.

For example, Kuroda (1989) illustrates how the voluntary provision game of non-additive public services with spillovers results in the inefficient Nash equilibrium (s). Koide (1988), on the other hand, analyzed the optimal locations of the public facilities whose benefits are additive. Although it is highly contentious whether Kuroda's model speaks to the reality (Axelrod 1984;Buchanan and Tullock 1962), his model clearly shows by explicitly considering the 'spatial' and 'territorial' dimension that there exists an opportunity for the exploitation by selfish agents, so that there is a necessity to coordinate the actions taken by competitive local governments.

The current study closely follows the model settings of Kuroda (1989) and Koide (1988). However, the former is different from the latter two, because it analyzes the location problem of the obnoxious facilites exhibiting the summation type of externalities. In the following, we present the model settings and solve the location competition problem between two local governments, which is followed by a brief explanation of the analytical results.

2. The Model

Imagine two cities, A and B, located side by side. For simplicity, suppose that they are formed on a long narrow line J of width 1 and length 1. The cities are inhabited by a continuum of homogeneous households. Without loss of generality, let us assume that City A is located to the left of City B. Denote the political boundary between the two cities by b. Then, City A's jurisdiction J_A spans over [0, b] and City B's jurisdiction J_B, [b, 1].

Suppose that each city is contemplating the construction of an obnoxious facility such as municipal incinerators. To simplify the story, assume that the facility is totally financed by the grants from the central government and that its scale was already determined. Therefore, the only decision to be made is where to locate the facility. However, by the nature of the facility, it generates public bads such as smoke and bad smell. The facility of City

A cannot be located within City B's jurisdiction, vice versa.

Suppose that wind blows equally in both directions and carries the bads all over City A and B. Obviously, the level of bads originating from a facility decreases in distance from the facility location, but increases in the operating scale of the facility. For example, we might express the level of bads at x, E(x) as follows :

$$E(x)=K_A-a\mid y_A-x\mid+K_B-a\mid y_B-x\mid \quad \forall x\in J \qquad (1)$$

where $K_i(i=A, B)$ is the operating scale of City i's facility located $x=y_i$ in its own jurisdiction and a constant greater than zero. To make the analysis simple, let us assume that any location $x\in[0, 1]$ is affected by the public bads from both cities. That is, we assume

$$K_A-a\mid y_A-x\mid\geq 0,\ K_B-a\mid y_B-x\mid\geq 0 \quad \text{for all } x\in[0, 1]$$

Now, let us describe how the location competition is played. First, observe that because the bads spill over into another jurisdiction, one city's location decision of the facility necessarily affects the welfare of its neighbors. Therefore, there emerges a dependency structure in the facility location decision problem. The game is played in two stages. In the first stage, each city decides the facility location and in the second stage given the facility locations each household makes its decision. To make this statement more precise, we need to specify the household's and city government's objective function. Suppose that the facility locations (y_A, y_B) are somehow given from the first stage. Then, each household living at City i (i=A, B) maximizes its utility as follows :

$$\max_{z,s,x\in J_i} u_i(z,s,E(x))=\alpha\log z+\beta\log s-v(E(x)) \qquad (2)$$
$$\text{subject to } z+R(x)s=I,$$

where z is the composite good whose price is one, s the lot size, E(x) the level of bads at x, v(E(x)) the function of E(x), R(x) the land rent at x, and I the household income. For simplicity, assume in (2) that

$$\alpha+\beta=1,\ \alpha,\ \beta>0$$

Observe in (2) that the household is allowed to live only within its current city. The residential equilibrium corresponding to a certain configuration of facility locations is achieved when all the households living in the same city enjoys the same level of (indirect) utility.

Suppose this equilibrium level of utility is given by u_i at city i. Then, city i decides the facility location given the number of its own residents, N_i and the other city's facility location y_{-i} such that its decision maximizes the common utility of its own homogeneous residents. Therefore, the city government's problem may be written as follows :

$$\max_{y_i\in J_i} u_i^*(y_i\mid y_{-i},N_i) \qquad (3)$$

In (3), $u_i^*(y_i\mid y_{-i},N_i)$ is the level of (indirect) utility which would be realized at the residential equilibrium corresponding to (y_A, y_B).

3. Solution to the Problem

Let us solve the household's maximization problem of the second stage first. To this end, assume that the land market is competitive. Define the bid rent at $x \in [0, 1]$ by

$$\Psi(x) = \max_{s,z} \frac{I-z}{s} \quad (4)$$
$$\text{subject to } u(z,s,E(x)) = u,$$

Therefore, by the very meaning of $\Psi(x)$ the market rent at x, R(x) should be given by this $\Psi(x)$ at equilibrium. Observe that given the market rent $R(x)=\Psi(x)$ at each x, the utility maximizing choice of lot size s in (2) simultaneously solves the maximization problem in (4). The same also holds for the composite good z.

Suppose that N_A and N_B households reside in City A and B, respectively at the present moment. Then, at equilibrium each household at each city obtains the same level of utility, say, u_i the land market clears; and N_A and N_B households reside in City A and B, respectively. Consequently, with the log-linear utility function in (2) we can characterize the residential equilibrium corresponding to (y_A, y_B) at City i as follows. For each $x \in J_i$

$$z = z(x) = aI$$
$$s = s(x) = \alpha^{-\alpha/\beta} I^{-\alpha/\beta} Exp(u_i/\beta) Exp(\beta^{-1} v(E(x)))$$
$$R(x) = \Psi(x) = \alpha^{\alpha/\beta} \beta I^{\alpha/\beta} Exp(-u_i/\beta) Exp(-\beta^{-1} v(E(x)))$$
$$n = n(x) = \alpha^{\alpha/\beta} I\gamma^{\alpha/\beta} Exp(-u_i/\beta) Exp(-\beta^{-1} v(E(x)))$$
$$N_i = \int_{J_i} n(x)dx. \quad (5)$$

Next, let us solve first stage problem. Because each city behaves as a utility maximizer, we first need to find the indirect utility function u_i(i=A and B). From (5),

$$N_i = \int_{J_i} n(x)dx = \alpha^{\alpha/\beta} I^{\alpha/\beta} Exp(-u_i/\beta) \int_{J_i} Exp(-\beta^{-1} v(E(x))dx \quad (6)$$

Solving this equation with respect to u_i, we obtain the following expression :

$$u_i = \beta \log(N_i^{-1} \alpha^{\alpha/\beta} I^{\alpha/\beta}) + \beta \log Z_i(y_i; y_{-i}),$$

where

$$Z_i(y_i; y_{-i}) = \int_{J_i} Exp(-\beta^{-1} v(E(x)))dx. \quad (7)$$

with $-i$'s facility location y_{-i} being given.

Since u_i is increasing in Z_i, the former is maximized when Z_i is maximized. Therefore, we only need to investigate how Z_i behaves with respect to y_i.

Recall that the public bads produced by both facilities reach every inch in J_i. Therefore, to rewrite the level of bads E (x) at each $x \in [0, 1]$,

$$E(x) = \begin{cases} K_A + K_B - a(y_A + y_B) + 2ax & \text{for } 0 \le x \le y_A \\ K_A + K_B + a(y_A - y_B) & \text{for } y_A \le x \le y_B \\ K_A + K_B - a(2x - y_A - y_B) & \text{for } y_B \le x \le l. \end{cases} \quad (8)$$

Thus, from (7) and (8)

$$Z_A(y_A; y_B) = \int_0^{y_A} Exp(-\beta^{-1} v(K_A + K_B - a(y_A + y_B) + 2ax))dx + \int_{y_A}^{b} Exp(-\beta^{-1} v(K_A + K_B + a(y_A - y_B)))dx \quad (9)$$

Before we characterize the Nash equilibrium location pair(y^*_A,y^*_B), observe that locating the obnoxious facility inside one's own jurisdiction is not preferable to the periphery locations, 0, b, and 1 (see Fujita 1986).

Define a new function f(x) as follows :

$$f(x) = Exp(-\beta^{-1} v(K_A + K_B - a(b + y_B) + 2ax)). \quad (10)$$

Differentiate f(x) with respect to x.

$$f'(x) = -2a\beta^{-1}v' f(x) < 0 \qquad (11)$$
$$f''(x) = 4a^2\beta^{-1}f(x)(\beta^{-1}(v')^2 - v'') \qquad (12)$$

When v is concave in E(x), f(x) is strictly convex in x, so that

$$Z_A(b;y_B) = \int_0^b Exp(-\beta^{-1}v(K_A+K_B-a(b+y_B)+2ax))dx$$
$$> \int_0^b Exp(-\beta^{-1}v(K_A+K_B-ay_B))dx = Z_A(0;y_B).$$

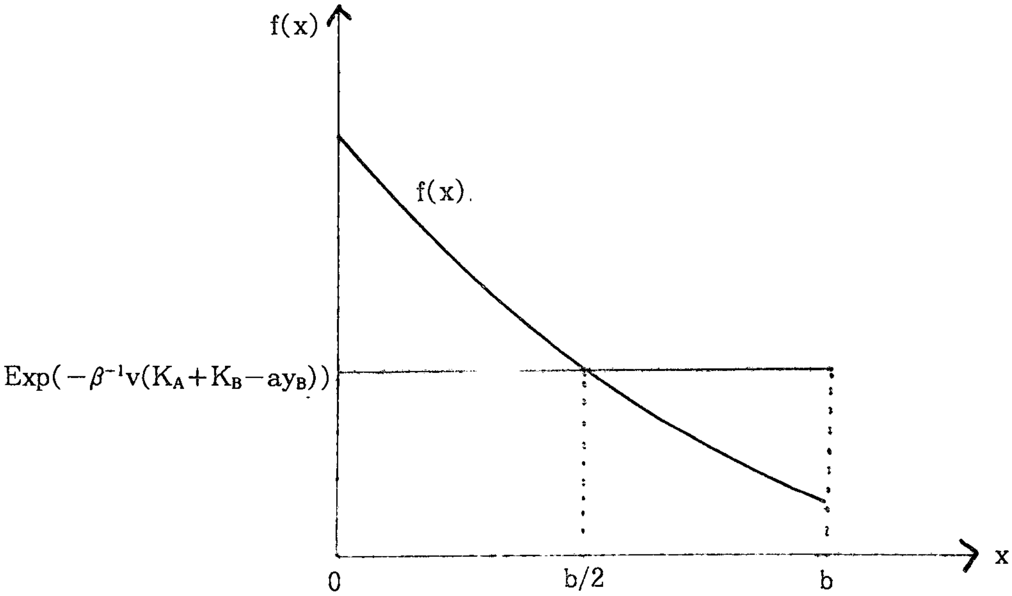

Figure 1. Environmental Quality in City A

Similary, observe that when f is strictly concave in x, we can claim that no matter where B locates the facility it is preferable for A to put the facility at $x=0$. Therefore, by symmetry we can state as follows :

Proposition 1 : Suppose that each city is concerned only about the welfare of its own residents. Then, the Nash equilibrium location pair(y^*_A, y^*_B) is

(1) (b,b) if v is concave in E(x) and
(2) (0,1) if $\beta^{-1}(v')^2 - v'' < 0$

4. Optimal Facility Locations

We consider how the regional government might locate the facilities. The regional government is assumed to treat the area [0, 1] as a single region. It determines the facility scales with K_A+K_B given and their locations with a single purpose of maximizing the welfare of City A and B. The residents of both cities are free to move within the region [0, 1]. At equilibrium, a common level of utility u will prevail corresponding to the facility location vector y. Here, y is not necessarily a singleton. Similar to (5) before, we can characterize the residential equilibrium corresponding to y with minor adjustments.

Since periphery location(s) of the facility(s) is preferable, we only consider whether to put the facility(s) at one or both ends of [0, 1] and how large they should be if they are put at both ends of the interval J. A moment's thought suggests that the facilities should be scaled similarly such that the environmental quality is uniform all over [0, 1]. For example, in Figure 2 by scaling down K_A and scaling up K_B, $E(x)$ stays constant at K_A+K_B-al, which is an improvement over the interval [0, c]. This means that two facilities must be provided with similar scale at both ends of [0, 1]. To be more precise, the optimal facility locations and scales are given as follows :

$(y^*_A, y^*_B)=(0, 1)$
$K^*_A \geq K^*_B - al$ and $K^*_B \geq K^*_A - al$.

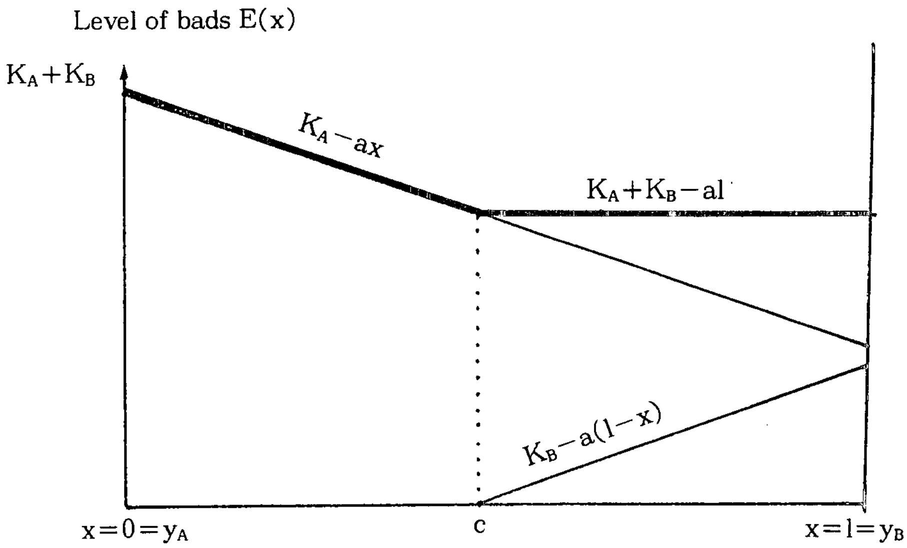

Figure 2. Environmental Quality and Facility Scales

Proposition 2 : Assume $K_A \geq K_B - al$ and $K_B \geq K_A - al$. Then,

(1) when v is concave in $E(x)$, the Nash equilibrium location pair (o,l) is inefficient and
(2) when $\beta^{-1}(v')^2 - v'' < 0$, the Nash equilibrium location pair is efficient.

Consequently, when the marginal disutility of public bads diminishes(i.e.,$v''<0$), the equilibrium locations are always inefficient even with the obnoxious facilities with summation type of interjurisdictional externalities. The analysis suggests that when v(E(x)) is concave in E(x) each city behaves as if it maximizes the negative externalities for the neighboring jurisdictions.

However, the analysis demonstrates that when the marginal disutility of the bads increases fast enough such that $\beta^{-1}(v')^2 - v'' < 0$, the resulting spatial configuration is optimal. This result is in stark contrast with the conventional wisdom and Kuroda's analysis(1989).

5. Concluding Remarks

The existence of the spatial externalites between jurisdictions is not necessarily the cause of inefficiency in providing local public goods. If this is the case, for some type of public services even with spillovers can rely on the decentralized provision by competing jurisdictions. This is certainly a good news. There is an ample reason for respecting the dynamics between autonomous localities.

However, the reality is not be that simple. Facility locating is, without doubt, a controversial issue in the intergovernmental relation. Besides, it usually involves the tug of war between government and residents in site and is frequently riddled with the exercise of physical prowess in the clash process. This is why we cannot comfort ourselves with the rather rare analytical result. On the contrary, it is the time demanding our serious efforts for the successful adjudication of conflicting interests.

This is certainly an interesting but practically important question. The challenge seems to involve not only technical but also mental reorientation of the planning and administrative community and the residents in general. It must be a burgeoning field of research and the promising niche for the would-be practioners.

Returning back to the paper, it will be interesting to analyze the self-financed case, variable facility size, and other types of cumulative effects.

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