地域硏究 第9巻 第1號
1993年 6月, pp.101～106

Adjustment Speed and Role of the Central Government in a System of Regions : A Note*

Chang-Ho Shin
Seoul Development Institute

1. Introduction

Myers (1990) argued that a Nash competing equilibrium as a consequence of local government's behavior was Pareto optimal, and so there was no role for the central government in providing interregional transfers. Local governments make the set of optimal transfers to obtain a preferred size of regional population.

Hercowitz and Pines (1991) reexamined Myers' work using a dynamic migration model with a costly mobility, and demonstrated the necessity of the central government intervention to obtain the socially optimal population allocation. When migration is costly, a decentralized competitive Nash equilibrium cannot be optimal. If there is only a small difference in fiscal capacity between two regions, fiscally rich government has no incentive to transfer resource to fiscally poor government. If the difference is large, the former will transfer some portion of it to the latter, but never the socially optimal amount. In this situation, it is required for the central government to intervene so as to achieve the social optimum.

In this note we wish to discuss whether there will be really no role for the central government in a system of regions even if there is no migration cost. For that purpose, we analyze the adjustment time which is required to reach the goal starting from an autarky market equilibrium in two cases, i.e. the Nash competing equilibrium and central government intervention. It will be shown that the adjustment speed with the central government's intervention is much faster than that of the Nash competing equilibrium.

2. Social optimum

We assume that there is an economy which consists of two regions. The total economy's population is assumed to be fixed as $\bar{N}$ which is the sum of N_1 and N_2, and is homogeneous. Individuals are assumed mobile freely between two regions.

Preferences are defined by a strictly quasi-concave utility function given by $U_i = U(C_i, G_i)$, where C_i is per capita amount of the composite good consumed, G_i is the consumption of local public good by an individual residing in region $i(i=1, 2)$.

Each region has a production function, $F_i(N_i)$, where N_i is the population of region i, and it is assumed that $F_i'>0$, and $F_i''<0$.

Let the marginal rate of transformation between the composite good C and the local public good G be unity so that G_i denotes the cost of production of local public good in region i. It is also assumed that the local public good is pure and there is no spill-over of G_i across regions.

The social planner allocates C_i, G_i, N_i(i

*The author wishes to thank Professor Noboru Sakashita of the University of Tsukuba for his valuable suggestions.

$=1, 2$) to maximize the utility of a representative individual in region i subject to the following constraints. The first constraint in (1) is that the utility level should be equated across two regions to ensure social stability. The second constraint is national feasibility for all purpose goods. The last constraint requires that all individuals be located in a specific region.

The social planner's problem is to

$$\max_{\{C_1,C_2,G_1,G_2,N_1,N_2\}} U_1=U(C_1,G_1)$$

subject to

$$U(C_2,G_2)-U(C_1,G_1)=0 \quad (1)$$

$$F_1(N_1)+F_2(N_2)-N_1C_1-N_2C_2-G_1-G_2=0$$

$$\bar{N}-N_1-N_2=0.$$

From the first order conditions for the above problem, we have condition (2) which is the Samuelsonian condition for the supply of local public good in each region :

$$N_i\frac{U_{Gi}}{U_{Ci}}=1, \qquad i=1, 2 \qquad (2)$$

where $U_{Ci}\equiv\partial U/\partial C_i$, $U_{Gi}\equiv\partial U/\partial G_i$. This is the familiar result that the sum of the marginal rates of substitution equals the marginal rate of transformation. The solution to (2) for G_i determines how regional output is divided between the composite good and the local public good in each region.

We have also a social net benefit condition of migration which states that the gap between marginal productivity of labor and per capita consumption takes the same value in each region :

$$F'_1-C_1=F'_2-C_2 \qquad (3)$$

where $F'_i\equiv\partial F_i/\partial N_i(i=1, 2)$. Equation(3) states that the net marginal benefit from locating in region $i(i=1, 2)$ must be equated between regions for the purpose of an efficient allocation of population.

In this model we have six equations (2), (3), and three constraints in (1) for six unknowns (C_i, G_i, N_i ; $i=1, 2$). A social optimum solution for resource allocation can be achieved when these conditions are simultaneously satisfied.

3. Nash equilibrium of voluntary local governments

Myers(1990) argued that when regional governments, which are endowed with relatively more resource-tax revenue, voluntarily donate some of it to the less regions, thus correcting the distorted incentive and deterring the inefficient immigration. We use the same model as Myers' (1990) except for the production function, the sole input of which is the regional population.

Local government $i(i=1, 2)$ decides a composite private good C_i, a public good G_i, regional population N_i, and resource transfer from region i to j, $S_{ij}(j=2, 1)$ in order to maximize a representative resident's utility function $U(C_i, G_i)$ subject to equal utility constraint between two regions, regional feasibilities, and a given national population for a given amount of inverse transfer and local public good supply in another region, S_{ji} and G_j. Local government will be assumed to transfer resources to another region in order to "purchase" a preferred regional population size (see Myers, 1990, p.114). Local governments determine the optimal population size, and then transfer resources to another region in order to keep up the size. Therefore, the choice of regional population size and the resource transfer must be practiced jointly.

Local government l's problem is to

$$\max_{\{C_1,G_1,N_1,S_{12}\}} U_1=U(C_1,G_1)$$

subject to

$$U(C_2,G_2)-U(C_1,G_1)=0$$

$$F_1(N_1)-G_1-N_1C_1-S_{12}+S_{21}=0,$$

$$F_2(N_2)-G_2-N_1C_1-S_{21}+S_{12}=0, \quad (4)$$

$$\bar{N}-N_1-N_2=0,$$

$$G_1\geq0,\ S_{12}\geq0, \text{ and } N_1\geq0,$$

for given G_2 and S_{21}. A similiar expression holds for region 2.

By solving this maximizing problem simultaneously for the two regions we have the following conditions :

a Samuelsonian condition in each region :

$$N_i \frac{U_{Gi}}{U_{Ci}} = 1, \qquad i=1, 2, \tag{5}$$

social net benefit condition :

$$F'_1 - C_1 = F'_2 - C_2. \tag{6}$$

Here we have seven equations, (5), (6), and four constraints in (4), for eight variables (C_i, G_i, N_i, S_{ij} ; $i=1, 2$) so that the solution is only unique up to the difference $S=S_{12}-S_{21}$. We have no unique solution for the interregional resource transfers. If we consider possible transaction cost of interregional resource transfer between two regions, the best solution must be what one region chooses zero transfer to another region. The decentralized competitive Nash equilibrium can be Pareto optimal when conditions (5) and (6) with the constraints are simultaneously satisfied.

4. A numerical simulation of the Nash equilibrium

We have known that Nash equilibrium of voluntary local governments is efficient by the strategic set (G_i, S_{ij}). From a dynamic point of view, we compare the solution of the decentralized competitive Nash equilibrium with that of central government intervention by a numerical simulation, and discuss the policy implication.

Local governments can adjust the sizes of their own residents through their interregional resource transfers. This process can be modelled as a simultaneous game of public good supply and resource transfer by the two local governments.

1) Decentralized competitive Nash equilibrium

Suppose that the dynamic adjustment equations of the strategic variables for a decentralized competitive Nash game can be written as

$$\begin{aligned} \dot{G}_1 &= \lambda_1\{G_1^*(G_{2,t}, S_{21,t}) - G_{1,t}\} \\ \dot{S}_{12} &= \mu_1\{S_{12}^*(G_{2,t}, S_{21,t}) - S_{12,t}\} \\ \dot{G}_2 &= \lambda_2\{G_2^*(G_{1,t}, S_{12,t}) - G_{2,t}\} \\ \dot{S}_{21} &= \mu_2\{S_{21}^*(G_{1,t}, S_{12,t}) - S_{21,t}\} \end{aligned} \tag{7}$$

where $G_{i,t}^*$, $S_{ij,t}^*$($i=1,2$; $j=2,1$) is an optimal value of $G_{i,t}$, $S_{ij,t}$ at time t, respectively, and λ_i, μ_i($i=1,2$) represent the speeds of adjustment, and the dot indicates differentiation with respective to time.

Alternatively, we may also formulate the above differential equations in terms of the following system of difference equations :

$$\begin{aligned} G_{1,t+1} &= \lambda_1\{G_1^*(G_{2,t}, S_{21,t}) - G_{1,t}\} + G_{1,t} \\ S_{12,t+1} &= \mu_1\{S_{12}^*(G_{2,t}, S_{21,t}) - S_{12,t}\} + S_{12,t} \\ G_{2,t+1} &= \lambda_2\{G_2^*(G_{1,t}, S_{12,t}) - G_{2,t}\} + G_{2,t} \\ S_{21,t+1} &= \mu_2\{S_{21}^*(G_{1,t}, S_{12,t}) - S_{21,t}\} + S_{21,t}. \end{aligned} \tag{8}$$

When we are dealing with discrete time, the strategy variables will change their values only when the variable t changes from one integer value to the next. The amount of the strategy variables of one region in the (t+1)th period will depend on the amounts of its own strategy variables and another region's variables of the preceding period t and the adjustment speeds.

2) Nash equilibrium with central government intervention

Suppose that the central government intervenes the adjustment process by keeping an optimal value of S_{12} and S_{21}, i.e., $\bar{S}_{12}$ and $\bar{S}_{21}$, constantly. This intervention can be done because the central government has more abundant information than local governments in real world.

The dynamic adjustment equations of

local public goods can be formulated as

$$\dot{G}_1=\lambda_1\{G_1^*(G_{2,t}, \bar{S}_{21,t})-G_{1,t}\}$$
$$\dot{G}_2=\lambda_2\{G_2^*(G_{1,t}, \bar{S}_{12,t})-G_{2,t}\} \quad (9)$$

where $\bar{S}_{12}$, $\bar{S}_{21}$ is an optimal amount of S_{12}, S_{21} given by the central government, respectively. The above differential equation can be reformulated as the following system of difference equations :

$$G_{1,t+1}=\lambda_1\{G_1^*(G_{2,t}, \bar{S}_{21,t})-G_{1,t}\}+G_{1,t}$$
$$G_{2,t+1}=\lambda_2\{G_2^*(G_{1,t}, \bar{S}_{12,t})-G_{2,t}\}+G_{2,t} \quad (10)$$

where λ_1 and λ_2 are the same adjustment speeds as those of(8).

3) Comparison of adjustment speed between two equilibriums

Here, it is assumed that a representative resident's utility function in time period t is specified by a log-linear type, $U_{i,t}=\alpha \log C_{i,t}+(1-\alpha)\log G_{i,t}+2(0\leq\alpha\leq1)$, and the regional production function in time period t is specified as $F_{i,t}=A_iN_{i,t}^{\beta}(0\leq\beta\leq 1)$. For concreteness, we analyze those adjustment functions by a numerical simulation. We assume here that the values of the parameters are given as $\alpha=0.8$, $\beta=0.6$, $A_1=1.2$, $A_2=1.0$, $\bar{N}=100$, $\lambda_1=\lambda_2=0.7$, and $\mu_1=\mu_2=0.8$ over time.

In this case, socially optimal solution and autarky market equilibrium solutions are given as Table 1 (see Appendix for the concept of an autarky market equilibrium). In a process of the decentralized competitive Nash equilibrium, the reaction functions of the two local governments for the supply of local public good are labelled G_1 and G_2 in Figure 1 for given values of (S_{12}, S_{21}), i.e., (0,0.3575). The crossing point E is the Nash equilibrium when the strategies consist of the supply of local public good only. The reaction functions for the interregional resource transfers are labelled S_{12} and S_{21} in Figure 2 for given values of (G_1, G_2), i.e. (3.3688, 1.1780). Because two curves are coincided on the right hand side of $S_{21}=0.3575$, the (S_{12}, S_{21}) pair have no unique solution. However, there is no doubt that $(S_{12}, S_{21})=(0, 0.3575)$ is the best one when we consider possible transaction cost of interregional resource transfers between two local governments. It is certain that a decentralized competitive Nash equilibrium with interregional transfers is Pareto optimal.

Now, we wish to know what time path of the solution of the dynamic system, (8), starting from the autarky market equilibrium in which $G_1=3.1059$, $S_{12}=0$, $G_2=1.4978$, and $S_{21}=0$, converges to the decentralized competitive Nash equilibrium where $G_1=3.3688$, $S_{12}=0$, $G_2=1.1780$, and $S_{21}=0.3575$. Table 2 summarizes the resultant time paths of strategy variables when the local governments react Nash competitively. As shown in this table, the process needs at least 15 time periods before reaching the Nash equilibrium.

On the other hand, if the central government intervenes the process by keeping $\bar{S}_{12}=0$ and $\bar{S}_{21}=0.3575$ constantly, the time paths of the strategy variables of the two local governments formulated by (10) becomes one shown in Table 3. It is obvious from Table 2 and Table 3 that the adjustment speed with the central government's intervention is much faster than that without intervention.

5. Concluding remarks

In this note, we examined Myers' work (1990) with a numerical simulation.

From a long-run point of view, we compared the adjustment speeds to the desired state in two processes, which were the decentralized Nash equilibrium and the central government intervention equilibrium. Because the latter process is much faster than the former, we conjecture that the latter case will be more desirable than the former. Therefore, it is still required for the central government to intervene in the interregional transfers from a long-run point of view even if there is no migration cost.

References

Hercowitz, Z. and D. Pines., 1991, "Migration with Fiscal Externalities, "*Journal of Public Economics*, 46, pp.163-180.

Myers, G. 1990, "Optimality, Free Mobility and the Regional Authority in a Federation," *Journal of Public Economics*, 43, pp.107-121.

Sakashita, N. 1990, "Presidential Address : Spatial Interdependence and Externalities," *Papers of the Regional Science Association*, 67, pp.1-11.

Shin, C.H. 1992, *The Role of Local and Central Governments in a System of Regions*, Unpublished Ph.D.dissertation, University of Tsukuba.

Appendix : Autarky Market Equilibrium

Autarky market equilibrium implies a market equilibrium in which each local government maximizes the utility of a representative resident under the assumption of strictly no interregional resource transfers (see Sakashita, 1989, p.3). That is, the output produced by each region is allocated to the absorption of the residents and the government within the region. Migration of people is still possible in this equilibrium. However, local government i(i=1, 2) maximizes the utility of its own residents, taking the size of population in its region as given. The local government behaves myopically in the sense that it ignores the influence of its behavior on migration. The local government 1's behavior is formulated as follows :

$$\max_{\{C_1,G_1\}} \quad U_1=U(C_1,G_1), \qquad \text{(A.1)}$$

subject to

$$U(C_2,G_2)-U(C_1,G_1)=0,$$
$$F_1(N_1)-G_1-N_1C_1=0,$$
$$F_2(N_2)-G_2-N_2C_2=0,$$
$$\bar{N}-N_1-N_2=0$$

A similiar expression holds for region 2. The maximization problems for two regions yield the following equilibrium conditions :

a Samuelsonian condition in each region ;

$$N_i\frac{U_{Gi}}{U_{Ci}}=1, \qquad i=1,2 \qquad \text{(A.2)}$$

and the four constraints in (A.1) ; Six equations, (A.2) and four constraints in (A.1), are sufficient to determine the values of six unknown(C_i,G_i,N_i ; i=1,2).

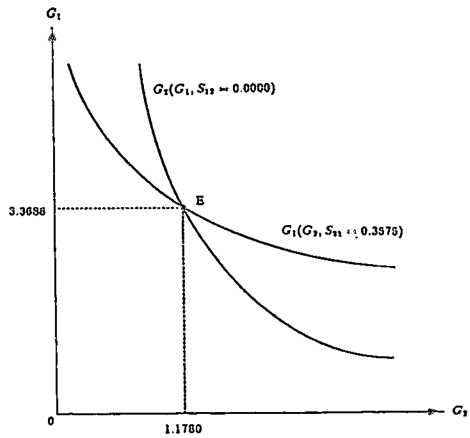

Figure 1. Reaction curves of G_1,G_2

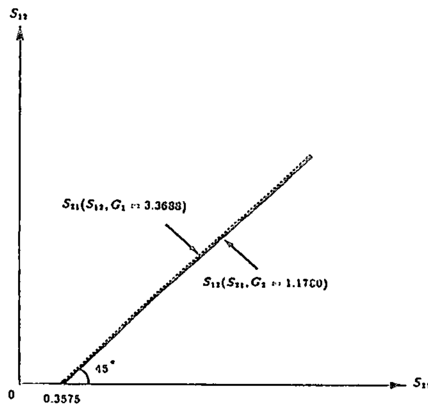

Figure 2. Reaction curves of S_{12},S_{21}

Table 1. Social Optimum and Autarky Market Equilibrium

	G_1	G_2
Social Optimum	3.3688	1.1780
Autarky Market Equilibrium	3.1059	1.4978

Table 2. Time Paths of Strategy Variables in Decentralized Competitive Nash Game

period t	$G_{1,t}$	$S_{12,t}$	$G_{2,t}$	$S_{21,t}$
0	3.1059	0.0000	1.4978	0.0000
1	3.1886	0.0000	1.3338	0.2154
2	3.2980	0.0000	1.2647	0.2820
3	3.3199	0.0000	1.2192	0.3246
4	3.3410	0.0000	1.2008	0.3387
5	3.3532	0.0000	1.1907	0.3468
6	3.3600	0.0000	1.1851	0.3515
7	3.3639	0.0000	1.1820	0.3541
8	3.3660	0.0000	1.1802	0.3556
9	3.3672	0.0000	1.1792	0.3564
10	3.3679	0.0000	1.1787	0.3569
11	3.3683	0.0000	1.1784	0.3571
12	3.3685	0.0000	1.1782	0.3573
13	3.3686	0.0000	1.1781	0.3574
14	3.3687	0.0000	1.1780	0.3574
15	3.3688	0.0000	1.1780	0.3575

Table 3. Time Paths of Strategy Variables in in Central Government Intervention Game⟨$\bar{S}_{12}=0$, $\bar{S}_{21}=0.3575$⟩

period t	$G_{1,t}$	$G_{2,t}$
0	3.1059	1.4978
1	3.3076	1.2757
2	3.3519	1.2074
3	3.3639	1.1868
4	3.3674	1.1806
5	3.3684	1.1787
6	3.3687	1.1782
7	3.3688	1.1780