지반과 구형 평판구조사이의 접촉응력에 적합한 형상함수

Proper Shape Fuction for the Contact Stress in the Soil-Plate Interaction Problems

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요 약

지반위에 얹혀진 구형평판을 에너지 방법으로 해석하는 방법에 대한 일반적인 전개이다. 본 논문에서는 기본적으로 지표면과 평판사이의 접촉응력을 가정한 다음 Boussinesque의 식을 적분하여 지표면 혹은 평판의 처짐을 구하는 방법을 시도하였다. 임의의 차수를 갖는 다항식과 Chebychev 다항식으로 접촉응력을 가정할 때 Boussinesque의 식을 적분하는 방법을 서술하고 그 결과를 에너지법에 이용하는 과정을 설명하였다. 해석결과 임의차수를 갖는 다항식을 접촉응력 함수로 적당하지 않고, Chebychev 다항식이 합당한 것으로 나타났으나 평판 Boundary의 Stress Singularity를 고려한 함수를 선택하면 훨씬 효과적일 것으로 판단되었다.

Abstract

General formulation to analyse the rectangular thin plate on a soil medium by energy method is developed. In the problem, Boussinesque's formular needs to be integrated after asssuming the contact stress distribution. Two different functions, i.e., power series and Chebychev polynomials are used to approximate the contact stress distribution. It was found that Chebychev polynomials are better function to describe the contact stress than power series. Chebychev polynomials considering stress singularity around plate boundary is recommended as the desirable shape function for future research.

Introduction

The analysis of a thin plate on a soil medium is a typical soil-structure interaction problem drawn the attention of both structural and geotechnical engineers. Numerical technique such as finite element method is a tool widely used to solve such problems. But this method would be expensive since 3-D elements are required to model the system properly. As an alternative way, energy method can be used for the problem.

To apply the energy method for the soil-plate interaction problem, choosing an ap-

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propriate function describing shape of the stress distribution between the plate and soil medium is a key in the success the energy method.

In this study, general formulation is developed to predict the flexural behavior of a rectangular thin plate resting on an elastic soil medium by energy method using two different shape functions, i.e., power series polynomial and Chebychev polynomials. Numerical results of these two methods are compared and discussed.

Analytical Formulations

The deformation function, $w(x, y)$ of the soil surface, which also represents the deflection function of the plate, is related to the contact stress distribution, $q(x, y)$, by the Boussinesque's formular(1) as follows :

$$w(x,y) = \frac{(1-\nu_s)}{2\pi G_s}\int_{-a}^{a}\int_{-b}^{b}\frac{q(\xi,\eta)}{[(\xi-x)^2+(\eta-y)^2]^{1/2}}d\xi d\eta \tag{1}$$

where, ν_s is the poisson's ration, G_s is the shear modulus of the soil medium and (ξ, η) is the point where a force is applied. Therefore, if we assume a contact distribution properly, the deflection of the plate and the total potential energy of the system can be determined. The problem is what function will make the Eq.1 be integrable for $w(x, y)$ and fit the stress distribution properly. Generally power series polynomial is used as a shape function in this kind of problem. In this study formulation using power series polynomial and chebychev polynomial as the contact stress distribution will be made and discussed.

If the plate of a size $2a \times 2b \times t$ is subjected to a uniform load of intensity p and rests on a soil medium, the contact stress distribution can be assumed as an even power series function of a spatial coordinates, x and y as follows :

$$q(x,y) = p\sum_{i,j=0}^{n} B'_{ij}x^{2i}y^{2j} \tag{2}$$

where, n is an arbitrary integer and B'_{ij}'s are the unknown coefficients. The even power is assumed because of the symmetricity of the problem about x and y axis.

Substituting Eq.2 to Eq.1 reduces

$$w(x,y) = \frac{(1-\nu_s)p}{2\pi G_s}\sum_{i=0}^{n}\sum_{j=0}^{n} B'_{ij}I_{ij} \tag{3}$$

where,

$$I_{ij} = \int_{-a}^{a}\int_{-b}^{b}\frac{\xi^{2i}\eta^{2j}}{[(\xi-x)^2+(\eta-y)^2]^{1/2}}d\xi d\eta$$

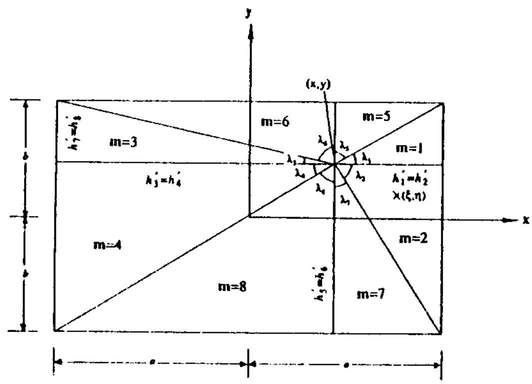


Fig.1 Plate Division Scheme for Integration

Using the polar coordinates and dividing the area as shown in Fig.1, Eq.3 can be integrated and the resultsing expression for $w(x, y)$ will take the following form(In the expression the spatial coordinates x and y are normalized to X and Y as $X = x/a$ and $Y = y/b$ respectively.) (2) :

$$q(X,Y) = p\sum_{i=0}^{n}\sum_{j=0}^{n} B_{ij}X^{2i}Y^{2j} \tag{4}$$

$$w(X,Y) = K\sum_{i=0}^{n}\sum_{j=0}^{n} B_{ij}H_{ij}(X,Y) \tag{5}$$

where,

$$B_{ij} = a^{2i}b^{2j}B'_{ij}$$

$$K = \frac{(1-\nu_s)bp}{2\pi G_s}$$

$$H_{ij}(X,Y) = \sum_{s=0}^{2i}\sum_{t=0}^{2j} C_{ijst}X^{2i-s}Y^{2j-t}\left[\beta^{t+1}\left(h_1^{s+t+1}I_t^1 + h_2^{s+t+1}(-1)^t I_t^2 + h_3^{s+t+1}(-1)^s I_t^3 + h_4^{s+t+1}(-1)^{s+t}I_t^4\right) + \chi^s\left(h_5^{s+t+1}I_s^5 + h_6^{s+t+1}(-1)^s I_s^6 + h_7^{s+t+1}(-1)^t I_s^7 + h_8^{s+t+1}(-1)^{s+t}I_s^8\right)\right]$$

$$\beta = a/b, \quad \chi = b/a$$

$$C_{ijst} = \frac{\binom{2i}{s}\binom{2j}{t}}{s+t+1}, \quad \binom{i}{s} = \frac{i!}{s!(i-s)!}$$

$$h_1 = h_2 = 1-X, \quad h_3 = h_4 = 1+X, \quad h_5 = h_6 = 1-Y, \quad h_7 = h_8 = 1+Y$$

$$I_t^m = \int_0^{u_m}\frac{u^t}{\sqrt{1+u^2}}du, \quad m = 1,2,3,\ldots 8$$

$$u_1 = \chi\frac{h_5}{h_1} = \frac{1}{u_5}, \quad u_2 = \chi\frac{h_7}{h_1} = \frac{1}{u_7}, \quad u_3 = \chi\frac{h_5}{h_3} = \frac{1}{u_6}, \quad u_4 = \chi\frac{h_7}{h_3} = \frac{1}{u_8}$$

To evaluate the strain energy stored in the plate system and to apply the boundary condition properly, Eq.5 needs to be easily differentiable. Eq.5 is so complicated that it is hardly differentiable. It needs to be expressed in an easily differentiable form. This can be achieved by approximating the function $H_{ij}(X,Y)$ be Chebychev polynomials(3).

After the approximation and simplification, Eq.5 can be written in the following form :

$$\begin{aligned} w(X,Y) &= K\sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{r=0}^{n}\sum_{s=0}^{n} D_{ijrs}B_{rs}X^{2i}Y^{2j} \\ &= K\sum_{i=0}^{n}\sum_{j=0}^{n} A_{ij}X^{2i}Y^{2j} \end{aligned} \tag{6}$$

where,

$$A_{ij} = \sum_{r=0}^{n}\sum_{s=0}^{n} D_{ijrs}B_{rs} = [D][B]$$

$$D_{ijrs} = \frac{1}{ND^2}\sum_{u=i}^{n}\sum_{v=j}^{n} C_{uv}t_{2u}^{u-i}t_{2v}^{v-j}\sum_{k=1}^{ND}\sum_{l=1}^{ND} T_{2u}(\gamma_k)T_{2v}(\gamma_l)\phi_u\phi_v H_{rs}(\gamma_u,\gamma_l)$$

$$C_{ij} = \sum_{r=0}^{n}\sum_{s=0}^{n} B_{rs}\sum_{k=1}^{ND}\sum_{l=1}^{ND} T_{2i}(\gamma_k)T_{2j}(\gamma_l)H_{rs}(\gamma_k,\gamma_l)$$

$$T_n(x) = \frac{1}{2}\sum_{k=0}^{[n/2]} t_{n-2k}^k x^{n-2k}$$

$$t_m^k = 2^m(-1)^k\frac{m-2k}{m-k}\frac{(m+k)!}{m!k!}$$

where, $[n/2]$ is $n/2$ if n is even and $(n-1)/2$ if n is odd, and $\phi_i = 1$ if $i \neq 0$ and $\phi_0 = \frac{1}{2}$.

Now the stress contact distribution function and deflection fuction can be differentiated as needed.

Boundary Condition

Along the edges of the plate, bending and twisting moments and the vertical shear force must vanish. The conditions can be written as follows(4) :

1. Zero moment conditions :

$$[M_x]_{x=\pm a} = -D_p\left[\frac{\partial^2 w}{\partial x^2} - \nu_p\frac{\partial^2 w}{\partial y^2}\right]_{x=\pm a} = 0$$

$$[M_y]_{y=\pm b} = -D_p\left[\frac{\partial^2 w}{\partial y^2} + \nu_p\frac{\partial^2 w}{\partial x^2}\right]_{y=\pm b} = 0$$

2. Zero shear force conditions :

$$[V_x]_{x=\pm a} = -D_p\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu_p)\frac{\partial^3 w}{\partial x\partial y^2}\right]_{x=\pm a} = 0$$

$$[V_y]_{y=\pm b} = -D_p\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu_p)\frac{\partial^3 w}{\partial x^2\partial y}\right]_{y=\pm b} = 0$$

where, $D_p = \frac{E_p t_p^3}{12(1-\nu_p)}$ represents the flexural rigidity of the plate. Taking the partial derivative of the function given in Eq.6 and substituting them into the boundary condition, gives $4(n+1)$ equations. They can be written in a matrix form as follows :

$$[W][A] = 0 \tag{7}$$

where, $[A]$ is the unknown coefficient matrix of size $N^3 \times 1$, i.e., $[A] = [A_{00}, A_{01}, \cdots A_{0n}, A_{11}, \cdots A_{nn}]^T$. The elements of $[W]$ are functions of i, j, ν_p, β, χ.

One important point to be noted here is that the minimum number of terms, i.e., n(or N), to be considered to satisfy the prescribed boundary conditions should be greater than the number of boundary conditions, that is the relationship $N=n+1 \geq 5$, must be satisfied. Eq.7 represents $4N$ number of linear simultaneous equations with N^2 number of unknown coefficients (generalized coordinates). Using tthese relationships, $4N$ number of generalized coordinates can be eliminated by expressing them in terms of the remaining N^2-4N number of unknowns (independent generalized coordinates). Therefore, reducing the number of unknowns and retaining the independent coordinates according to Eq.8 can be regarded as imposing the boundary conditions. Eq.7 can be written as follows ; $[W][D][B]=0$, where, $[WD]=[W][D]$. It can be partitioned as

$$[[WD_1][WD_2]]\begin{bmatrix}[B_1]\\ [B_2]\end{bmatrix} = 0$$

It gives the relations ;

$$[B_1] = [T][B_2], \qquad [T] = -[WD_1]^{-1}[WD_2] \tag{8}$$

which represents that $4N$ number of unknowns can be written by the N^2-4N number of unknowns. By this operation total number of unknowns reduces from N^2-2N and $[B_2]$ becomes the independent generalized coordinates.

Also to reduce the number of indices from two to one, Eq.4 will be rewritten as follows :

$$\frac{q(X,Y)}{p} = B_1 ZQ(1) + \cdots + B_N ZQ(N^2)$$

where $ZQ(IK)=X^{2i}Y^{2j}$ in which i and j are related to IK by the formula ; $IK=iN+j+1$ for $0 \leq i \leq N-1$ and $0 \leq j \leq N-1$. Considering Eq.8, it can be written as

$$\frac{q(X,Y)}{p} = \sum_{I_i=1}^{N^2-4N} G(I_i)B_{4N+I_i} \tag{9}$$

where,

$$G(I_i) = \sum_{k=1}^{4N}((ZQ(k)T(k,I_i) + ZQ(4N+I_i))$$

where $T(k, I_i)$ is the elelent at k-th row and I_i-th column of the transformation matrix $[T]$ in Eq.8. Similarly, $w(X, Y)$ can be written in terms of the independent generalized coordinates. For this purpose we define a new vector ZW as follows :

$$ZW(II) = \sum_{r=0}^{n}\sum_{s=0}^{n} D_{rsij}X^{2r}Y^{2s} = \sum_{I_k=1}^{n^2} D(I_k.II)X^{2r}Y^{2S}$$

where, II varies form 1 to $4N$, and r and s are related to I_k by the formula, $I_k=rN+s+1$. Then $w(X, Y)$ can be expressed as follows :

$$\frac{w(X,Y)}{K} = \sum_{I_i=1}^{N^2-4N} F(I_i)B_{4N+I_i} \tag{10}$$

where

$$F(I_i) = \sum_{I_i=1}^{N^2-4N}(ZW(k)T(k.I_i) + ZW(4N+I_i))$$

and $T(K, I_i)$ was defined previously.

Minimization of Total Potential Energy

The total potential energy functional, U_T, of the plate-soil medium system is composed of three contributions, i.e., strain energy stored in the plate due to bending, strain energy stored in the soil medium and work done by externally applied load. Each of these are given by the following expressions :

1. The strain energy stored in the plate, U_p :

$$U_p = \frac{D_p}{2}\int_{-1}^{1}\int_{-1}^{1}\left[\frac{1}{a^3b^3}\left(b^4w_{XX}^2 + a^4w_{YY}^2 + 2a^2b^2w_{XX}w_{YY}\right) + 2(1-\nu_p)\frac{1}{ab}\left(w_{XY}^2 - w_{XX}w_{YY}\right)\right]dXdY$$

where, w_{XX} is the second derivative of w with respect to X, x_{YY} is the second derivative of w with respect to Y, w_{XY} is the partial derivative of w with respect to X and Y respectively.

2. The strain energy stored in the soil medium, U_s is equal to the work done by the contact stress at the plate-soil medium interface.

$$U_s = \frac{1}{2}ab\int_{-1}^{1}\int_{-1}^{1} q(X,Y)w(X,Y)dXdY$$

3. The work done by the externally applied load p, U_ℓ is given by :

$$U_l = -pab\int_{-1}^{1}\int_{-1}^{1} w(X,Y)dXdY$$

Differentiating Eq.10 properly and substituting them into the potential energy terms, total potential energy terms will be written in terms of the N^2-4N number of independent coordinates. To find the unknowns, set eqaul to zero the first derivative, with respect to each of the independent generalized coordinates, i.e.,

$$\frac{\partial U_t}{\partial B(I_i)} = 0 \qquad (11)$$

This will give the following equation :

$$[X][B] = [F] \qquad (12)$$

$$\begin{aligned}
X(I_i, I_j) = {} & \frac{D_p bK^2}{a^3}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16rl(2r-1)(2l-1)}{(2(r+l)-3)(2(m+s)+1)} \\
& + \frac{D_p\nu_p K^2}{ab}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16ls(2s-1)(2l-1)}{(2(r+l)-1)(2(m+s)-1)} \\
& + \frac{D_p aK^2}{b^3}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16ms(2s-1)(2m-1)}{(2(r+l)-1)(2(m+s)-3)} \\
& + \frac{D_p\nu_p K^2}{ab}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16rm(2r-1)(2m-1)}{(2(r+l)-1)(2(m+s)-1)} \\
& + 4(1-\nu_p)\frac{K^2}{ab}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{64rsim}{(2(r+l)-1)(2(m+s)-1)} \\
& + \frac{1}{2}pKab\sum_{i=1}^{N^2} CRS(i,I_i)\left[\frac{4}{(2(r+i_1)-1)(2(s+j_1)+1)}\right. \\
& \qquad \left. + \sum_{k=1}^{4} NT(k,I_j)\frac{4}{(2(r-i_2)+1)((2(s-j_2)+1)}\right] \\
& + \frac{1}{2}pKab\sum_{i=1}^{N^2} CRS(i,I_j)\left[\frac{4}{(2(r+i_3)+1)(2(s-j_3)+1)}\right. \\
& \qquad \left. + \sum_{k=1}^{4} NT(k,I_i)\frac{4}{(2(r-i_2)-1)((2(s+j_3)+1)}\right]
\end{aligned}$$

where,

$$CRS(i,I_i) = D(i,4N+I_i) + \sum_{k=1}^{4N} T(k,I_i)D(i,k)$$

and r, s, l, m, i_1, j_1, i_2, j_2, i_3, and j_3, are integrers, which are obtained from the following relationships :

$$i = rN + s + 1$$

$$j = lN + m + 1$$

$$4N + I_i = i_1N + j_1 + 1$$

$$k = i_2N - j_2 + 1$$

$$4N + I_j = i_3N + j_3 + 1$$

and

$$F(I_i) = \sum_{i=1}^{N^2} CRS(i,I_i)\frac{4}{(2r+1)(2s+1)}$$

Now the independent generalized coordinates can be obtained from.

$$[B] = [X]^{-1}[F]$$

Finally, substituting the coefficient, $[B]$ into Eq.9 and Eq.10, the contact stress distribution and the deflection of the plate can be evaluated.

Formulation Using Chebychev Polynomials for the Contact Stress

To find the better shape function for the contact stress distribution, analytical formulation is made for the same problem when the contact stress distribution itself is approximated by Chebychev polynomials. This time the contact stress distribution will be as follows :

$$q(X,Y) = p\sum_{i=0}^{n}\sum_{j=0}^{n} B_{ij}T_{2i}(X)T_{2j}(Y) \qquad (13)$$

Exactly the same procedure as before will be taken and the elements of the matrix $[X]$ in Eq.12 changes as shown in Appendix B.

$$\begin{aligned}
X(I_i,I_j) = {} & \frac{D_p b K^2}{a^3}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16rl(2r-1)(2l-1)}{(2(r+l)-3)(2(m+s)+1)} \\
& + \frac{D_p \nu_p K^2}{ab}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16ls(2s-1)(2l-1)}{(2(r+l)-1)(2(m+s)-1)} \\
& + \frac{D_p a K^2}{b^3}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16ms(2s-1)(2m-1)}{(2(r+l)+1)(2(m+s)-3)} \\
& + \frac{D_p \nu_p K^2}{ab}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{16rm(2r-1)(2m-1)}{(2(r+l)-1)(2(m+s)-1)} \\
& + 4(1-\nu_p)\frac{K^2}{ab}\sum_{i=1}^{N^2}\sum_{j=1}^{N^2} CRS(i,I_i)CRS(j,I_j)\frac{64rslm}{(2(r+l)-1)(2(m+s)-1)} \\
& + \frac{1}{2}pKab\sum_{i=1}^{N^2} CRS(i,I_i)\left[\sum_{I_k=0}^{i_1}\sum_{I_l=0}^{j_1} t_{2I_k}^{i_1-I_k} t_{2I_l}^{j_1-I_l}\frac{1}{(2(I_k+r)+1)(2(I_l+s)+1)}\right. \\
& \left. + \sum_{i=1}^{4N} T(k,I_j)\sum_{I_k=0}^{i_2}\sum_{I_l=0}^{j_2} t_{2I_k}^{i_2-I_k} t_{2I_l}^{j_2-I_l}\frac{1}{(2(I_k+r)+1)(2(I_l+s)+1)}\right] \\
& + \frac{1}{2}pKab\sum_{i=1}^{N^2} CRS(i,I_j)\left[\sum_{I_k=0}^{i_1}\sum_{I_l=0}^{j_1} t_{2I_k}^{i_1-I_k} t_{2I_l}^{j_1-I_l}\frac{1}{(2(I_k+r)+1)(2(I_l+s)+1)}\right. \\
& \left. + \sum_{i=1}^{4N} T(k,I_i)\sum_{I_k=0}^{i_2}\sum_{I_l=0}^{j_2} t_{2I_k}^{i_2-I_k} t_{2I_l}^{j_2-I_l}\frac{1}{(2(I_k+r)+1)(2(I_l+s)+1)}\right]
\end{aligned}$$

The elements of matrix $[F]$ will remain the same.

Numerical Results and Discussions

The analytical formulation presented so far is numerically investigated for an isolate square plate foundation resting on an elastic soil medium. The plate is assumed to be subjected to an uniform load and thin plate theory is used. The results thus obtained are compared with the results by a three dmensional finite element solution using SAP Ⅳ(5). Also, the flexural moments obtained are compaed with the results reported by Gorbunov-Posadov(6) and Zamman and Faruque(7).

$$K_g = \frac{12\pi(1-\nu_p^2)}{(1-\nu_s)}\frac{E_s}{E_p}\left(\frac{a}{t_p}\right)^2\left(\frac{b}{t_p}\right)^2$$

where, a, b and t_p represent the dimensions of the plate, E is the modulus of elasticity, ν is the poisson's ratio and subscrips p and s represent the quantities pertaining to plate and the soil medium, respectively. In addition, the plate deflection, $w(X, Y)$, the contact distribution, $q(X, Y)$, and the flexural moments of the plate, M_X and M_Y along the X and Y axis are nondimensionalized as follows :

$$\overline{w}(X,Y) = \frac{E_s}{pa(1-\nu_s)}w(X,Y)$$

$$\overline{q}(X,Y) = \frac{q(X,Y)}{p}$$

$$\overline{M}_X(X,Y) = M_X(X,Y)$$

$$\overline{M}_Y(X,Y) = M_Y(X,Y)$$

A square plate ($a/b=1$) in smooth contact on an isotropic elastic soil medium is selected as an example. The plate is subjected to a uniform load of $q=1psi$. This is solved by finite element method using SAP Ⅳ, and using energy method assuming a power series to approximate the contact stress distribution. In the finite element method, three-dimensional eight-noded isoparametric element with three translational degree of freedom per node is used. Due to the symmetricity of the problem, only one quater part of the plate is modeled using the finite elements(Fig.2). The effective zone of the soil medium considered is bounded by 5 times th plate dimension in the z direction and 1.5 times the plate dimension in the x and y direction, respectively(Fig.3). This boundary is consistent with the values reported by Boussinesque and Westergaad. The nodes of the planes of symmetry and those on the planes defining the boundary of the soil medium are constrained such that no translation normal to these planes occurs. Nondimensional plate deflection and the contact stress distribution are compared with the results obtained by the three-dimensional finite

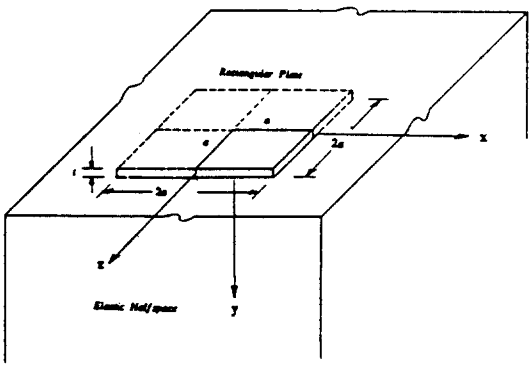


Fig.2 Square Plate Analysed

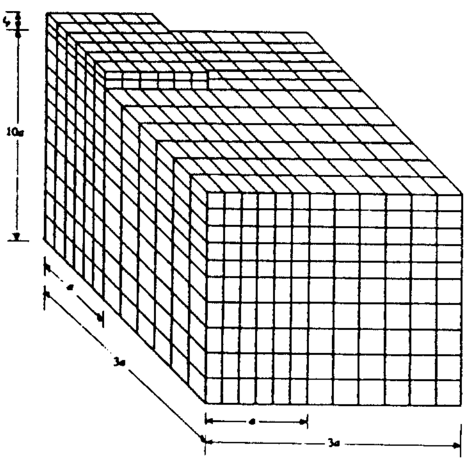


Fig.3 Finite Element Mesh Used for the Square Plate Problem

element analysis. Flexural moments are compared only with two results reported by Zaman and Faruque and Gorbunov-Posadov.

A comparison of the non-dimensional plate deflection for $K_g = 10$ is shown in Fig.4. It is observed that the deflection obtained by the method developed in this study does not change significantly as the number of terms in the power series are increased from 4 to 5. Also the present analysis predicted the deflection at the plate center which was 4.5% less compared to the results obtained by the finite element nalysis. In Fig.5 the non-dimensional flexural moments for a plate with $K_g = 10$ are compared with the existing solutions. The results obtained for the moment at the plate center by the method developed in this study is 29.7% higher than the value reported by Gorbunov-Posadov and 12.2% higher than the value reported by Zaman and Faruk. But the boundary moment predicted in this study converges to zero, as they should, but those reported by Zaman and Faruk did not do so. Fig.6 shows the contact stress distribution obtained by the power series approximation when $n = 4$ and $n = 5$. In this figure the results are compared with those obtained by the

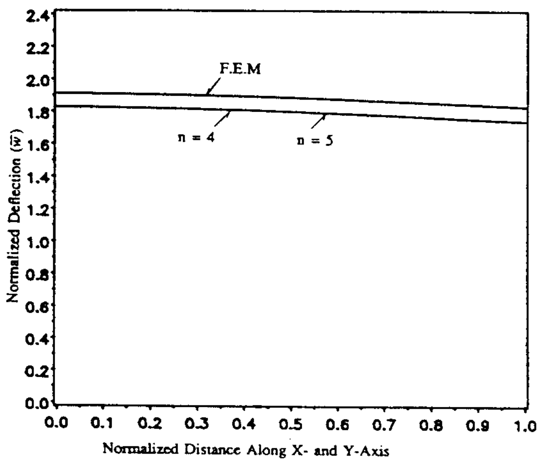


Fig.4 Comparison of the Plate Deflection Variation Obtained Using the Power Series Approximation an That Reporte in the Literature for $K_g = 10$

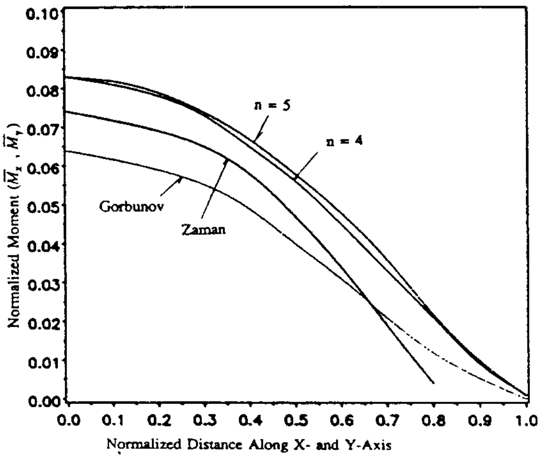


Fig.5 Comparison of the Moment Variation Obtained Using the Power Series Approximation and That Reported in the Literature for $K_g = 10$

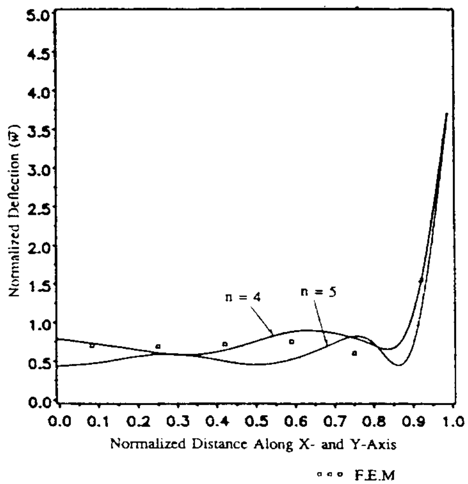
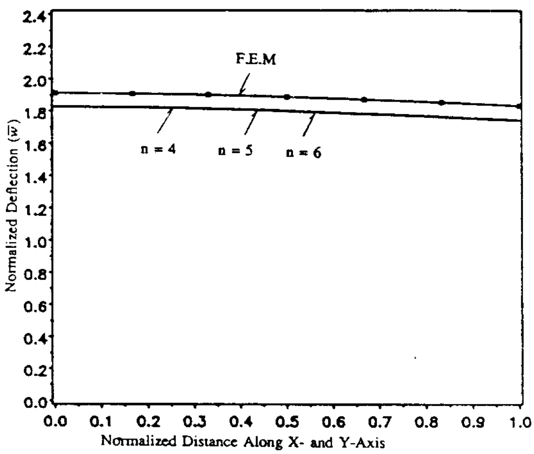


Fig.6 Contact Stress Distribution Obtained Using the Power Series Approximation and the Finite Element Analysis for $K_g = 10$

three-dimensional finite element analysis. When $n = 5$, the matrix $[X]$ in Eq.12 was found to be ill-conditioned making it difficult to invert it. Because of this trouble, contact stress distribution in Fig.6 shows some wave form which should not be so.

Fig.7 shows the deflection of the plate obtained by the formulation using the Chebychev polynomials for the contact stress distribution. It is found that the deflection does not change significantly as n increases beyond 4. The variance of the flexural moment for $K_g = 10$ along the x and y axis obtained by using the Chebychev polynomial approximation is compared with the results reported by Zaman and Faruque and Gorubnov-Posadov in Fig.8. As shown in this figure, the results obtained by the Chebychev approximation for $n = 4$, 5, 6 more or less overlap each other, showing that flexural moments.predicted do not change significantly as n is increased beyond 4. Fig.9 shows the contact stress distribution obtained by using the Chebychev approximation along with the results obtained by three-dimensional finite element analysis. From the figure, it can be seen that the variation of the contact stress predicted near the center of the plate by using the Chebychev polynomials does not change significantly as n is increased, whereas, the variation of the contact stress gradient near the boundary increases rapidly as n is increased, which implies that as the number of the Chebychev polynomial terms is increased, better prediction for the contact stress distrbution can be expected.



Fing.7 Plate Deflection Obtained Using Chebyshev Polynomial and the Finite Element Analysis for $K_g = 10$

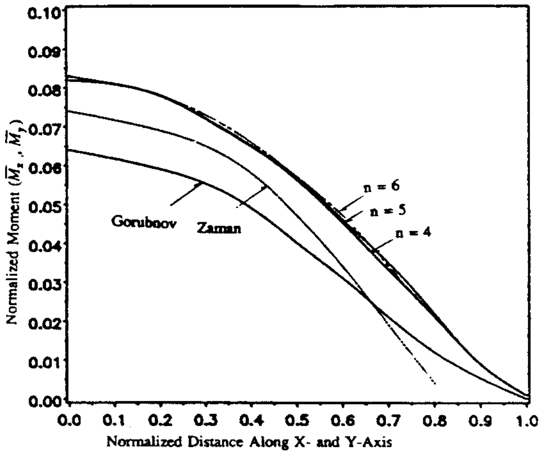


Fig.8 Comparison oth the Moment Variation Obtained Using the Chebyshev Polynomial and That Reported in the Literature for $K_g = 10$

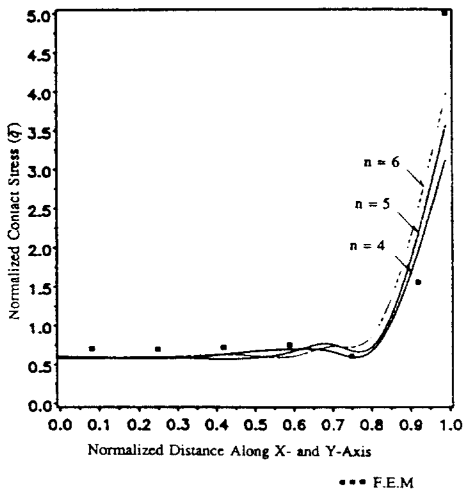


Fig.9 Comparison of the Contact Stress Distr. Obtained Using the Chebyshev Polynomial and the Finite' Element Analysis for $K_g = 10$

Conclusions and Recommendations

From the results presented, it can be concluded that the simple power series polynomial does not give a good representation of the contact stress distribution. The Chebychev polynomial approximation for the contact stress distribution results in a better control over the solution, making it possible to invert the matrix $[X]$ in Eq.12 up to the case when the number of the polynomial terms, *n*, is equal to 6. On the other hand, the simple power series approximation results in an ill conditioned matrix $[X]$ when the number of the polynomial terms, n, is increased beyond 4. This address the importance of selecting a proper shape function to approximate the contact stress distribution in the plate-soil interaction problem. Finally it is recommended that the contact stress distribution $q(X, Y)$, be approximated by a new function which incorporates the stress singularity terms around the boundary of the plate. It can be achieved by dividing the Chebychev approximation function in Eq.13 by $(\sqrt{1-X^2}\sqrt{1-y^2})$.

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