좌굴을 고려한 원통쉘 보강재의 최적설계에 대하여

Optimum Design of Stiffeners in the Stiffened Cylindrical Shells based on Structural Stability

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요 약

Ring과 Stringer로 보강된 원통형 Shell이 길이 방향 압축력과 횡압력을 받을 경우의 국부 및 전체 좌굴강도를 효율적으로 해석하고, 최적보강재의 치수를 설계하는 방법을 제안했다. 즉, 보강재의 이산성을 고려하고 각 보강재 설치 방식에 따라 변위함수를 적절히 선정하여 좌굴 Mode를 조사함으로써 국부 및 전체좌굴 현상의 규명이 가능함을 밝혔다. 또한 국부좌굴 및 전체좌굴이 동시에 일어나는 조건으로부터 최적 보강재의 치수를 결정할 수 있음을 보였다.

Abstract

An efficient approach to the buckling analysis of stiffened cylndrical shells with rings and stringers under the axial and the lateral pressure loadings is presented. By this approach, the local buckling as well as overall buckling behavior has been investigated considering the discretness of stiffeners and appropriate adoption of displacement functions. Some design criteria based on structural stability to determine optimum scantlings of stiffeners are also suggested. It is shown that the optimum scantlings of stiffeners can be designed from the condition of equal local and overall buckling strength.

1. Introduction

Cylindrical shell structures have been widely used because of their light weight and effective enhancement of stiffness due to the geometric characteristics. But they frequently suffer from lacking in structural stability. To efficiently complement this weakness, the strengths such as membrane strength and bending strength have been usually increased by the longitudinal stiffners (stringers) and the circumferential stiffeners (rings).

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이 논문에 대한 토론을 1994년 3월 30일까지 본학회에 보내주시면 1994년 9월호에 그 결과를 게재하겠습니다.

For optimum design of cylindrical shell structures based on structural stability, the local and overall buckling analysis with the effect of discreteness of stiffeners will be needed. Yang et.al.[1] has treated stiffened cylindrical shells by means of energy method, but didn't consider the discreteness of stiffeners sufficiently. Subbiah et.al.[2] has performed the calculation of shell stiffened only with rings considering the discreteness of stiffeners. He calculated the overall and local buckling load by the combination of F.E.M. and analytical method. Kunoo[3] carried out the minimum weight design of cylindrical shell with multiple stiffener size under buckling constraints, which needed tremendous computational efforts and cost.

In this paper, the discreteness of stiffeners was considered by using the energy method, Trefftz criteria, in the model stiffened with rings or stringers. To save the computing time, the displacement functions were expanded in Fourier series. The local and overall buckling mode could be expressed by adopting appropriate displacement functions in the longitudinal or circumferential direction. And the effect of size of stiffeners on overall and local buckling was also investigated.

The result obtained above gives the important criteria to design the optimum stiffener size. Consequently, in a given shell scantling, the optimum stiffener size based on structural stability can be easily designed by the present approach.

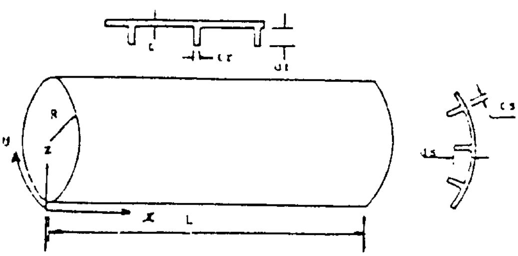


Fig.1 Geometry and coordinate system of a stiffened cylindrical shell.

2. Basic Formulation

The geometry and coordinate system of a stiffened cylindrical shell are shown in Fig.1. Let u, v and w be the displacement components in x, θ, and z directions respectively. The kinematic relations between the displacements and strain components in the middle plane are

$$\varepsilon_x = u_{,x} + \frac{1}{2}\beta_x^2$$

$$\varepsilon_\theta = \frac{1}{r}(v_{,\theta} - w) + \frac{1}{2}\beta_\theta^2$$

$$\gamma_{x\theta} = (\frac{1}{r}u_{,\theta} + v_{,x}) + \beta_x \beta_\theta \qquad (1)$$

$$\beta_x = w_{,x}$$

$$\beta_\theta = \frac{1}{r}(v + w_{,\theta})$$

The strain and curvature component at the distance z from middle plane are

$$\overline{\varepsilon_x} = \varepsilon_x - z\kappa_x \qquad \kappa = \beta_{x,x}$$

$$\kappa_\theta = \frac{1}{r}\beta_{\theta,\theta}$$

$$\overline{\varepsilon_\theta} = \varepsilon_\theta - z\kappa_\theta \qquad (2)$$

$$\overline{\gamma_{x\theta}} = \gamma_{x\theta} - 2z\kappa_{x\theta}$$

$$\kappa_{x\theta} = \frac{1}{2}(\frac{1}{r}\beta_{x,\theta} + \beta_{\theta,x})$$

Letting the symbols σ_x, σ_θ and $\tau_{x\theta}$ denote the corresponding normal and shear stress component, the total potential energy of the cylindrical shell stiffened with stringers and rings is given as follows :

$$\Pi = U_1 + U_2 + U_3 + \Omega \qquad (3)$$

where U_1, U_2 and U_3 are the strain energy of shell skin, stringers and rings respectively, and Ω is the potential energy of external loads on the shell.

$$U_1 = \frac{C}{2}\int_0^L\int_0^{2\pi}(\varepsilon_x^2 + \varepsilon_\theta^2 + 2\gamma\varepsilon_x\varepsilon_\theta + \frac{1-\nu}{2}\gamma_{x\theta}^2)\, r d\theta dx + \frac{D}{2}\int_0^L\int_0^{2\pi} (\kappa_x^2 + \kappa_\theta^2 + 2\nu\kappa_x\kappa_\theta + 2(1-\nu)\kappa_{x\theta}^2)\, r\, d\theta\, dx$$

$$U_2 = \frac{1}{2}\sum_{k=1}^{n_s}\int_0^L [\int_{A_{sk}} E_{sk}\, \overline{\varepsilon_x}^2 dA_{sk} + \frac{G_{sk}J_{sk}}{r^2} w_{,x\theta}^2]_{\theta=\theta_k} dx$$

$$U_3 = \frac{1}{2}\sum_{K=1}^{n_r}\int_0^L [\int_{A_{rk}} E_{rk}\, \overline{\varepsilon_\theta}^2 dA_{rk} + \frac{G_{rk}J_{rk}}{r^2} w_{,x\theta}^2]_{x=x_k}\, r\, d\theta \qquad (4)$$

where

$$C = \frac{E\,t}{2(1-\nu^2)}, \quad D = \frac{E\,t^3}{12(1-\nu^2)}$$

E,A and GJ are The Young's modulus, the cross-sectional area, the torsional rigidity and the subscripts s and and r are used to denote the stringers and the rings respectively.

The potential energy has been considered for 3 different loadings. One is the uniform axial loading in the longitudinal direction and the other is the hydrostatic pressure loading around the circumferential direction.

The potential energy for an axial loading case is

$$\Omega_L = \int_0^L\int_0^{2\pi}(\sigma_x t)u_{,x} r\, d\theta\, dx + \sum_{k=1}^{n_s}\int_0^L [\int_{A_{sk}} u_{,x}\, dA_{sk}]_{\theta=\theta_k} dx \qquad (5)$$

and the potential energy of fluid-pressure loading case considering following force effect [4] is

$$\Omega_{Fluid-Pressure} = pr\int_0^L\int_0^{2\pi}[-w + \frac{1}{2r}(v + wv_{,\theta} - vw_{,\theta} + w^2)]\, d\theta\, dx \qquad (6)$$

Now let us apply the Trefftz criteria such that the buckling stress can be calculated when the second variation of the total potential energy in Eq.(3) has an extreme value, that is $\delta(\delta^2\Pi)=0$.

To investigate the possible existence of adjacent equilibrium configurations u, v and w, we give small perturbations u_1, v_2 and w_1 due to buckling to the prebuckling deformations u_0, v_0 and w_0.

$$u = u_0 + u_1$$
$$v = v_0 + v_1 \qquad (7)$$
$$w = w_0 + w_1$$

The total potential energy Π is obtained by substituting Eq.(1) into Eq.(3).

In the equation of Π obtained just above, the second order terms of u_1, v_1 and w_1 in product with u_0, v_0 and w_0 are arranged to express the stresses of the state. These stresses are σ_x, σ_θ and $\tau_{x\theta}$ as the stresses of shell skin, and σ_{xs} and σ

$\sigma_{\theta r}$ as the stresses of the stringer and the ring, respectively. Since the torsional loading is not included in this paper, the term of $\tau_{x\theta}$ is omitted.

The boundary condition is a simple support at each end. Displacement functions are assumed to meet this boundary condition.

$$u = \sum_{m=1}^{k_m} \sum_{n=1}^{k_n} U_{mn} \sin n\theta \cos \frac{m\pi x}{L}$$
$$v = \sum_{m=1}^{k_m} \sum_{n=1}^{k_n} V_{mn} \cos n\theta \sin \frac{m\pi x}{L} \qquad (8)$$
$$w = \sum_{m=1}^{k_m} \sum_{n=1}^{k_n} W_{mn} \sin n\theta \sin \frac{m\pi x}{L}$$

These displacement functions are substituted into $\delta^2 \Pi$ and the Trefftz criteria $\delta(\delta^2 \Pi = 0)$ are obtained in Eq.(9) by differentiating $\delta^2 \Pi$ with respect to U_{mn}, V_{mn} and W_{mn}.

$$\frac{\partial}{\partial U_{mn}} (\delta^2 \Pi) = 0$$
$$\frac{\partial}{\partial V_{mn}} (\delta^2 \Pi) = 0 \qquad (9)$$
$$\frac{\partial}{\partial W_{mn}} (\delta^2 \Pi) = 0$$

where $m = 1, \cdots\cdots, K_m$ and $n = 1, \cdots\cdots, K_n$

Eq.(9) can be rewritten in matrix form as Eq.(10) of eigen value problem where $[C_{ij}]$ denotes the stiffness matrix and $[D_{ij}]$ denotes the geometric stiffness matrix which contains initial stresses σ_x, σ_{xs}, σ_θ, $\sigma_{\theta r}$ and external loading.

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{bmatrix} \qquad (10)$$

To solve above eigen value problem, it is neccessary to know the initial stresses under given external loading.

The problem is how the unknowns, σ_x, σ_{xs}, σ_θ, $\sigma_{\theta r}$ and p, in $[D_{ij}]$ should be treated. For the loading case in Fig.2, to derive the relationship between the external load, N_x and p, and internal stress, σ_x, σ_{xs}, σ_θ, $\sigma_{\theta r}$, we assume that both the shell skin and the stiffeners are subject to the uniform stress or the uniform strain. By using this assumption, all unknowns can be expressed in terms of the load per unit length N_x and hydrostatic pressure p. Consequently Eq.(10) can be converted into the eigen value problem containing the only one unknown loading parameter which yields buckling load, since the relationship between N_x and p is given in each loading case.

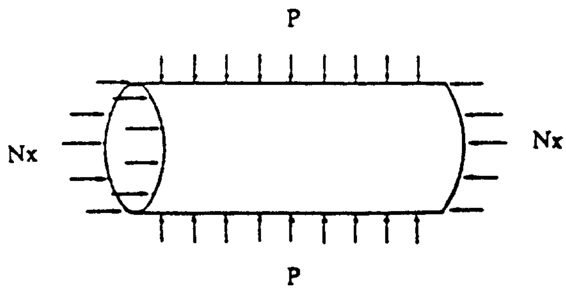


Fig.2 A cylindrical shell under combined axial load and lateral hydrostatic pressure.

3. Numerical Results

The loading cases used in this paper are classified as in Fig.3 for convenience.

1) Cylindrical Shell Stiffened with Stringers

For this structure only Case-Ⅰ was applied for the buckling analysis. The calculation was performed using the displacement functions of

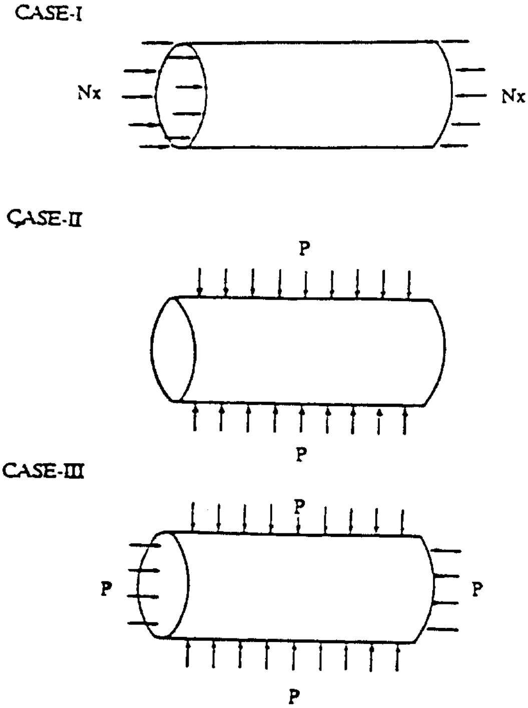


Fig.3 Typical loading cases.

several Fourier series in the direction of circumference and one term in the direction of x.

$$u = \left(\sum_{n=1}^{k_n} U_{mn} \sin n\theta \right) \cos\frac{m\pi x}{L}$$

$$v = \left(\sum_{n=1}^{k_n} V_{mn} \cos n\theta \right) \sin\frac{m\pi x}{L} \qquad (11)$$

$$w = \left(\sum_{n=1}^{k_n} W_{mn} \sin n\theta \right) \sin\frac{m\pi x}{L}$$

To find the local buckling and overall buckling, each value of m was tried from 1 to 15 with the fixed kn of 15. The values of buckling stress have the tendency of Fig.4 in accordance with changing m. In Fig.4 it can be easily found that the overall buckling occures at m_1 and the local buckling occures at m_2 which is the

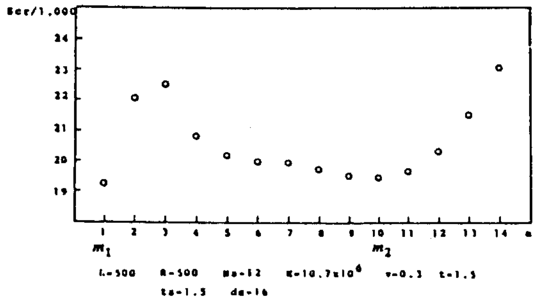


Fig.4 Effect of longitudinal wave number m on buckling strength.

second local minimum. Actually the value of m_2 slightly changes according to the scantlings of the structure. The overall and local buckling modes are shown in Fig.5. In Fig.5 the radial displacement along circumferential direction occures independently of stiffeners in the overall buckling, however, the shell skin deforms only between stiffeners in the local buckling.

To investigate the effect of stiffeners on the

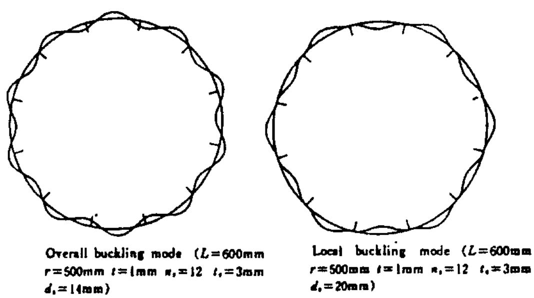


Fig.5 Buckling modes of cylindrical shell stiffened with stringers.

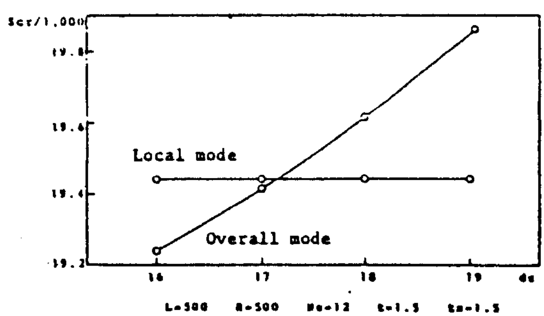


Fig.6 Overall and local buckling strength vs. stringer height.

local and overall buckling, the tendency of the change in each buckling stress was considered. These results are plotted in Fig.6. In Fig.6 the overall buckling stress increases rapidly by enlarging the depth of stiffeners, but local buckling stress hardly changes regardless of increase in the size of stiffness of the stiffeners. If the height of stringer ds is increased to the intersection value of local and overall modes, the buckling stress will increase, but there is no profit in increasing stringer height over the intersection value, for the local buckling would occur over the value.

2) Cylindrical Shell Stiffened with Rings

For this model, Case-II and Case-III were taken into account. The calculations were performed using the displacement functions of several Fourier series in the direction of x and one term in circumferential direction.

$$u = \left(\sum_{m=1}^{k_m} U_{mn} \cos\frac{m\pi x}{L} \right) \sin n\theta$$
$$v = \left(\sum_{m=1}^{k_m} V_{mn} \cos\frac{m\pi x}{L} \right) \cos n\theta \qquad (12)$$
$$w = \left(\sum_{m=1}^{k_m} W_{mn} \sin\frac{m\pi x}{L} \right) \sin n\theta$$

Differing from the former case, this case adopted the scheme that each value of n was tried from 1 to 15 with the fixed value of k_m to find the local and overall buckling. The change of the buckling stress has a similar tendency to Case-II and Case-III according to changing in the value of n. The result of Case-III is plotted in Fig.7

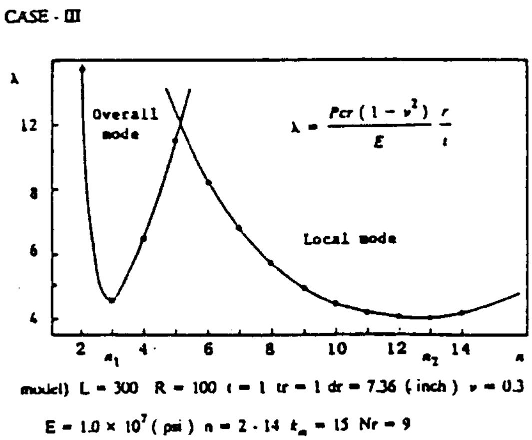


Fig.7 Effect of circumferential wave number n on buckling strength.

In Fig.7 it can be found that the overall and the local buckling occurs at n_1 and n_2 respectively. The results obtained by the Authors have been compared with Subbiah's results[2]. T-type ring stiffener was used in this calculation. This comparison is plotted in Fig.8.

As shown in Fig.8, the results of this paper give slightly higher values than Subbiah's. This difference comes from the reason that several Fourier series were assumed as dis-

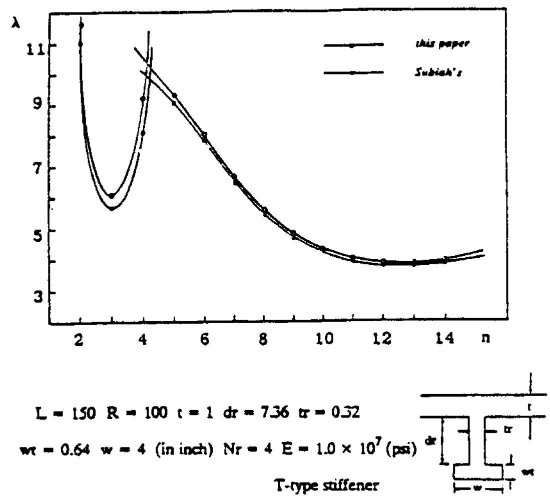
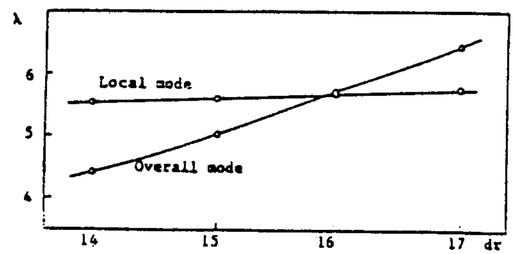


Fig.8 Comparison of buckling strength.

placement functions in this paper but Subbiah used F.E.M. assuming first-order polynomial.

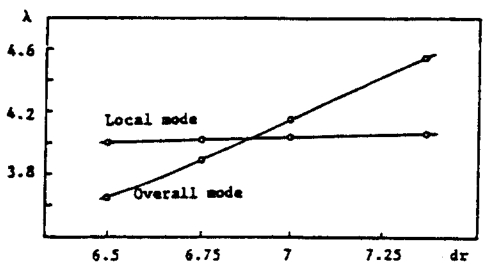
The buckling mode obtained by Subbiah seems to be closer to the real buckling mode than that of this paper. But the tendency of both results is nearly the same and the difference between the values of two results is small. By the present approach, it is possible to considerably improve the computational efficiency due to great reduction of degrees of freedom.



model) L = 300　R = 100　t = 1　dr = 14 - 17　tr = 1 (inch)　ν = 0.3
E = 1.0 × 10^7 (psi)　Nr = 9

Fig.9 Overall and local buckling strength vs. ring stiffener height(Case-Ⅱ)

The tendency of the changes in local and overall buckling according to increasing the scantling of ring stiffeners was investigated in Case-Ⅱ and Case-Ⅲ. These results are plotted in Fig.9 and Fig.10. The intersection point of



model) L = 300　R = 100　t = 1　dr = 6.5 - 7.25　tr = 1 (inch)　ν = 0.3
E = 1.0 × 10^7 (psi)　Nr = 9

Fig.10 Overall and local buckling strength vs. ring stiffener height(Case-Ⅲ)

overall and local buckling strength curves gives the optimum scantlings of ring stiffeners. Thus, it is possible to perform optimum design of stiffeners such as stringers, rings in the cylindrical shells based on structural stability.

4. Conclusions

In the cylindrical shell stiffened with stringers or rings, the local and overall buckling can be found by adopting appropriate displacement functions.

In the model stiffened with stringers, the overall and local buckling can be expressed by investigating the change of buckling load and the buckling mode according to the change of the displacement function in the longitudinal direction.

In the model stiffened with rings, the overall and local buckling can be found by changing the radial displacement function along circumferential direction like the model stiffened with stringers.

In any model stiffened with stringers or rings, the local buckling stress remains unchanged in spite of enlarging the height or thickness of stringers, but the overall buckling stress increases considerably. Consequently the local and overall buckling occur concurrently at some value of the size of stiffeners. From these results, an approach to the optimum design of stiffeners in the cylindrical shells has been suggested.

For the practical design use, further studies on the optimization with the effect of manufacturing imperfections are recommended since the buckling strength is very sensitive to the imperfections.

Acknowledgement

The Authors are very grateful to Mr. Jae Seon Yum, Department of Naval Architecture & Ocean Engineering, Seoul National University, for his assistance during the preparation of this paper.

Part of this work was supported by Seoul National University Sammi Resarch Fund.

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(접수일자 : 1992. 12. 15)