충격성잡음과 비선택적 페이딩 매체에서 한 주파수 도약 확산 대역 계통의 오류확률

Error Probability of an FHSS System in Impulsive Nonselective Fading Channels

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Abstract

In this paper, noncoherent reception performance of a slow frequency hopping spread spectrum communication system operated in channels with impulsive nois and nonselective multipath fading characteristics is investigated. For the impulsive noise, the ϵ – contaminated mixture model is used. The expressions for bit error rate as functions of channel and system parameters are obtained.

요 약

이 논문에서는 충격성 잡음과 비선택적 다중경로 페이딩 특성이 있는 매체에서 주파수 도약대역 확산 통신 계통의 성능을 살펴보았다. 충격성 잡음을 나타내는 데에는 ϵ 혼합 모델을 썼고, 통신 매체와 계통의 여러가지 매개 변수를 바꾸어 가며 이 통신 계통의 오류 확률을 수치 계산 방식으로 얻었다.

I. Introduction

The performance of FHSS communication sys tems in Gaussian noise multipath fading channels has been investigated by several authors. In [1], for example, the FHSS communication system with binary FSK (BFSK) modulation was investigated. The channel model under consideration was a noisy multipath channel with very slow fading. In [2], assuming that the channel is jammed by intentional jamme whose jamming power resource is Gaussian noise, numerical results of error rates are obtained of signal-to-jamming power ratio.

It is well-known that in some cases the Gaussian noise assumption can not be entirely justified. For example, the non-Gaussian nature of atmospheric noise wac clearly shown in [3]: the atmospheric noise can be represented as the sum of a normal flucturation and a pulse component. In several studies the effects of non-Gaussian impulsive manmade noise bave been discussed [*e. g.*, 4]. Certain non-Gaussian noise has important implication for receiver design and evaluation of system performance. For instance, the ϵ-contaminated

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mixture noise model was used in the detection of signals in non-Gaussian noise [*e. g.*, 5, 6].

In this paper, when a signal is transmitted by the FHSS-BFSK communication system through nonselective Rayleigh fading channels with impulsive noise, we will investigate the performance of two nonlinear detectors, the squarer and limiter-squarer detectors.

II. System model

The system model considered in this paper is shown in Figure 1. The transmitted signal is given by

$$S(t) = A\cos[2\pi(f_c + f_k + a_k\Delta)t + \theta_k],\ kT \le t < (k+1)T, \quad (1)$$

where $A = \sqrt{2E_b/T}$ is the amplitude of $S(t)$, f_c is the carrier frequency, f_k is the hopping frequency in the interval $kT < t < (k+1)T$, a_k is a rectangular pulse of duration T, which may assume value -1 or $+1$ with eqnal probability, Δ is one-half the spacing between two FSK tones and satisfies $\Delta = \frac{l}{2T}$ for some integer l [1], the phase angle θ_k is a random variable which is uniformly distributed between 0 and 2π, and E_b is the energy per bit. It is assumed that both the hopping and data rates are equal to $1/T$. For each time interval of duration T, the hopping frequency f_k takes on one value from the frequency set $H = \{F_0, F_0 + C/T, F_0 + 2C/T, \ldots, F_0 + (K-1)C/T\}$, where F_0 is a frequency which satisfies the condition $F_0 \gg (K-1)C/T$, C is a positive integer, and K denotes the number of requencies used in hopping.

In this paper the transmitted signal is assumed to be propagated through a noisy multipath fading channel and the noise is assumed to be modeled by the ϵ-contaminated mixture noise model, for which the probability density function (pdf) is

$$f(x) = (1-\epsilon) f_N(x) + \epsilon f_I(x). \quad (2)$$

In (2) f_N is a Gaussian pdf with zero mean and variance σ_N^2, and f_I is in general a zero mean pdf.

2.2 The receiver and received signal

The received signal can be written as, for $kT \le t < (k+1)T$,

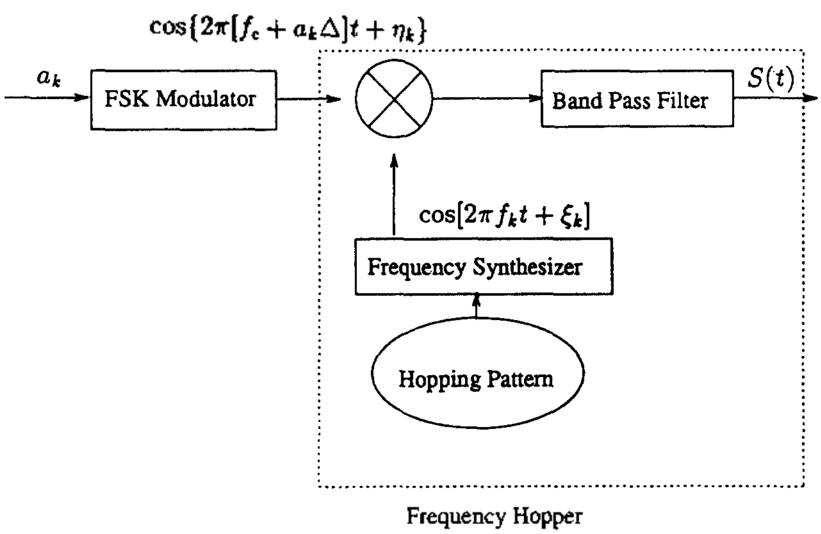

Figure 1. A block diagram of the FHSS transmitter.

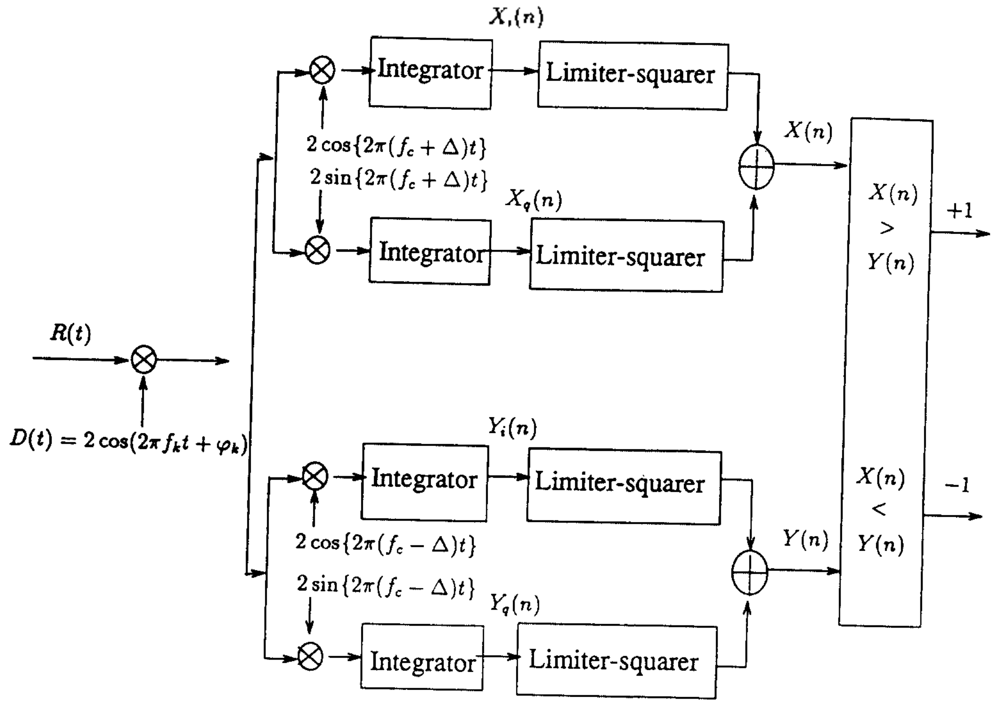

Figure 2. A block diagram of the limiter-squarer receiver.

$$R(t) = A\sum_{m=0}^{M-1} r_m \cos[2\pi(f_c+f_k+a_k\Delta)(t-\tau_m)+\psi_m]+N(t), \tag{3}$$

where the multipath strength r_m has the Rayleigh pdf

$$P_{rm}(r) = \frac{2r}{b_m}\exp\{-\frac{r^2}{b_m}\},\ r \ge 0, \tag{4}$$

the random variable τ_m is the path delay of the m th multipath signal reative to the time reference $\tau_0=0$, and the random phase ψ_m is uniformly distributed over $[0, 2\pi]$. It is assumed that r_m, τ_m, and ψ_m, $m=0, 1, \ldots, M-1$, are statistically independent of each other. The white noise $N(t)$ is assumed to be zero-mean and statistically independent of r_m, τ_m, and ψ_m. It is also assumed that the random variables in $S(t)$ and $R(t)$ are independent of each other.

The receiver using a limiter-squarer (i. e., the limiter-squarer receiver) is shown in Figure 2, with the input-output characteristic of the limiter-squarer detector shown in Figure 3. The square-law receiver is obtained if we replace the limiter squarers in Figure 2 with squarers.

In Fgure 2,

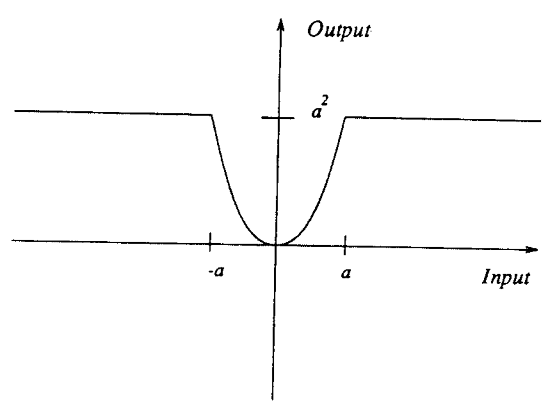

Figure 3. The input-output characteristic of a limiter-squarer.

$X(n) =$

$$\begin{cases} X_i^2(n) + X_q^2(n), & \text{if } |X_i(n)| \le a,\ |X_q(n)| \le a; \\ X_i^2(n) + a^2, & \text{if } |X_i(n)| \le a,\ |X_q(n)| > a; \\ a^2 + X_q^2(n), & \text{if } |X_i(n)| > a,\ |X_q(n)| \le a; \\ 2a^2, & \text{if } |X_i(n)| > a,\ |X_q(n)| > a; \end{cases} \quad (5)$$

with a similar expression for $Y(n)$. The $X_i(n)$ and $Y_i(n)$ are the outputs of the in-phase branches, and the $X_q(n)$ and $Y_q(n)$ are the outputs of the quadrature branches. Denoting the dehopping signal by

$$D(t) = 2\cos(2\pi f_n t + \varphi_n),\ nT \le t < (n+1)T, \quad (6)$$

where the random variable φ_n is uniformly distributed over $[0, 2\pi]$, $X_i(n)$ and $X_q(n)$ are given by

$$X_i(n) = \frac{2}{T}\int_{nT}^{(n+1)T} R(t)\,D(t)\cos[2\pi(f_c+\Delta)t]dt \quad (7)$$

and

$$X_q(n) = \frac{2}{T}\int_{nT}^{(n+1)T} R(t)\,D(t)\cos[2\pi(f_c+\Delta)t]dt \quad (8)$$

with similar expressions for $Y_i(n)$ and $Y_q(n)$.

3. Detector output pdf

The pdf of $X(n)$ for the sqare-law receiver can be shown to be [7]

$$f_X(n) = \frac{(1-\epsilon)^2}{2\alpha_N^2}\exp\left\{-\frac{x}{2\alpha_N^2}\right\} + \frac{\epsilon^2}{2\alpha_T^2}\exp\left\{-\frac{x}{2\alpha_T^2}\right\} + \frac{\epsilon(1-\epsilon)}{\alpha_N\alpha_T}\exp\left\{-\frac{x}{2\alpha_S^2}\right\} I_o\left(-\frac{x}{2\alpha_S^2}\right), \quad (9)$$

where

$$\alpha_N^2 = \left(\frac{1+a_n}{2}\right)E_b(b_o+S_o) + \sigma_N^2,$$

$$\alpha_T^2 = \left(\frac{1+a_n}{2}\right)E_b(b_o+S_o) + \sigma_T^2,$$

$$\frac{2}{\alpha_S^2} = \frac{1}{\alpha_N^2} + \frac{1}{\alpha_T^2},$$

$$\frac{2}{\alpha_D^2} = \frac{1}{\alpha_N^2} - \frac{1}{\alpha_T^2},$$

and

$$I_o(x) = \frac{1}{\pi}\int_0^{\pi}\exp\{-x\cos\theta\}\,d\theta \quad (10)$$

is the modified Bessel function. The pdf of $Y(n)$ is similar to the above expression for $X(n)$ except that a_n is replaced by $-a_n$.

For the limiter-squarer receiver shown in Figure 2, the pdf $f_{X(n)}(x)$ is [7]

$$f_{X(n)}(x) = \frac{(1-\epsilon)^2}{2\alpha_N^2}\exp\left\{-\frac{x}{2\alpha_N^2}\right\} + \frac{\epsilon^2}{2\alpha_T^2}\exp\left\{-\frac{x}{2\alpha_T^2}\right\} + \frac{\epsilon(1-\epsilon)}{\alpha_N\alpha_T}\exp\left\{-\frac{x}{2\alpha_S^2}\right\} I_o\left(-\frac{x}{2\alpha_D^2}\right) \quad (11)$$

for $0 < x \le a^2$,

$$\begin{aligned} f_{X(n)}(x) = {} & \frac{(1-\epsilon)^2}{2\alpha_N^2}\exp\left\{-\frac{x}{2\alpha_N^2}\right\}\left[1+\frac{4}{\pi}\left\{\frac{\gamma_{NN}}{\sqrt{x-a^2}} - \theta_l(x)\right\}\right] \\ & + \frac{\epsilon^2}{2\alpha_T^2}\exp\left\{-\frac{x}{2\alpha_T^2}\right\}\left[1+\frac{4}{\pi}\left\{\frac{\gamma_{TT}}{\sqrt{x-a^2}} - \theta_l(x)\right\}\right] \\ & + \frac{\epsilon(1-\epsilon)}{\alpha_N\alpha_N}\left[\exp\left\{-\frac{x}{2\alpha_S^2}\right\} I_o\left(-\frac{x}{2\alpha_D^2}\right) + \frac{2}{\pi}\left\{\frac{1}{\sqrt{x-a^2}}\left[\gamma_{NT}\exp\left\{-\frac{x}{2\alpha_N^2}\right\} + \gamma_{TN}\exp\left\{-\frac{x}{2\alpha_T^2}\right\}\right]\right.\right. \\ & \left.\left. -\exp\left\{-\frac{x}{2\alpha_S^2}\right\}\int_0^{2\theta_l(x)}\cosh\left(-\frac{x\cos\theta}{2\alpha_D^2}\right)d\theta\right\}\right] \end{aligned} \quad (12)$$

for $a^2 < x < 2a^2$,

$$f_{X(n)}(x) = \left[(1-\epsilon)\operatorname{erfc}\left(\frac{a}{\sqrt{2}\,\alpha_N}\right) + \epsilon\operatorname{erfc}\left(\frac{a}{\sqrt{2}\,\alpha_N}\right)\right]^2 \delta(x-2a^2), \quad (13)$$

for $x=2a^2$, and

$$f_{X(x)}=0 \tag{14}$$

for $x\leq 0$ and $x>2a^2$, where

$$\mathrm{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp\{-t^2\}dt \tag{15}$$

is the complementary error function,

$$\theta_l(x)=\arcsin\sqrt{1-\frac{a^2}{x}}, \tag{16}$$

and

$$\gamma_{ij}=\sqrt{\frac{\pi\alpha_i^2}{2}}\exp\left\{\frac{a^2}{2\alpha_i^2}\right\}\mathrm{erfc}\left(\frac{a}{\sqrt{2}\alpha_i}\right) \tag{17}$$

with $i, j=N, T$.

Ⅳ. Evaluation of error probability

The probability of bit error is

$$P_e=\frac{1}{2}Pr\{X(n)>Y(n)|a_n=-1\}$$
$$+\frac{1}{2}P_r\{X(n)<Y(n)|a_n=1\}. \tag{18}$$

Using the results in Section 3, we have

$$P_e=\frac{1}{2}\int_0^{\infty}\int_y^{\infty}u_X(x)\,u_Y(y)|_{a_n=-1}dxdy$$
$$+\frac{1}{2}\int_0^{\infty}\int_y^{\infty}u_X(x)\,u_Y(y)|_{a_n=1}dydx \tag{19}$$

for the square-law receiver, where u_X represents the pdf $f_{X(n)}$ given by (9) and $u_Y(\cdot)$ is the same as u_X except that a_n is replaced by $-a_n$. Similarly we have

$$P_e=\frac{1}{2}\int_0^{2a^2}\int_y^{2a^2}v_X(x)\,v_Y(y)|_{a_n=-1}dxdy$$
$$+\frac{1}{2}\int_0^{2a^2}\int_x^{2a^2}v_X(x)\,v_Y(y)|_{a_n=1}dydx \tag{20}$$

for the limiter-squarer receiver, where v_X is the pdf (11)-(14), and v_Y is obtained from v_X by substituting a_n with $-a_n$. We will use numerical calculation to compute the probability of bit error.

To show the probability of bit error in various cases, let us define the threshold to noise ratio (TNR) as

$$\mathrm{TNR}=\frac{a}{\sigma_N^2}, \tag{21}$$

the ratio of the noise variances as

$$\mu=\frac{\sigma_I^2}{\sigma_N^2}, \tag{22}$$

and the signal to interference ratio (SIR) as

$$\mathrm{SIR}=10\log_{10}\left(\frac{S_0}{b_0}\right), \tag{23}$$

where $S_0=\sum_{m\in N_0}b_m$ with N_0 denoting all the paths for which the path delays lie in $[0, T]$.

Figures 4 and 5 show the probability of bit error as a function of SNR for various values of μ, where SNR $=10\log_{10}[E_b/\{(1-\epsilon)\sigma_N^2+\epsilon\sigma_I^2\}]$. Since we consider nonselective fading channels, it is assumed that $S_0=1$ [1]. In Figure 4, the probability of bit error is shown for $\mu=10.0$ and SIR $=0$ dB, when the values of TNR are 5.0, 10.0, and 15.0. In Figure 5, the impulsiveness is higher than in Figure 4 : the value is now $\mu=50.0$.

When the SNR is low, the performance of the limiter-squarer receiver is better than that of the square-law receiver under noise environment. For example, in Figure 5 (b), the limiter-squarer receiver with TNR = 20 has approximately 8dB SNR gain over the square-law receiver when $P_e=10^{-1}$. The reason for this is that the limiter with proper value of TNR reduces the effects of impulsive (large-valued) noise.

When the SNR is high, on the other hand, the square-law receiver has better performance than the limiter squarer receiver if the value of TNR is small. For example, as we can see in Figure 4 (a), to attain the probability of bit error of 10^{-5}, the limiter-squarer receiver with TNR 5.0 requires 54dB SNR, the limiter-square receiver with TNR 10.0 requires 52dB SNR, and the square-law receiver requires approximately 50dB SNR. This can be explained as follows. In the case of high SNR, the limiting property of the limiter-squarer detector prevents the detector from fully exploiting the information of the large-valued transmitted signal since the effects of the large amplitude signal is limited at the maximum to the threshold level a. Therefore, the limiter-squarer receiver may have worse performance than the square-law receiver in that case if the TNR is not chosen properly.

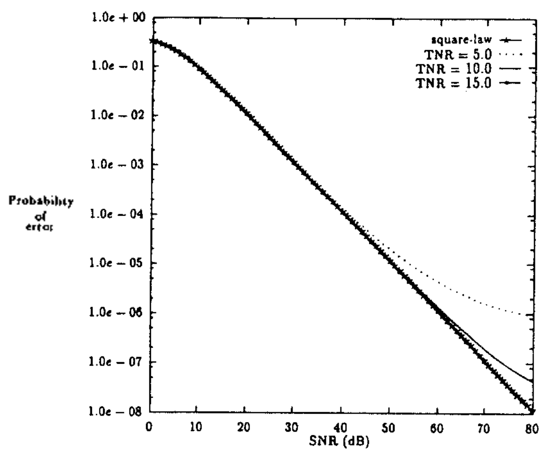

Figure 4. (a) The probability of bit error when $\epsilon = 0.01$, $\mu = 10.0$ and SIR = 0dB.

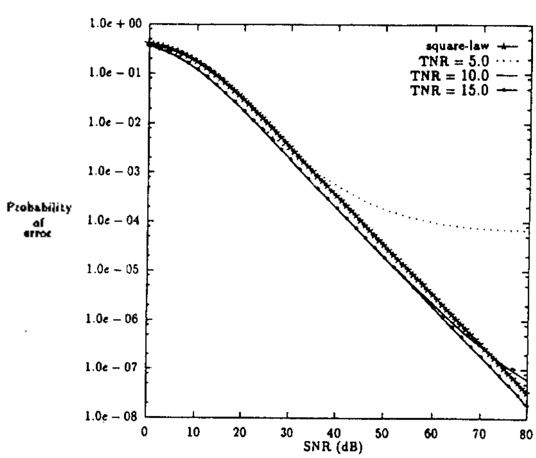

Figure 4. (b) The probability of bit error when $\epsilon = 0.1$, $\mu = 10.0$ and SIR = 0dB.

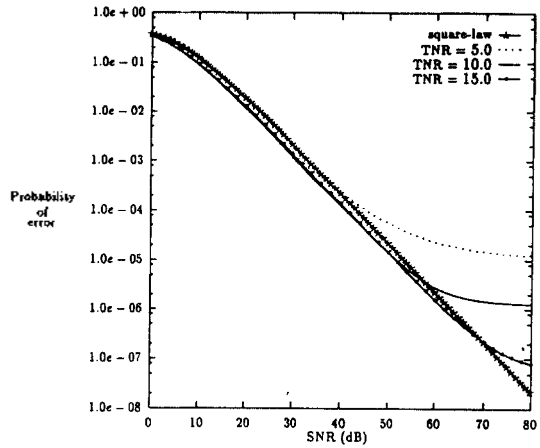

Figure 5. (a) The probability of bit error when $\epsilon = 0.01$, $\mu = 50.0$ and SIR = 0dB.

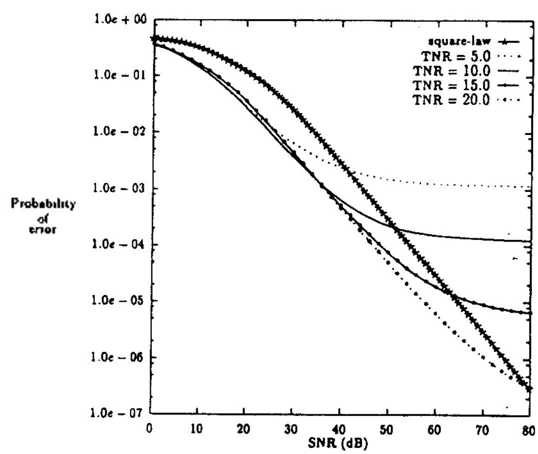

Figure 5. (b) The probability of bit error when $\epsilon = 0.1$, $\mu = 50.0$ and SIR = 0dB.

V. Summary

In this paper, we investigated the performance characteristics of the FHSS-BFSK communication system using limiter-squarer detectors and that using square-law detectors.

The probabilities of bit error of the FHSS-BFSK communication systems are obtained by numerical analysis in various cases of the ϵ-contaminated mixture noise.

The performance of the limiter-squarer receiver is shown to be better than that of the square-law receiver if the TNR is chosen properly or if SNR is low. At high SNR, the square-law receiver has better performance than the limiter-squarer receiver when the value of TNR is small.

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