Migration Characteristics in Sine-Wave Type Rivers

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ABSTRACT/This paper presents a model on the migration charactristics which is developed by using the equations for conservation of mass, momentum, and for lateral stability of the streambed. This model enables prediction of the magnitude and the location of near-bank bed scour as well as rates and direction of meander migration in the sine-wave type rivers (SWR) of small sinuosity. It is evident from this study that the transverse bed slope factor B′ and transverse mass flux factor play significant roles in predicting migration charactristics, and their values of $B' = 4.0$ and $\alpha = 0.4$ seem reasonable. This model will produce a useful guidelines in planning, design, construction, and development of SWR basin projects.

1. Introduction

The annual mean precipitation of Korea is 1, 274mm which is 1.3 times of the world average value (970mm). However, two thirds of this amount is concentrated in flood season (June to September), which causes frequent flood damage(2). Especially, the abnormally heavy rainfalls during recent several years innundated natural rivers.

Most natural rivers have meandering patterns. Hydraulic engineers have been trying to find a way to stabilize the rivers by quantifying these meandering characteristics. And the need for developing the theoretical model to predict the migration characteristics has been increased.

The meander migration causes problems on river-related human activities such as navigation, flood damage mitigation, irrigation, etc. Lacey(18) suggested the width-depth ratio to be less than 6~10 for stable channels. Since for most natural rivers, however, tho ratio is greater than 6~10, they are unstable and get through meandering process to the stable condition. Chang(9), Leopold and Wolman(19) studied relationship between the meander characteristics such as meander planform and meander wavelength, and flow characteristics such as bank-full discharge(27), depth and top width,

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for development of a model to predict the meander characteristics. Hooke[12] suggested an empirical migration equation which could be obtained from the relationship between magnitude and width of the bank erosion.

The migration of the small sinuosity SWR has been studied by several investigators. Ikeda et al.[14, 15], Kitanidis and Kennedy[17], Blondeaux and Seminara[8] investigated the size of the bank erosion caused by the secondary flow. Odgaard[20, 21] introduced the phase lag between channel curvature and secondary flow to determine the direction of the meander migration. Others among those who studied this topic were Dietrich and Smith[10], Howard and Knutson[13], Shen et al.[24], Yen[29] and in Korea, Ko[3], Yoon[4], Lee and Yoon[5], Cha[6, 7] et al.

By using the equations for conservation of mass, momentum, and for lateral stability of the stream-bed to know the migration characteristics, flow characteristics along the channel centerline was applied to derive linear equation of the transverse bed slope, and then its solution was used to develope the theoretical model with the small wave theory.

2. Governing Equations

The SWR model is for two dimensional flow in mildly curved alluvial channels with uniform bed particles. It applies to the central region of the channel cross section, of width B, where the effects of bank resistance on the flow pattern are insignificant. The banks effect the flow pattern within a distance of about one water depth from the banks, and the channel width is assumed to be constant. The variables in the basic equations are shown in Fig. 2.1 using orthogonal curvilinear coordinate system.

In Fig.2.1, the X-axis is along the channel centerline, positive in the streamwise direction, the Y-axis is perpendicular to the X-axis and positive toward the concave bank, and the Z-axis is vertically upward from the streambed. The velocity components in the X-, Y-, and Z-directions are denoted as U, V, and W, respectively. The equations of motion are[23, 29]

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}+W\frac{\partial U}{\partial Z}+\frac{UV}{R}=-\frac{1}{\rho}\frac{\partial Pr}{\partial X}+F_X \tag{2.1}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}+W\frac{\partial V}{\partial Z}-\frac{U^2}{R}=-\frac{1}{\rho}\frac{\partial Pr}{\partial Y}+F_Y \tag{2.2}$$

$$U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}+W\frac{\partial W}{\partial Z}+g=-\frac{1}{\rho}\frac{\partial Pr}{\partial Z}+F_Z \tag{2.3}$$

in which R = local radius of curvature; Pr = pressure; F_X, F_Y, and F_Z = friction terms in the X, Y, and Z-directions, respectively; g = acceleration due to gravity; ρ = fluid density. The continuity equations for flow and sediment, and equations for vertical and horizontal velocity profiles are represented as the following equations respectively.

$$\frac{\partial U}{\partial X}+\frac{1}{R}\frac{\partial (VR)}{\partial Y}+\frac{\partial W}{\partial Z}=0 \tag{2.4}$$

$$\frac{\partial Q_X}{\partial X}+\frac{1}{R}\frac{\partial(Q_Y R)}{\partial Y}=0 \tag{2.5}$$

$$\frac{U}{U_m}=\left(\frac{p+1}{1}\right)\left(\frac{Z}{d}\right)^{1/p} \tag{2.6}$$

$$V=V_m+2V_s'\left(\frac{Z}{d}-\frac{1}{2}\right) \tag{2.7}$$

in which Q_X and Q_Y=volumetric bedload transport per unit width in the X- and Y-directions, respectively. And p is a velocity profile exponent; $p=U_m/U_*=(8/f)^{1/2}$; U_m=averaged flow-velocity; $U_*[=(\tau_o/\rho)^{1/2}]$=shear velocity; τ_o=shear stress; κ=von Karman's constant($\cong$0.4), f=Darcy-Weisbach's friction factor, and V_m is mean velocity of V and V_s' transverse velocity due to centrifugal force in the water surface.

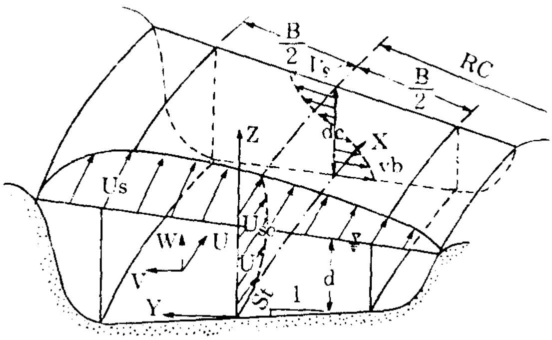

Fig. 2.1 Definition Sketch for Sinusoidal-Channel Section(Idealized).

3. Simplification of Flow Equations

By applying the above-mentioned assumption and experiment results of the Rozovskii[23], friction terms are equal to $F_X=(1/\rho)\partial\tau_X/\partial Z$, and $F_Y=(1/\rho)\partial\tau_Y/\partial Z$, where τ_X and τ_Y=shear stresses in the X and Y directions, respectively. As Eq.(2.3) is reduced to the hydrostatic condition, the pressure terms in Eqs.(2.1) and (2.2) can be written in terms of S_L and S_R, the slopes of the water surface, in the streamwise and transeverse directions respectively, and by applying the kinematic boundary conditions at the free surface and bed, the equations of motion and continuity for the averaged depth then read

$$U_m\frac{\partial U_m}{\partial X}+V_m\frac{\partial U_m}{\partial Y}+\frac{(UV)_m}{R}=gS_L-\frac{\tau_{bX}}{\rho d} \tag{3.1}$$

$$U_m\frac{\partial V_m}{\partial X}+V_m\frac{\partial V_m}{\partial Y}-\frac{U_m^2}{R}=gS_R-\frac{\tau_{bY}}{\rho d} \tag{3.2}$$

$$\frac{\partial(U_m d)}{\partial X}+\frac{1}{R}\frac{\partial(V_m R d)}{\partial Y}=0 \tag{3.3}$$

in which τ_{bX} and τ_{bY} =bed shear stresses in the X and Y-directions, respectively. The velocity profile in Eq. (2.2) can be obtained by computing Eqs. (2.6) and (2.7) at the water surface. By eliminating S_R at Eq. (3.2), Eq. (2.2) the following equation yields

$$U_X\frac{\partial U_s}{\partial X}-U_m\frac{\partial V_m}{\partial X}+V_s\frac{\partial V_s}{\partial Y}-V_m\frac{\partial V_m}{\partial Y}=\frac{U_s^2-U_m^2}{R}+\frac{\tau_{bY}}{\rho d}+\frac{1}{\rho}\left(\frac{\partial\tau_Y}{\partial z}\right)_s \tag{3.4}$$

The ∂_Y near the water surface(subscrpit s) is determined as $\tau_Y=\varepsilon(\partial V/\partial Z)$, and assuming that eddy viscosity, ε is isotropic and V given by Eq. (2.7), as τ_X is to become linearly distributed, that is, $\tau_X=\tau_{by}(1-z/d)$, we can obtain the following expression.

$$\left(\frac{\partial\tau_Y}{\partial Z}\right)_s=-\frac{2\rho\kappa V_s U_*}{d}\quad\frac{p}{p+1} \tag{3.5}$$

The value of τ_Y at the bed is determined as a fraction of τ_{bx} instead of τ_{bx}. Because the near-bed deflection angle of τ_{bx} is the same as that of velocity due to the effect of the net(depth-averaged) transverse fluid flux V_m and the centrifugally induced skewing of the velocity distribution, i.e

$$\frac{\tau_{bY}}{\tau_{bx}}=\frac{V_b}{U_b}=\frac{V_m-U_s'}{U_m} \tag{3.6}$$

By using Eq.(2.7), Eq.(3.1), and Eq.(3.6), we can calculate the velocity components in X and Y direction, respectively. In fully-developed bend region, where V_m and $\partial/\partial X$ terms are zero, the following equations are given

$$U_m^2=\frac{p^2}{\kappa^2}gS_L d \tag{3.7}$$

$$\frac{V_s'}{U_m}=\frac{1}{\kappa^2}\quad\frac{(2p+1)(p+1)}{p+1+2p^2}\quad\frac{d}{R} \tag{3.8}$$

Eqs.(3.7) and (3.8) are experimentally well verified. Eq.(3.7) is the traditional Darcy-Weisbach relationship[26]. Eq.(3.8) agrees with laboratory data reported by Falcon[11], Kikkawa et al.[16], and Odgaard and Bergs[24]. In Eq. (3.8), the factor of d/R is essentially constant, varying only between 7.2 and 8.0 when p varies between 3 and 6. By simplifying these equations, we can determine the unknowns of the governing equation, that is, U_m, V_m, d, and V_s'.

4. The derivation of Linear Equations

To simplify equations, variables U_m and d can be linearized by assuming them to be centerline values respectively. According to the experiment and field data by Zimmermann and Kennedy[30], Thorne et al.[22, 23], both $(U_c)_m$ and d_c are essentially constant along the centerline of channels, and their variation in transverse direction is nearly linear over the centerline. The following equations can be obtained

$$\frac{U_m}{(U_c)_m}=1+\frac{Y}{d_c}U_{lc} \qquad (4.1)$$

$$\frac{d}{d_c}=1+\frac{Y}{d_c}S_{lc} \qquad (4.2)$$

in which S_{lc}=transverse bed slope at the centerline ; and U_{lc}=normalized transeverse velocity gradient at the centerline.

To simplify equations further, V_m and V_s' are evaluated along the channel centerline. Using the continuity equation for $(V_c)_m$, integration yields

$$(V_c)_m=\frac{\alpha}{8}(U_c)_m\ \frac{B^2}{d_c}\ \frac{d}{dX}(S_{lc}+U_{lc}) \qquad (4.3)$$

in which α=correction factor transverse–mass flux to accomodate the error introduced as a result of linearization of Eq.(4.3). By assumpting of linear, transeverse variarion of both $(U_c)_m$ and d_c, the value α is defined by comparing it with the integral of Eq.(4.3). In case of field data, as they are reported that have a range of 0.2~0.8, we used 0.4 in this paper.

If we integrated the transverse bed slope S_{lc} by the contiuity equation for sediment, Q_{Yc} is described as

$$Q_{Yc}=Q_{Xc}\frac{\beta}{8}\frac{B^2}{d_c}M'\frac{dU_{lc}}{dX} \qquad (4.4)$$

in which β=a correction factor of the same order, introduced for the same reason, as in Eq.(4.3).

The relationship between Q_X and Q_Y is obtained by the balance of forces, such as gravitational action between bed particles, drag force of fluid, lift, and friction associated with the loss of momentum through collisions with bed surface(19). In a curved channel, the centrifugally induced secondary current causes both fluid drag and friction to be in directions different from that of the channel axis. By resolving these forces in the X– and Y–directions and then applying incipient–motion concepts(16) and dimensionless critical bed shear stress for simplification, higher order terms are neglected, and the following linear equations yields

$$\frac{dU_{lc}}{d\Psi}+A_1U_{lc}=\frac{1}{2}A_1S_{lc} \qquad (4.5)$$

$$\frac{d^2U_{lc}}{d\Psi^2}+A_2\frac{d^2U_{lc}}{d\Psi^2}+A_3\frac{dS_{lc}}{d\Psi}+A_4\frac{dU_{lc}}{d\Psi}+A_5S_{lc}=A_6 \qquad (4.6)$$

in which Ψ=X/B and A_1, A_2, A_3, A_4, A_5, A_6 are linear coefficients. By eliminating S_{lc} with Eq.(4.5), Eq.(4.6) is as follows

$$\frac{d^3U_{lc}}{d\Psi^3}+H_1\frac{d^2U_{lc}}{d\Psi^2}+H_2\frac{dU_{lc}}{d\Psi}+H_3U_{lc}=H_4 \qquad (4.7)$$

in which $H_1=A_1+0.5A_1A_2+A_3$, $H_2=A_1A_3+0.5A_1A_4+A_5$, $H_3=A_1A_5$, $H_4=0.5A_1A_6$.

It is apparent that the system[U_{lc} and S_{lc} in the Eqs.(4.6) and (4.7)] are derived by the damped oscillation forced by H_4. In a fully developed bend, where $d/d\Psi=0$,Eqs.(4.6) and (4.7) yield

$$\frac{U_m}{(U_c)_m}=\left(\frac{d}{d_c}\right)^{1/2} \qquad (4.8)$$

$$S_{tc}=H F_{DC} \frac{d}{R_c} \tag{4.9}$$

in which F_{DC}=particular densimetric Froude number defined as $F_{DC}=(U_c)_m/\Delta g D_{50})^{1/2}$, and $H=(2p+1)(p+1)/[B'\kappa(\theta)^{1/2}p(p+1+2p^2)]$, where B'is function of Coulomb's friction factor and the ratio of lift force to drag force of a bed particle. By Ikeda et al.[14], as values of B'are reported to be between 3 and 6, we used 4.0 for the value. By Ikeda[14] and Odgaard[21], as $d^2U_{tc}/d\Psi^2$ is negligibly small, Eqs.(4.5) and (4.6) yield[28]

$$\frac{d^2S_{tc}}{d\Psi^2}+(A_3+\frac{1}{2}A_4)\frac{dS_{tc}}{d\Psi}+A_5S_{tc}=A_6 \tag{4.10}$$

5. Analysis and Applications

5.1 Analysis

In order to know the solution of a channel alignment perturbation in such as the form of a traveling sinusoid shown in Fig. 5.1, the following equation yields

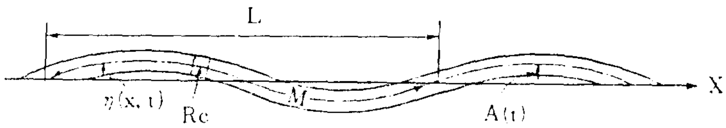

Fig. 5.1 Form of Traveling Sinusoid for the Stability of the System.

$$\eta(X, t)=A(t) \sin[k (X-ct)] \tag{5.1}$$

in which X is a distance along the channel axis;$k=2\pi/L$ is the wave number;L=meander wavelenth;t=time;c=celerity of sine wave. The channel-centerline displacement $\eta(t)$ is limited to values much smaller than the meander wavelength. The centerline curvature is then

$$\frac{1}{R_c}=-\frac{d^2\eta}{dX^2}=k^2 A(t) \sin[k(X-ct)] \tag{5.2}$$

The differntial equation for U_{tc} can be obtained by substituting Eq. (5.1) into Eq. (4.7). The solution, which is periodic and independent of the initial condition, is

$$U_{tc}=\frac{N B k^2 A}{(E_1^2+E_2^2)^{1/2}} \sin [k(X-ct)-\gamma] \tag{5.3}$$

in which $N=8\kappa^2(2p+1)/[\alpha p3(p+2)]$;$E_1=H_3-2H_1k^2B^2$ and $E_2=H_2kB-k^3B^3$. The phase shift between Utc and the channel-axis displacement is $\gamma=\tan^{-1}(E_2/E_1)$, and the equivalent transverse bed slope is obtainbed by substituting Eq. (5.3) into Eq. (4.5)

$$S_{tc}=\frac{2 N B k^2 A}{(E_1^2+E_2^2)^{1/2}} [1+(\frac{Bk}{A_1}{}^2)]^{1/2} \sin [k(X-ct)-\phi] \tag{5.4}$$

in which $\phi=-\tan^{-1}(Bk/A_1)$. In order to determine A(t), an equation is introduced that describes the rate of lateral shifting of the channel axis due to erosion of the concave bank and deposition of the convex bank. As shown in Fig.5.2, assuming that the rate of bank retreat, V_c is proportional to the erosion rate of bank, d_{bk} the equation of V_c can be derived as

$$V_c=E'(U_c)_m\left(\frac{d_{bk}}{d_c}-1\right) \tag{5.5}$$

in which E'=parameter describing the erodibilty of the bank material. Because of the assumed mild curvature of the channel, V_c may be equaled to the rate of change of channel alignment, $\partial\eta/\partial t$. After reduction of some relationships for amplitude-growth rate, $\partial A/\partial t$, and celerity c the following equations yield

$$\frac{1}{A}\frac{\partial A}{\partial t}=\frac{2E'(U_c)_m}{B}KBk\left[1+\left(\frac{Bk}{A_1}\right)^2\right]^{1/2}\cos\phi \tag{5.6}$$

$$c=2E'(U_c)_m K\left[1+\left(\frac{Bk}{A_1}\right)^2\right]^{1/2}\sin\phi \tag{5.7}$$

in which $K=0.5(NB/dc)[kB/(E_1^2+E_2^2)^{1/2}]$.

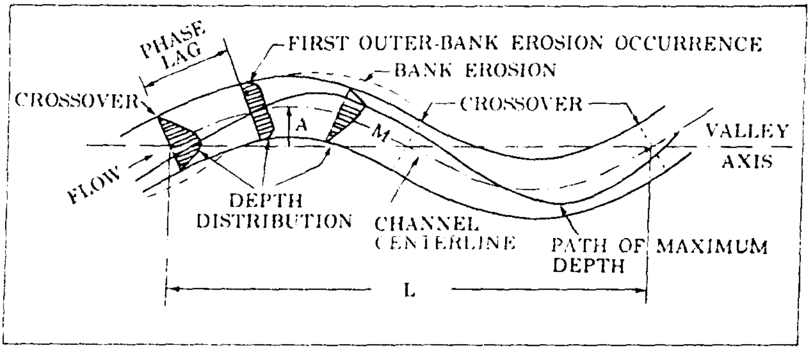

Fig. 5.2 Concepts for Bank Erosion Model.

5.2 Applications

The verification of this model shows Fig.5.3 (scale:1/12000). We verified the two local streams, for this model Kum Stream and Gilsan Stream, both of which are tributaries to Guem River in Choongnam[1].

As shown in Fig.5.4, we displayed a B/R_c disribution from each start point of both streams to two subreaches of KU_1 and KU_2 in Kum Stream case and to three subreaches GI_1, GI_2 and GI_3 for Gilsan Stream case. It was shown that the spacing of each subreach at Gilsan and Kum Stream was appeared 45~90m and 60~155m, respectively.

Because this stream is lack of a serviceable data and flow records, bank-full discharge was obtained by sustituting the flow equation(Gauckler-Manning equation) in the literlature(27). The primary flow data is presented in Table 5.1[1].

As shown in Fig.5.5, the distribution of L/B in the Kum Stream have a range of 5 to 35 wavelength, and the results are vague. On the other hand, we verified this model through Gilsan stream and this shows a concentrated attribution from 5 to 15.

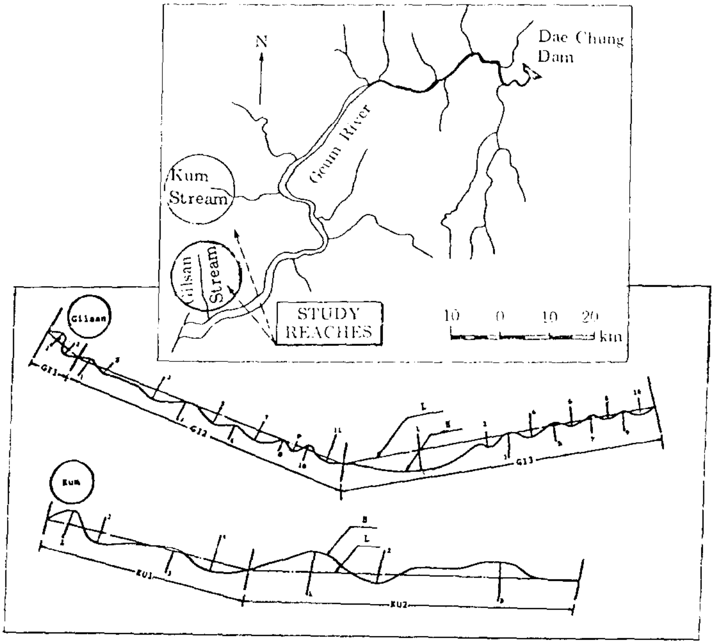

Fig.5.3 Plan View of Study Reaches.

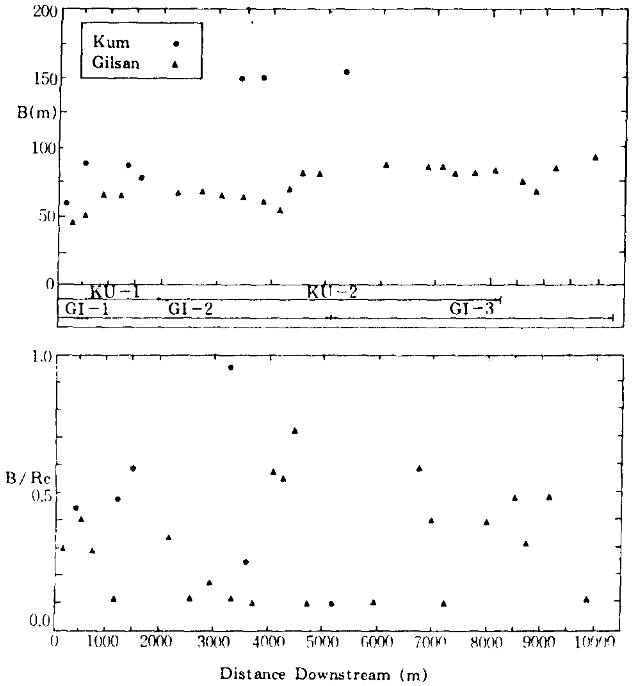

Fig.5.4 Summary of Data on Bends Reaches (in Kum and Gilsan Stream).

Table 5.1. Flow Data of Study Reaches.

Stream	Sinuosity (M/L)	Longitudianl Slope of Water Surface $S_L(\times 10^{-4})$	Bends Number	Bank-Full Conditins					
				Median Grain Size D_{50}(㎜)	Top Width B(m)	Flow Depth d_c(m)	Discharge Q(cms)	Friction Parame-ter p	Particle Froude Number F_{DC}
Kum	1.15	4.545 ~9.531	7	0.62 ~0.64	60 ~155	2.78 ~3.38	440 ~770	3.77	17.83
Gilsan	1.18	3.725 ~8.241	23	0.25 ~1.02	45 ~90	2.80 ~3.92	450 ~1180	3.79	25.22

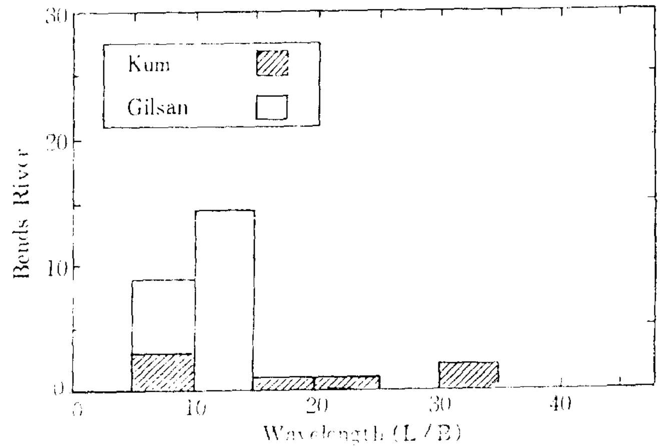

Fig.5.5 Histogram of Meander Wavelength of Bends in Two Stream Study Reaches.

Table 5. 2. Computation of Lateral and Down-Valley Migration Rates for Gilsan Stream.

Parameters	Migration Rates	Parameters	Migration Rates
A_1	0.583	kB	0.2490
A_2	-1.482	E_1	0.0061
A_3	1.348	E_3	0.0550
A_4	-2.861	N	0.0870
A_5	0.329	K	5.1230
H_1	1.499	ϕ_3	1.0560
H_2	0.281	$(1/A)\partial A/\partial t$	$3.960\times10^{-8}sec^{-1}$
H_3	0.192	c	$2.287\times10^{-7}m/s$

In this paper, the values of $B'=4.0$, $E'=4.8\ 10^{-7}$, $M'=3.0$, and $\alpha=0.40$ were used to compute the lateral and down-valley migration rates $\partial A/\partial t$ and c for Gilsan Stream. By using, as before, p=3.79, B/dc=26.17, it agrees well in the general case of curvature channel(B=10.0~60.0, p=3.0~5.0). F_{DC} was 25.22 which is larger than the mean value(5~15). The computed values of these variables are shown in Table 5.2. Down-valley migration rates are obtained by Eqs.(5.6) and (5.7).

And Fig. 5.6 shows a comparision between the predicted values and the measured ones of Gab River by Cha et al.[7] in which $t=0$, $k=2\pi/L=k^2A=1/Rm$, $B'=6.0$, $M'=3.0$, and $\alpha=0.4$. The values of $(U_c)_m=1.76$m/sec, $B=35.86$m, $d_c=2.05$m, $S_L=0.00994$, $p=3.78$, $R_m=118$m, and $M=1000$m were used herein. In this paper, we obtained the lateral velocity distribution of water-depth through transverse bed slope in the flow-centerline. At the case of the variation of flow-direction the predicted values agreed with the measured ones relatively well.

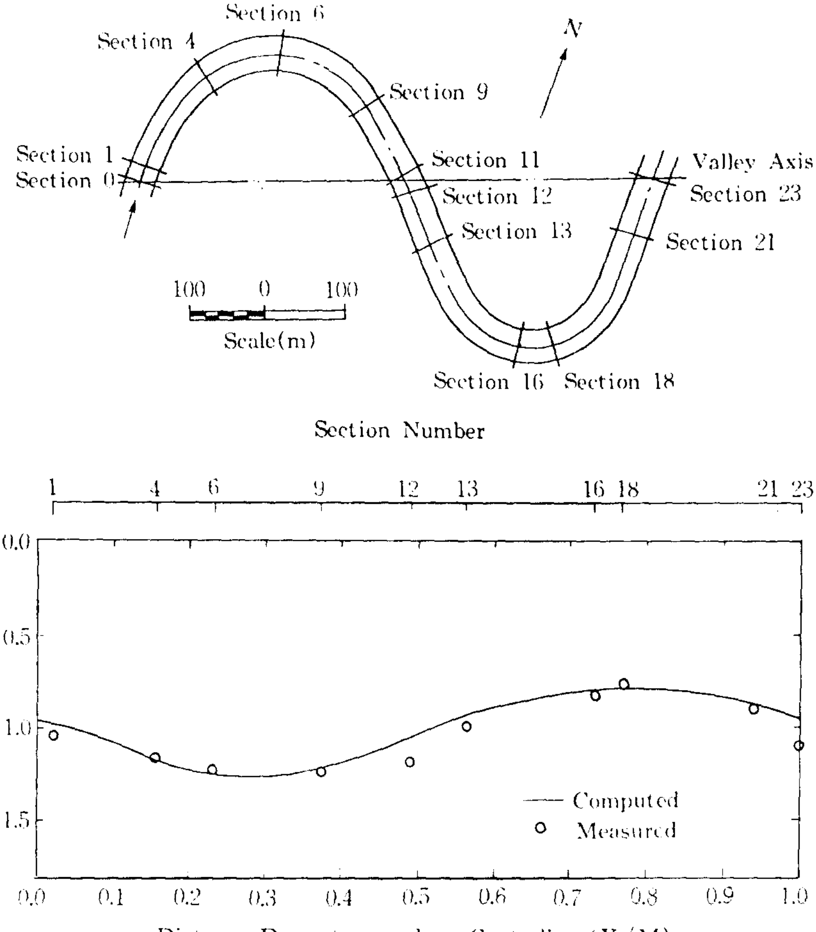

Fig. 5.6 Simulation of Near-Bank Depth in Bends of Gab River.

6. Conclusions

This study developed a model on the Meander-Migration Characteristics in sine-wave type rivers which obtained the solution with small-wave theory, after it was linearized by deriving flow characteristics for the transverse bed slope in the channel-centerline to damped oscillation system.

With the equations for conservation of mass, momentum, and lateral stability of the streambed, we could estimate the magnitude and direction of meander migration which relates primary variables of flow and the rate of bank erosion to the difference between near-bank and centerline depths. It is evident from this anaysis that transverse bed slope factor B′ and the transverse-mass flux factor α play significant roles. In determining them, their reasonable values were turned out to be B′=4.0 and $\alpha=0.40$, respectively.

This study will offer useful informations for the development and design in the river channels with small sinuosity.

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