Capillary Hysteresis Model in Unsaturated Flow: State of the Art

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Abstract/The purpose of this study is to classify existing hysteresis models and to discuss a possibility of a new type of the hysteresis model. The existing hysteresis models are classified into three types: the interpolation model, the scaling model and the domain model, of which only domain model is to simulate hysteresis curves based on the theoretical approach. It is useful to develop a hysteresis model that requires only one branch of hysteresis curves for the model calibration because obtaining hysteresis curves by experiments is expensive and time-consuming. These types of models are developed based on the independent domain concept by many investigators, however their models are not successful to accurately simulate real data of Rubicon Sandy Loam and Dune Sand. There is a possibility that a new model is based on the dependent domain concept considering the weighting factor, $P_a(\theta)$, which accounts for the pore blockage effect against air entry. Conclusively, a new model where the weighting factor $P_a(\theta)$ in Model Ⅲ-1 (Mualem, 1984) reduces to a known variable through an appropriate method is an alternative model which requires only one branch of main curves for the model calibration.

1. Introduction

Infiltration phenomenon is that water infiltrates into the soil through the soil surface and its feature is characterized by the unsaturated flow in the porous medium, which is an important component in the hydrologic cycle especially in the rainfall-runoff relation. Many hydrologic researches are focused on the establishment of the rainfall-runoff relation, but the infiltration plays the most uncertain role in the rainfall-runoff relation. Thus, the rainfall-runoff model has more physical basis through promoted appreciation of the infiltration phenomenon. Singh (1989) reported that it is generally held that approximately 70 percent of annual precipitation infiltrates into the soil in continental United States, and 30 percent becomes streamflow. However, this percentage of precipitation in-

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filtration may vary widely for individual storms ranging from approximately 30 percent (high runoff) to 100 percent (no runoff). Effective rainfall (or cumulative infiltration) is the most sensitive parameter in the rainfall-runoff model (Singh & Woolhiser, 1976). The research on the infiltration phenomenon is a relatively unexhausted field in the rainfall-runoff model, while the amount of infiltrated water cannot be neglected compared with the amount of rainfall. Therefore, the analysis of the unsaturated flow and related problems such as cumulative infiltration, infiltration rate and the moisture content profile in the soil must be focused in hydrology.

The exact simulation of infiltration rate is an important component in the establishment of irrigation plan. The exact analysis of the moisture profile in the soil resulting from irrigation water is necessary so that irrigation water can be prohibited from infiltrating under roots of vegetation. Therefore, the research on the unsaturated flow in the porous medium is the elementary component in the irrigation plan, and the more exact analysis of the unsaturated flow means the establishment of the more efficient irrigation plan.

Contaminating materials such as fertilizer, agricultural chemicals and exhausted industrial materials deteriorate the quality of water in the channel and the ground. Groundwater may become important gradually as agricultural water, industrial water and portable water. Contaminating materials move with infiltrated water from the soil surface to the groundwater surface through the unsaturated zone. Therefore, the analysis of solute transport may not be expected without the basis on the research of the unsaturated flow in the soil. It is reported by many researchers that the contamination of groundwater becomes serious. To cope with the contamination of groundwater efficiently, researches on the unsaturated flow may be concerned continuously.

In the view of watershed, agricultural and environmental hydrology as explained above, it is important to analyze and appreciate the unsaturated flow more exactly in the porous medium. One of factors to affect the unsaturated flow is the hysteresis phenomenon because in natural the unsaturated flow is governed by hysteresis. Thus, the purpose of this study is to review previous hysteresis models, to classify and compare models, and to discuss a possibility of a new type of the hysteresis model which requires only one branch of the hysteresis loop.

2. Background

Hysteresis phenomenon in the unsaturated flow is such that the water retention function, $\theta(\Psi)$, or the hydraulic conductivity, $K(\Psi)$, has different values in the drying process and the wetting process, where the former is called capillary pressure hysteresis and the second is called hydraulic conductivity hysteresis. Kool and Parker (1987) stated that it would seen that ignoring hysteresis is in many cases not based on knowledge that its effects are negligible, but rather on expediency motivated by a lack of sufficient data to adequately calibrate a hysteresis model. The purpose of modeling hysteresis is ultimately to analyze the unsaturated flow more exactly because the unsaturated flow is governed by hysteresis phenomenon in natural.

The governing equation of the unsaturated flow is expressed as Richards(1931)' equation where the dependent variable is the capillary pressure,

$$C(\Psi)\frac{\partial\Psi}{\partial t}=\frac{\partial}{\partial Z}\left[K(\Psi)\frac{\partial\Psi}{\partial Z}\right]-\frac{\partial K(\Psi)}{\partial Z},\ C(\Psi)=\frac{\partial\theta(\Psi)}{\partial\Psi} \tag{1}$$

where $C(\Psi)$ is the water capacity, Ψ is the capillary pressure, θ is the moisture content, $K(\Psi)$ is the hydraulic conductivity, t is time and z is positive-downward distance. The first term in the right-hand side of Eq.(1) expresses the change of the moisture content by the capillary pressure, and the second term expresses the change by the gravity.

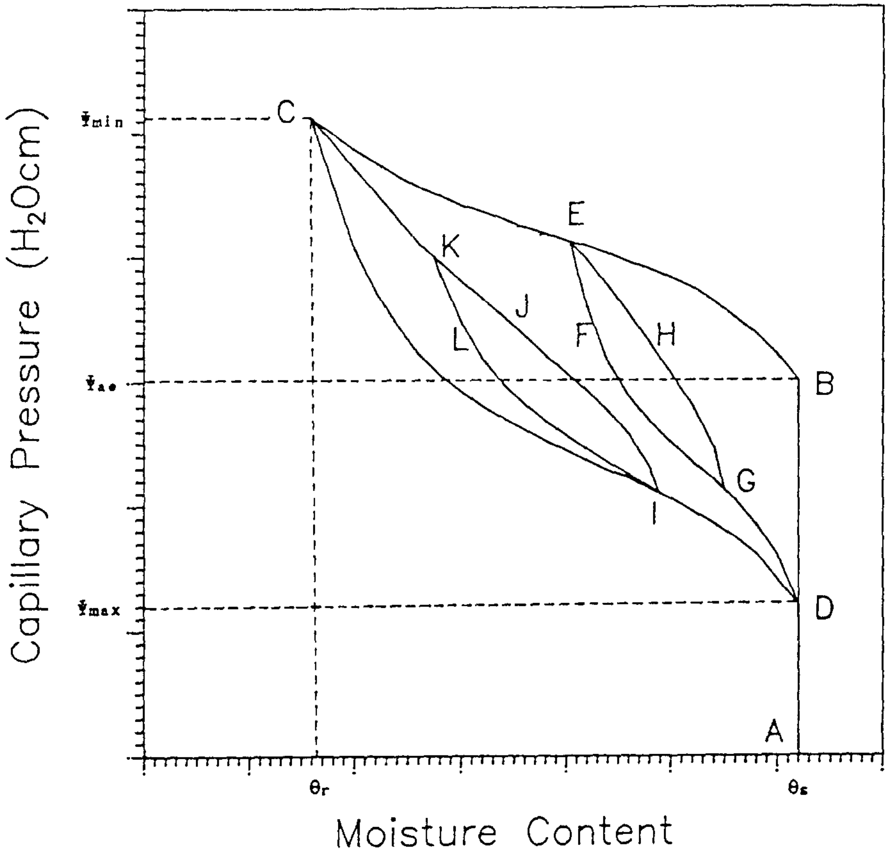

Fig.1 Typical $\theta(\Psi)$ Hysteresis for Sand

-Curve ADBEC : MDC, Curve CIDA : MWC, Curve IJKC : PDC
Curve EFGD : PWC, Curve GHE : SDC, Curve KLI : SWC
M : Main, P : Primary, S : Secondary, D : Drying, W : Wetting

-MDC, MWC are after experimental data of Poulovassilis and Childs(1971), while scanning curves are calcurated using Model III-1(Mualem, 1984).

-Coordinates : $E(\theta^d_1, \Psi^d_1)$, $I(\theta^w_1, \Psi^w_1)$, $K(\theta^d_2, \Psi^d_2)$, $G(\theta^w_2, \Psi^w_2)$

Data on (Ψ) and K(Ψ) for a given soil are needed to analyze Eq.(1). Topp et al. (1966, 1967), Topp (1969, 1971a), Talsma (1970), Poulovassilis (1970a), Vachaud and Thony (1971) observed considerable hysteresis in θ-Ψ relation and negligible hysteresis in K-Ψ relation through laboratory experiments. Watson et al. (1975), Royer and Vachaud (1975) observed non-neglecting capillary hysteresis through in-situ experiments. Beese and Pleog (1976), Hoa et al. (1977) showed that a numerical simulation in which the capillary hysteresis effect is ignored may involve important errors compared with experimental data. Watson and Sardana (1987) showed that the computer-based configuration of moisture content profiles in soils during redistribution considering hysteresis shows marked differences compared with those neglecting hysteresis. Hopmans and Dane (1986) reported that the hysteresis effect dominates the temperature effect on the simulation of the water movement in the unsaturated flow. Therefore, only capillary hysteresis is considered in this study. Typical hysteresis curves for sand (Poulovassilis and Childs, 1971) are shown in Fig.1.

Previous hysteresis models are classified into three types : the interpolation model proposed by Whisler and Watson (1969), Klute and Heermann (1974), Curtis and Watson (1984), the scaling model proposed by Dane and Wierenga (1975), Scott et al. (1983), Parker and Lenhard (1987) and the domain model. The interpolation model is such that when the reversal occurs in a main curve, the primary curve can be obtained through interpolating two experimental-given primary curves or one primary curve and one main curve surrounding the reversal point of the wanted primary curve. This model is not efficient because many experimental data are required to calibrate the model, and there is a drawback that the pumping effect occurs in this model. The scaling model is such that scanning curves can be obtained through scaling experimental-given main drying and wetting curves appropriately (or arbitrarily). This model must require two hysteresis curves for the model calibration in terms of the model structure, and does not explain hysteresis phenomenon physically. There is also a drawback that the pumping effect occurs except for Parker and Lenhard (1987)'s model. The pumping effect is such that the hysteresis curve starting at the main drying (or wetting) curve approaches the main wetting (or drying) curve when reversals occur in succession between two given moisture contents, This pumping effect is not a physical phenomenon characterized in the unsaturated flow but a phenomenon occurring based on the structure of the model. A model where the pumping effect occurs is not desirable one because the model does not simulate scanning curves as closed loops. On the other hand, the domain model simulates hysteresis curves under the physical basis. Thus, discussion focused on the domain model is continued. The features of these three types of hysteresis models are summarized in Table 1.

Because the domain model which is a conceptual model considers the porous medium as a set of porous domains containing characteristic drying and wetting porous radii, the distribution of these radii determines the hysteresis behavior of the medium. The domain model is classified into two types : the independent domain model (IDM) and the dependent domain model (DDM). IDM and DDM explain hysteresis phenomena under the assumption that each pore behaviors independently when water enters into the pore and each pore is affected by neighboring pores, respectively.

Table 1 Classification of Hysteresis Models

Model	Author(s)	Data[1]	Feature
Interpolation Model	Whisler & Watson(1969) Klute & Heermann(1974) Curtis & Watson(1984)	2MC & many PC	No Physical Basis, Pumping Effect
Scale Model	Dane & Wierenga(1975) Scott et al.(1983) Parker & Lenhard(1987)	2MC	No Physical Basis, Pumping Effect except for P & L(1987)' Model
Domain Model	Mualem(1973, 74, 77, 84) Parlange(1976, 1980) Mualem & Dagan(1975) Park & Sonu(1992)	1MC to 2MC & 2PC	Physical Basis, No Pumping Effect

1) Data needed for Model Calibration
MC : Main Curve, PC : Primary Curve

Poulovassilis (1962) explained moisture content-capillary pressure hysteresis on the basis of IDM, and Poulovassilis (1971a) reported that IDM simulates hysteresis curves in good agreement with data obtained by laboratory experiments on the sand. But Topp and Miller (1966) showed that it is not appropriate to simulate hysteresis curves using IDM through experiments on the glass-beed because of the well-defined air entry value (Ψ_{ac}), and that the reason why experimental hysteresis data of Poulovassilis (1962) well agree with calculated values using IDM is due to the fact that Ψ_{ac} is approximately zero in experimental data. It is not also appropriate to simulate hysteresis curves using IDM through Vachaud and Thony (1971)'s experiments on the sand and Topp (1971a)'s experiments on the silt loam and the clay loam. Talsma (1970) argued that it is appropriate to simulate experimental data on the coarse sand using IDM; while it is not to simulate data on the fine sand, which may be improved using DDM. Since models proposed above researchers require many experimental data (two main curves and many primary curves) for the model calibration, these models are not efficient to analyze the unsaturated flow. Therefore, Mualem (1973, 1974, 1977), Parlange (1976, 1980), Mualem and Morel-Seytoux (1978), Hogarth et al. (1988) proposed simpler modified models based on the independent domain concept in order to apply hysteresis models in the unsaturated flow efficiently.

Topp (1971b) explained hysteresis phenomena using DDM, whose model is not efficient because his model needs many experimental data for the model calibration and the pore water distribution function should be solved by the trial-and-error method. Poulovassilis and Childs (1971) defined the pore water distribution function consisting of α-element accounting for the drying process in the porous medium and β-element accounting for the wetting process. However, their model is not efficient because of requiring many experimental data to determine this distribution function. On the other hand, Mualem and Dagan (1975) developed simpler models which are based on generalization of Topp (1971b)'s basic concept. Mualem and Miller (1979), Mualem (1984) and Park and Sonu (1992) proposed simpler modified DDMs in order to apply the hysteresis model in analyzing the unsaturated flow than Model Ⅳ of Mualem and Dagan (1975). Table 2 summarizes types of domain models, author(s) and the feature of each model.

Table 2 Classification of Domain Models

Type[1)]	Model	Author(s)	Data Needed[2)]	Feature[3)]
IDM	Model I	Mualem(1973)	2MC	Neel, f=h·m
	Model I-1	Mualem(1977)	1MC	Neel, f=h·h
	Parlange's Model	Parlange(1976, 1980)	1MC	Neel, f=m h=1
	Model II	Mualem(1974)	2MC	Mualem, f=h·m
	Model II-1	Mualem(1977)	1MC	Mualem, f=h·h
DDM	Model III	Mualem & Dagan(1975)	2MC, 1PDC	$P_w=1$, $P_a\leq1$ Mualem, f=h·m
	Model III-1	Mualem(1984)	2MC	$P_w=1$, $P_a\leq1$ Mualem, f=h·h
	Model IV	Mualem & Dagan(1975)	2MC, 1PDC, 1PWC	$P_w\leq1$, $P_a\leq1$ Mualem, f=h·m
	Model IV-1	Park & Sonu(1992)	2MC, 1PDC	$P_w\leq1$, $P_a\leq1$ Mualem, f=h·h

1) IDM=Independent Domain Model, DDM=Dependent Domain Model
2) MC=Main Curve, PDC=Primary Drying Curve, PWC=Primary Wetting Curve
3) Neel=using Neel diagram, Mualam=using Mualam diagram
f=pore distribution function
P_a, P_w =weighting factor accounting for pore blockage against air, water entry
$f=h\cdot h$, $f=h\cdot m$ mean $f(r,\rho)=h(r)\cdot h(\rho)$ $f(r,\rho)=h(r)\cdot m(\rho)$, respectively.

Researches to analyze the unsaturated flow considering hysteresis phenomena using domain models are reviewed in the followings. Nieber and Walter (1981) compared experimental data with simulated results which were obtained by analyzing the 2-dimensional unsaturated flow considering hysteresis with Model III. Stauffer (1982) also compared experimental data with calculated results through analyzing the 2-dimensional saturated-unsaturated flow considering hysteresis with Model III. Milly (1982) researched the numerical scheme of water and heat transport using the water retention function of Haverkamp et al. (1977) to simulate main curves and using Model II-1 to simulate scanning curves. Hopmans and Dane (1986) made a study of the effects of temperature and hysteresis using Model III-1 on the unsaturated flow. Mualem (1976) developed the hydraulic conductivity, $K(\Psi)$, hysteresis model based on the concept of Model III-1. As mentioned above, the domain model has been used widely to analyze all sorts of problems connected with the unsaturated flow where not neglecting hysteresis phenomena are not neglected.

3. Classification of Domain Model

Since a hypothetical porous medium is considered as a set of groups of the pores which are distinguished by the radii (r^*) of the openings of the pores in the group and the radii (ρ^*) of the pores within the group (Mualem, 1974), hysteresis phenomenon of the porous medium can be explained by the pore water distribution function, $f(r^*, \rho^*)$, which is the characteristics of the porous medium. Therefore, the essence of the domain model is how to know this distribution function. Now it is necessary to define pore radii r^* and ρ^* more physically. Since the capillary law is expressed as $\Psi=\alpha/R^*$ where Ψ is a negative constant and R^* is a characteristic pore radius, the radius r^*(or ρ^*) is R^* corresponding to the capillary pressure Ψ^d (or Ψ^w) in the drying (or wetting) process, respectively. There are two capillary pressures (Ψ^d and Ψ^w) corresponding to the drying and wetting process for an arbitrary-given effective moisture content, $\Theta(=\Theta-\Theta_r)$, where Θ is the moisture content and Θ_r is the residual moisture content. Thus, parameters characterizing hysteresis phenomenon are Ψ^d and Ψ^w, that is, r^* and ρ^* by the capillary law, which can be normalized such that

$$r=\frac{(r^*-R_{min})}{(R^*_{max}-R^*_{min})},\ \rho=\frac{(\rho^*-R_{min})}{(R^*_{max}-R^*_{min})} \tag{2}$$

respectively, where r, ρ are the dimensionless pore radii, R^*_{max} is the radius corresponding to the capillary pressure $\Psi_{max}(=\Psi_{we})$ at the water entry value and R^*_{min} is the radius corresponding to the capillary pressure Ψ_{min} at the residual moisture content (reference Fig.1). Now it is time to define the pore water distribution function $f(r^*, \rho^*)$ by which hysteresis phenomenon can be explained. When an arbitrary effective moisture content θ increases to $\theta+d\theta$, $d\theta$ is the change of corresponding to the moisture content contained in the porous medium due to the change of pore radii in the manner that $r\rightarrow r-dr$ (that is, $\Psi^d\rightarrow\Psi^d-d\Psi^d$), $\rho\rightarrow\rho+d\rho$ (that is, $\Psi^w\rightarrow\Psi^w+d\Psi^w$). Since $d\theta$ can be expressed as $f(r, \rho)drd\rho$, the effective moisture content contained in the porous medium is expressed as followings :

$$\theta=\int\int f(r,\rho)drd\rho \tag{3}$$

where the integration is performed over dimensionless characteristic pore radii(r and ρ) corresponding to the interval of capillary pressures within the pore occupied by water. The similarity hypothesis of Philip (1964) may be introduced to the distribution function,

$$f(r,\rho)=h(r)\cdot m(\rho) \tag{4}$$

where $h(r)$ is the volumetric porosity and $m(\rho)$ is the areal porosity. Eq.(4) means that the bivariate distribution function $f(r, \rho)$ can be expressed as a multiple of two single variate distribution functions $h(r)$ and $m(\rho)$. Thus, Eq.(3) becomes as following

$$\theta=\int\int h(r)\cdot m(\rho)drd\rho \tag{5}$$

The conceptual Mualem diagram or Neel diagram is used to integrate Eq.(5). Let's define $H(R)$ and $M(R)$ for simplicity.

$$H(R)\equiv\int_0^R h(r)dr,\qquad M(R)\equiv\int_0^R m(\rho)d\rho \tag{6}$$

It is the independent domain model(IDM) that simulates hysteresis phenomena by analyzing Eq.(5). Model I (Mualem, 1973) in Table 2 is to simulate hysteresis phenomena by analyzing Eq.(5) where the relation of $f(\Psi^d, \Psi^w)=h(\Psi^d).m(\Psi^w)$ is used and the Neel diagram considering only the hysteretic contribution satisfying $\Psi^w \geq \Psi^d$ (that is, $\rho \geq r$) is used to integrate Eq.(5). Model I-1 (Mualem, 1977) is by analyzing Eq.(5) where it is assumed that $h(r)=m(\rho)$ for the homogeneous porous medium, while Parlange's Model (Parlange, 1976, 1980; Mualem & Morel-Seytoux, 1978) is by analyzing Eq.(5) where it is assumed that $f(r, \rho)=m(\rho)$, $h(r)=1$ in Eq.(4) and Neel diagram is used. Model II (Mualem, 1974) is to simulate hysteresis phenomena by analyzing Eq.(5) where the relation of $f(r, \rho)=h(r).m(\rho)$ is used, the Mualem diagram considering both the hysteretic contribution and the reversible contribution is used to integrate Eq.(5), while Model II-1 (Mualem, 1977) is by analyzing Eq.(5) where it is assumed that $h(r)=m(\rho)$ in Eq.(4).

The dependent domain model (DDM) is introduced because IDM does not simulate scanning curves accurately compared with experimental data such as the sand with well-defined air entry value Ψ_{ae}. The meaning of DDM is IDM considering the weighting factor $P_a(\theta)$ and/or $P_w(\theta)$ accounting for the effect that each pore is affected by neighboring pores when air and/or water enters into the pore. Therefore, $P_a(\theta)$ is the weighting factor accounting for 'the pore blockage effect against air entry', which is a function of the effective moisture content and is defined as followings :

$$P_a(\theta)=\frac{\text{net amount of water extracted from pore}}{\text{amount of water when air has access to pore freely}} \tag{7}$$

which is greater than or equal to zero. Air enters into the pore hardly (or easily) because the pore blockage effect of neighboring pores against air entry is large (or small) if the pore contains large (or small) amount of the moisture content. The weighting factor $P_a(\theta)$ has the physical meaning such that $P_a(\theta)$ is approximately equal to 0 (or 1) due to the small (or equivalent) net amount of water extracted from the pore compared with the amount of water extracted from the pore when air has access to the pore freely. Similarly, the weighting factor accounting for the pore blockage effect against water entry can be defined. Therefore, DDM is to simulate hysteresis phenomena using the following Eq.(8), which considers the weighting factor $P_a(\theta)$ and/or $P_w(\theta)$ in Eq.(5),

$$\theta=\int\int f(r)\cdot m(\rho)\cdot P_a(\theta)\cdot P_w(\theta)\, dr d\rho \tag{8}$$

where DDM is same to IDM if $P_a(\theta)=P_w(\theta)=1$.

Model III (Mualem & Dagan, 1975) in Table 2 is to simulate hysteresis phenomena by analyzing Eq.(8) where only the pore blockage effect against air entry is considered ($P_a(\theta)\leq 1$, $P_w(\theta)=1$) and the Mualem diagram is used to integrate Eq.(8), while Model III-1 (Mualem, 1984) is by analyzing Eq.(8) where it is assumed that $h(r)=m(\rho)$ in Model III for the homogeneous porous medium. Model IV (Mualem and Dagan, 1975) is by analyzing Eq.(8) where $P_a(\theta)\leq 1$ and $P_w(\theta)\leq 1$, while Park and Sonu's Model (Park & Sonu, 1992) is by analyzing Eq.(8) where it is assumed that $h(r)=m(\rho)$ for the homogeneous porous medium. The above classification of DDM is summarized in Table 2.

Now let us consider the feature of each domain model shown in Table 2. When the similarity hypothesis is introduced to IDM, unknown variables in Eq.(5) which are expressed as forms of functions are H(R) and M(R) (that is, h(r) and m(ρ)) of Eq.(6), which require two main curves for the model calibration. At this time, Model I is to solve H(R) and M(R) by using the Neel diagram, while Model II is by using the Mualem diagram. When it is assumed that H(R) = M(R)(that is, h(r) = m(ρ)) or f(r, ρ) = m(ρ), there is only one unknown variable which can be solved using one branch of main curves. Therefore, Model I-1 and Parlange's Model are to solve one variable by using the Neel diagram, while Model II-1 is by using the Mualem diagram.

On the other hand, Model III is such that the similarity hypothesis is introduced to Eq.(8) where the weighting factor $P_w(\theta) = 1$ and $P_a(\theta)$ is the unknown variable. There are three unknown variables (that is, H(R), M(R) and $P_a(\theta)$) in Model III, which requires two main curves and one primary drying curve (since $P_a(\theta)$ is the unknown variable) for the model calibration. Three variables can be solved through integrating Eq.(8) using the Mualem diagram. There are two unknown variables in Model III under the assumption that H(R) = M(R) for the homogeneous porous medium, which becomes Model III-1. Thus, this model requires two main curves for the model calibration. Model IV is such that the similarity hypothesis is introduced to Eq.(8) where the weighting factor $P_a(\theta)$ and $P_w(\theta)$ are unknown variables. Thus, there are four unknown variables (that is, H(R), M(R), $P_a(\theta)$ and $P_w(\theta)$) in Model IV, which requires two main curves and two primary curves for the model calibration. Four variables can be solved through integrating Eq.(8) using the Mualem diagram. There are three unknown variables in Model IV under the assumption that H(R) = M(R) for the homogeneous porous medium, which becomes Park and Sonu's Model theoretically. Thus, this model requires two main curves and one primary curve for the model calibration.

4. Comparison and Discussion of Domain Model

The purpose of modeling hysteresis phenomenon is ultimately to analyze the unsaturated flow efficiently by means of developing the hysteresis model with high accuracy for simulating experimental data and/or with less data for the model calibration. These efforts have been continued since the development of Poulovassilis (1962)' domain model which needs two main curves and many primary curves for the model calibration. Here, the discussion is focused on the possibility of a new type model which requires only one branch of main curves for the model calibration and simulates hysteresis curves in good agreement with experimental data through the comparison of previous hysteresis models. Previous models can be classified into three types, of which only domain model has an advantage to be simplified by introducing an appropriate assumption to the domain model because the domain model simulates hysteresis curves based on the theoretical approach, while the interpolation model and the scaling model cannot be simplified in terms of structures of models. Therefore, the possibility of a new type model can be sought in the domain model.

Simplified domain models which require only one branch of main curves for the model calibration are Model I-1, Parlange's Model and Model II-1 based on the independent domain concept. Fig.2 and Fig.3 show the main drying curve (MDC) simulated from the main wetting curve (MWC) using these models for Rubicon Sandy Loam (Topp, 1969) and Dune Sand (Gillham et al., 1976), respectively. It may be concluded from Fig.2 and Fig.3 that simulated results using these models are not in good agreement with experimental data. Model Ⅰ-1 has a tendency to overestimate MDC compared with experimental data, and Model Ⅱ-1 has tendency to underestimate MDC. When MWC is simulated from MDC using these models, the simulated MWC passes the point B(θ_u, Ψ_{ac}) contained in MDC in Fig.1, which does not characterize the feature of hysteresis as shown in Fig.1. Therefore, Model Ⅰ-1 and Model Ⅱ-1 cannot simulate hysteresis curves physically. On the other hand, Parlange's Model cannot simulate the tendency as shown in MDC obtained by experiments, which results from the fact that the pore water distribution function f(r, ρ) is not always positive for all characteristic dimensionless pore radii, ρ, in Parlange's Model (Mualem & Morel-Seytoux, 1978). It is concluded that Model Ⅰ-1, Parlange's Model and Model Ⅱ-1 are not appropriate to the purpose of modeling hysteresis because these models do not simulate hysteresis curves accurately even so fully simplified.

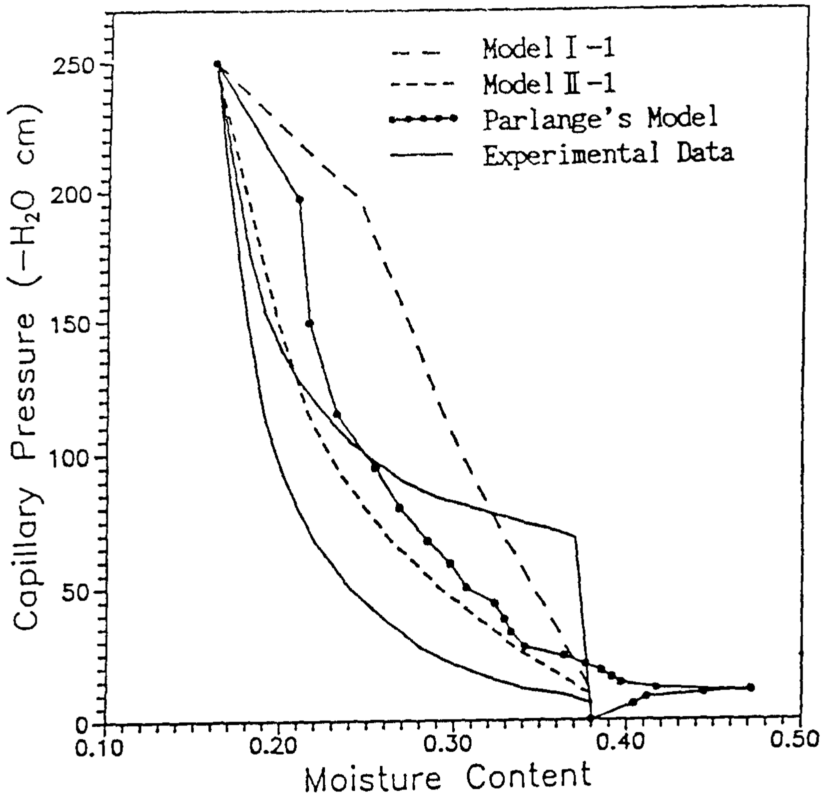

Fig 2 Comparison of Model Ⅰ-1, Parlange's Model & Model Ⅱ-1 with Experimental Data
[Rubicon Sandy Loam : after Topp(1969)]

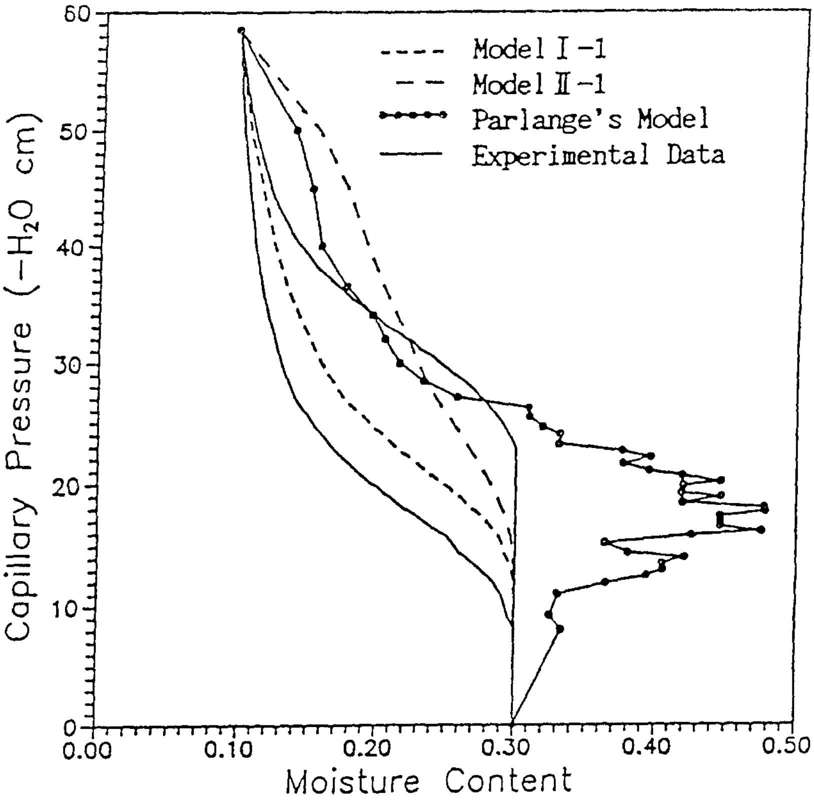

Fig 3 Comparison of Model Ⅰ-1, Parlange's Model & Model Ⅱ-1 with Experimental Data
[Dune Sand : after Gillham et al.(1976)]

In order to compare domain models in succession, IDM (Model Ⅰ and Model Ⅱ) and DDM (Model Ⅲ and Model Ⅲ-1) are to be selected. Fig.4 shows that each model simulates the primary drying curve (PDC) starting at $(\Psi^{w}_{1})=0.275$ for the sand (Poulovassilis & Childs, 1972), where θ (Ψ^{w}_{1}) is the moisture content corresponding to the capillary pressure Ψ^{w}_{1} at the reversal point where PDC starts. It is an acceptable result that PDC simulated by Model Ⅲ-1 is better agreement with experimental data than Model Ⅰ and Model Ⅱ, while three models mentioned above require two main curves for the model calibration. On the other hand, Model Ⅲ and Model Ⅲ-1 simulate PDC with almost same accuracy, while Model Ⅲ requires one more experimental data for the model calibration than Model Ⅲ-1 as shown in Table 2. Thus, Model Ⅲ-1 may be used to simulate hysteresis phenomena more efficiently. It can be concluded in comparison of Model Ⅰ, Model Ⅱ, Model Ⅲ and Model Ⅲ-1 with experimental data as shown in Fig.4 that Model Ⅲ-1 is most appropriate to the purpose of modeling hysteresis.

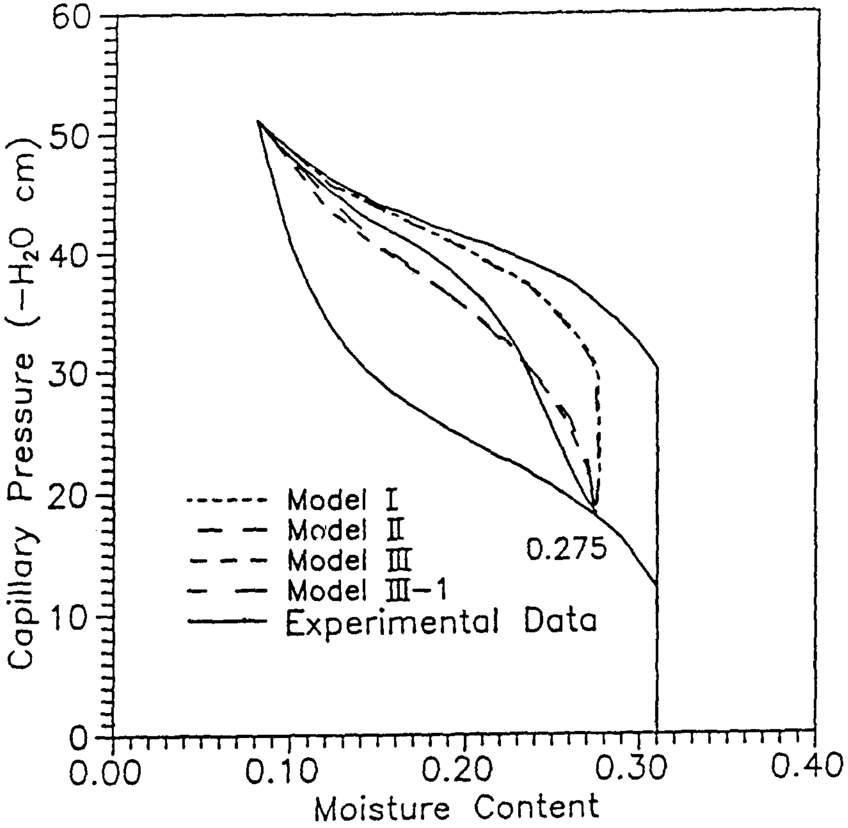

Fig 4 Simulated Primary Drying Curves using Various Domain Models
[Sand : after Poulovassilis & Childs(1971)]

Primary wetting curves (PWCs) simulated by Model III-1 are compared with experimental data denoted as circles in Fig.5, where the simulated PWCs are good agreement with data. As shown in Fig.1, Model III-1 does not cause the pumping effect because primary curves and secondary curves form closed curves. Therefore, it may be concluded that there is a possibility of a new type of the hysteresis model in DDM through the comparison of domain models.

Meanwhile, Model IV needs two main curves and two primary curves for the model calibration, and Park and Sonu's Model needs two main curves and one primary curve as shown in the Table 2. But these two models which are based on the dependent domain concept are not appropriate to the purpose of modeling hysteresis because of two reasons as discussed below. First, weighting factors $P_a(\theta)$ and $P_w(\theta)$ are coupled and two factors must be solved using the trial-and error method. Therefore, these models are not efficient because of complexity of the model calibration. Second, when the independent domain model (IDM) cannot simulate scanning curves physically, Model III-1 introducing the weighting factor $P_a(\theta)$ to IDM simulates the scanning curves with good accuracy as shown in Fig.5. Therefore, it is needless to introduce the weighting factor $P_w(\theta)$ causing complexity of the model.

Summarizing the above discussion on the dependent domain model, Model Ⅳ and Model Ⅳ-1 is not appropriate to the purpose of modeling hysteresis. While Model Ⅲ and Model Ⅲ-1 simulate scanning curves with approximately same accuracy, Model Ⅲ-1 is more efficient to analyze the unsaturated flow than Model Ⅲ because Model Ⅲ requires one more experimental data for the model calibration. Therefore, it is reasonable to seek a new type of the hysteresis model which needs only one branch of main curves in Model Ⅲ-1 by simplifying this model through the introduction of an appropriate assumption.

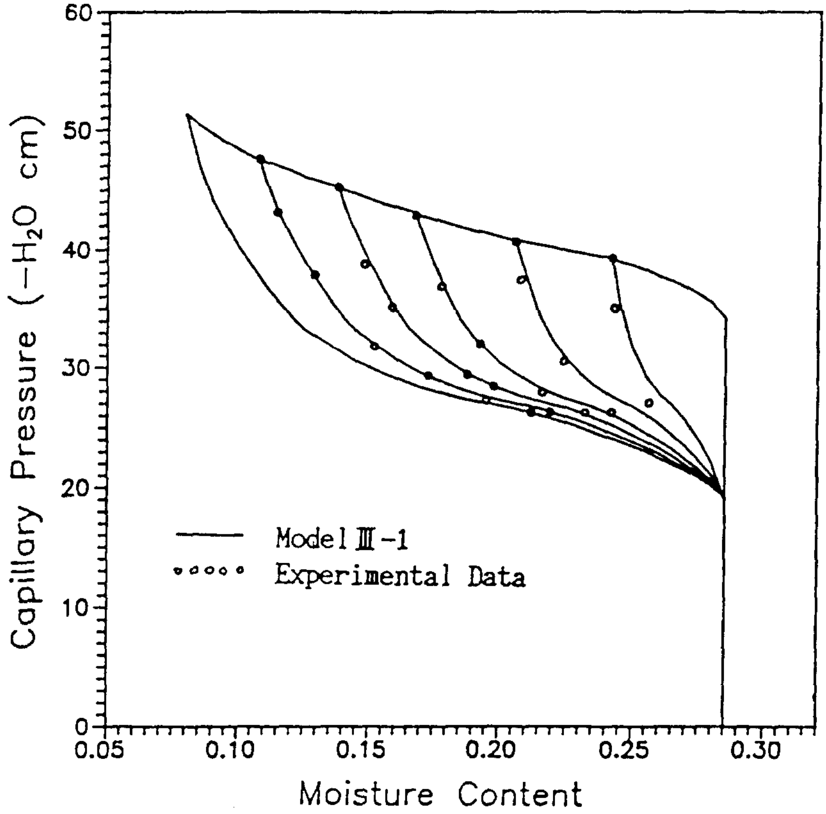

Fig 5 Comparison of Simulated PWCs using Model Ⅲ-1 with Experimental Data
[Mixed Sand of Two Fraction : after Poulovassilis & El-Ghamry(1977)]

5. Concluding Remarks

In the view of the purpose of modeling hysteresis, that is, to develop a model which requires less experimental data for the model calibration with keeping accuracy in simulating hysteresis curves and which is ultimately used to analyze the unsaturated flow efficiently, it is useful to develop a new model which requires only one branch of main curves because measuring hysteresis curves by experiments is expensive and time-consuming.

Published hysteresis models can be classified into three types: the interpolation model, the scaling model and the domain model, of which only domain model has theoretical background. Therefore, a new model can be sought in the domain model.

Previous models based on the independent domain concept such as Model I-1, Parlange's Model and Model II-1 require one branch of main curves for the model calibration. But these models are not appropriate to the purpose of modeling hysteresis because these models has a drawback to simulate experimental data of Rubicon Sand in Fig.2 and Dune Sand in Fig.3 with poor accuracy although fully simplified. Therefore, an alternative model can be sought in the dependent domain model (DDM). DDM is a model to introduce the weighting factor $P_a(\theta)$ and/or $P_w(\theta)$ accounting for the pore blockage effect against air and/or water entry to IDM. Among DDMs, Model IV and Park and Sonu's Model which consider both $P_a(\theta)$ and $P_w(\theta)$ are not appropriate to the purpose of modeling hysteresis because these models has a drawback that their efficiency is very low in calibrating models as discussed in the previous section.

On the other hand, Model III and Model III-1 which consider only $P_a(\theta)$ simulate experimental data on sand with almost same accuracy. But Model III needs one more data for the model calibration than Model III-1. Therefore, there is a possibility of a new type model in Model III-1. The weighting factor $P_a(\theta)$ accounting for the pore blockage effect against air entry has a physical meaning as discussed in the section 3. Therefore, if the unknown variable $P_a(\theta)$ is treated as a known variable by introducing an appropriate assumption based on a physical procedure, Model III-1 reduces to a new simplified model which contains only one unknown variable (reference Table 2). Conclusively, a new model where the weighting factor $P_a(\theta)$ reduces to a known variable through an appropriate method is an alternative one which requires only one branch of main curves for the model calibration, if the model does keep accuracy in simulating hysteresis curves compared with experimental data.

References

1. Beese, F. and van der Ploeg, R. R., (1976), "Influence of Hysteresis on Moisture Flow in an Undisturbed Soil Monolith", Jr. of Soil Sci. Soc. Am., 40, pp.480-484
2. Curtis, A. A. and Watson, K. K. V, (1984), "Hysteresis Affected Water Movement in Scale

Heterogeneous Profiles", W.R.R., Vol.20, No.6, pp.719-726

3. Dane, J. H. and Wierenga, P. J., (1975), "Effect of Hysteresis on the Prediction of Infiltration, Redistribution and Drainage of Water in a Layered Soil", Jr. of Hydrology, 25, pp.229-242
4. Gillham, R. W., Klute, A. and Heermann, D. F., (1976), "Hydraulic Properties of a Porous Medium: Measurement and Empirical Representation", Jr. of Soil Sci. Soc. Am., 40(2), pp.203-207
5. Haverkamp, R., Vauclin, M., Touma, J., Wierenga, P. J. and Vachaud, G., (1977), "A Comparison of Numerical Simulation Models for One-Dimensional Infiltration", Jr. of Soil Sci. Soc. Am., Vol.41, pp.285-294
6. Hoa, N. T., Gaudu, R. and Thirriot, C., (1977), "Influence of the Hysteresis Effect on Transient Flows in Saturated-Unsaturated Porous Media", W.R.R., 13(6), pp.992-996
7. Hogarth, W. L., Hopmans, J., Parlange, J.-Y. and Haverkamp, R., (1988), "Application of a Simple Soil-Water Hysteresis Model", Jr. of Hydrology, Vol.98, pp.21-29
8. Hopmans, J. W. and Dane, J. H., (1986), "Combined Effect of Hysteresis and Temperature on Soil-Water Movement", Jr. of Hydrology, 83, pp.161-171
9. Klute, A. and Heermann, D. F., (1974), "Soil Water Profile Development under a Periodic Boundary Condition", Soil Science, Vol.117, No.5, pp.265-271
10. Kool, J .B. and Parker, J. C., (1987), "Development and Evaluation of Closed-Form Expressions for Hysteretic Soil Hydraulic Properties", W.R. R, 23(1), pp.105-114
11. Milly, P. C. D., (1982), "Moisture and Heat Transport in Hysteretic, Inhomogeneous Porous Media: A Matric Head-Based Formulation and a Numerical Model", W.R.R., 18(3), pp.489-498
12. Mualem, Y., (1973), "Modified Approach to Capillary Hysteresis Based on a Similarity Hypothesis", W.R.R., Vol.9, No.5, pp.1324-1331
13. Mualem, Y., (1974), "A Conceptual Model of Hysteresis", W.R.R., Vol.10, No.3, pp.514-520
14. Mualem, Y., (1976), "Hysteretical Models for Prediction of the Hydraulic Conductivity of Unsaturated Porous Media", W.R.R., Vol.12, No.6, pp.1248-1254
15. Mualem, Y., (1977), "Extension of the Similarity Hypothesis Used for Modeling the Soil Water Characteristics", W.R.R., Vol.13, No.4, pp.773-780
16. Mualem, Y. and Morel-Seytoux, H. J., (1978), "Analysis of a Capillary Hysteresis Model Based on a One-Variable Distribution Function", W.R.R., 14(4), pp.605-610
17. Mualem, Y., (1984), "A Modified Dependent-Domain Theory of Hysteresis", Soil Science, Vol. 137, No.5, pp283-291
18. Mualem, Y. and Dagan, G., (1975), "A Dependent Domain Model of Capillary Hysteresis", W.R. R., Vol.11, No.3, pp.452-460
19. Mualem, Y. and Miller, E. E., (1979), "A Hysteresis Model Based on an Explicit Domain- Dependence Function", Jr. of Soil Sci. Soc. Am., Vol.43, pp.1067-1073
20. Nieber, J. L. and Walter, M. F., (1981), "Two-Dimensional Soil Moisture Flow in a Sloping Rectangular Region: Experimental and Numerical Studies", W.R.R., 17(6), pp.1722-1730
21. Park, C. K. and Sonu, J. H.,(1992) "Capillary Hysteresis Model in Unsaturated Flow: State of

The Art", Jr. of KAHS (Korea), Vol.25, No.3, pp.65-77

22. Parker, J. C. and Lenhard, R. J., (1987), "A Model for Hysteretic Constitutive Relations Governing Multiphase Flow: 1.Saturation-Pressure Relations", W.R.R., 23(12), pp.2187-2196
23. Parlange, J. -Y., (1976), "Capillary Hysteresis and the Relationship between Drying and Wetting Curves", W.R.R., 12(2), pp.224-228
24. Parlange, J. -Y., (1980), "Water Transport in Soils", Ann. Rev. of Fluid Mech., Vol.12, pp.77-102
25. Philip, J. R., (1964), "Similarity Hypothesis for Capillary Hysteresis in Porous Materials", Jr. of Geophysical Research, Vol.69, No.8, pp.1553-1562
26. Poulovassilis, A., (1962), "Hysteresis of Pore Water, an Application of the Concept of Independent Domains", Soil Science, Vol.93, pp.405-412
27. Poulovassilis, A., (1970a), "Hysteresis of Pore Water in Granular Porous Bodies", Soil Science, Vol.109, No.1, pp.5-12
28. Poulovassilis ,A., (1970b), "The Effect of the Entrapped Air on the Hysteresis Curves of a Porous Body and on its Hydraulic Conductivity", Soil Science, Vol.109, No.3, pp.154-162
29. Poulovassilis, A., (1977), "Hysteretic Steady State Soil Water Profile", W.R.R., 13(3), pp.549-557
30. Poulovassilis, A. and Childs, E. C., (1971), "The Hysteresis of Pore Water: The Non-Independence of Domains", Soil Science, Vol.112, No.5, pp.301-312
31. Richards, L. A., (1931), "Capillary Conduction through Porous Medium", Physics, Vol1, pp.318-333
32. Royer, J. M. and Vachaud, G., (1975), "Field Determination of Hysteresis in Soil-Water Characteristics", Proc. of Soil Sci. Soc. Am., 39(2), pp.221-223
33. Scott, P. S., Farquhar, G. L. and Kouwen, N., (1983), "Hysteretic Effects on Net Infiltration", Advances in Infiltration,Proc. of National Conference, ASAE, pp.163-170
34. Singh, V. P., (1989), Hydrologic Systems Volume : Watershed Modeling, Ch.5 Infiltration, Prentice Hall, New Jersey, pp.35-77
35. Singh, V. P. and Woolhiser, D. A., (1976), "Sensitivity of Linear and Nonlinear Surface Runoff Models to Input Errors", Jr. of Hydrology, 29, pp.243-249
36. Stauffer, F., (1982), "Numerical Simulation of Infiltration into Porous Media and Response of the Water Table", Finite Element in Water Resources, Proc. of 4th International Conference, Harnover, Germany, pp.(10)73-82
37. Talsma, T., (1970), "Hysteresis in Two Sands and the Independent Domain Model", W.R.R., Vol.6, No.3, pp.964-970
38. Topp, G. C., (1969), "Soil-Water Hysteresis Measured in a Sandy Loam and Compared with the Hysteretic Domain Model", Proc. of Soil Sci. Soc. Am., 33, pp.645-651
39. Topp, G. C., (1971a), "Soil Water Hysteresis in Silt Loam and Clay Loam Soils", W.R.R., Vol.7, No.4, pp.914-920

40. Topp, G. C., (1971b), "Soil-Water Hysteresis : The Domain Theory Extended to Pore Interaction Conditions", Proc. of Soil Sci. Soc. Am., Vol.35, pp.219-225

41. Topp, G. C., Klute, A. and Peters, D. B., (1967), "Comparison of Water Content-Pressure Head Data Obtained by Equilibrium, Steady-State and Unsteady-State Method", Proc. of Soil Sci. Soc. Am., 31, pp.312-314

42. Topp, G. C. and Miller, E. E., (1966), "Hysteretic Moisture Characteristics and Hydraulic Conductivities for Glass-Bead Media", Proc. of Soil Sci. Soc. Am., Vol.30, pp.156-162

43. Vachaud, G. and Thony, J. -L., (1971), "Hysteresis during Infiltration and Redistribution in a Soil Column at Different Intial Water Contents", W.R.R., Vol.7, No.1, pp.111-127

44. Watson, K. K. and Sardana, V., (1987), "Numerical Study of the Effect of Hysteresis on Post-Infiltration Redistributon", in Infiltration Development and Application, edited by Y-.S. Fok, pp. 241-250

45. Watson, K. K., Reginato, R. J. and Jackson, R. D., (1975), "Soil Water Hysteresis in a Field Soil", Proc. of Soil Sci. Soc. Am., 39, pp.242-246

46. Whisler, F. D. and Watson, K. K., (1969), "Analysis of Infiltration into Draining Porous Media", Jr. of Irrig. Drain. Div., Proc. of ASCE, Vol.95(IR4), pp.481-491