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Input Quantity Control in a Multi-Stage Production System with Yield Randomness, Rework and Demand Uncertainty

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Abstract

In this paper, we investigate the effects of yield randomness for lot-sizing in a multi-stage production system. The practical importance of incorporating yield randomness into production models has been emphasized by many researchers. Yield randomness, especially in semiconductor manufacturing, poses a major challenge for production planning and control. The task becomes even more difficult if the demand for final product is uncertain. An attempt to meet the demand with a higher level of confidence forces one to release more input in the fabrication line. This leads to excessive work-in-process (WIP) inventories which cause jobs to spend unpredictably longer time waiting for the machines. The result is that it is more difficult to meet demand with exceptionally long cycle time and puts further pressure to increase the safety stocks. Due to this spiral effect, it is common to find that the capital tied in inventory is the most significant factor undermining profitability. We propose a policy to determine the quantity to be processed at each stage of a multi-stage production system in which the yield at each stage may be random and may need rework.

1. Introduction

In this paper, we shall consider a multi-stage serial production system in which the yield(output) at each stage may be random and may need rework. Rework is allowed only once to make good output. The output which is still bad after rework is scrapped. The demand at the final stage is also uncertain. In this situation, we shall determine the quantity that should be processed at each stage as a product proceeds through a series of stations in its manufacturing sequence.

For multi-stage serial production systems without rework and with known demand, Lee and

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Yano [4] showed that the optimal input policy is of the critical number type. At each stage, if the output from an upstream stage exceeds some critical number, then only an amount equal to the critical number is used as input for the current stage, otherwise, all the available output is used. They assumed that all defective items are scrapped. However, as Wein [8] points out, some of the process steps in semiconductor fabrication have rework capabilities which are used extensively. In particular, the photolithography step is a major source of yield problems with rework capabilities.

We, thus, extend their work to allow for rework and demand uncertainty. We, however, use yield rate average from empirical or theoretical yield rate distribution to develop the model even though our approach may include yield rate distribution in itself with little difficulty. A single rework attempt is made at the rework center to make good output and average rework success rate is also used to develop the model. Wein [8] assumes that demand is given but we allow for demand uncertainty.

2. Problem formulation

The system considered here is an N-stage serial production system in which the yield at each stage may be random and may need rework. We assume that we have yield rate average at each stage from empirical yield rate data. We, however, assume that yield rate distributions at the various stages are mutually independent and that each has a range between zero to one. Beta distribution is usually considered as one of likely yield rate distributions. Throughout the paper, we use the term yield rate to refer to the fraction of good parts.

We assume that there is a single product and that the product has a predetermined routing through the production system represented conceptually as a serial system. There is demand uncertainty for the finished product and the problem is to determine the optimal input quantity at each manufacturing stage to minimize the expected total cost consisting of production cost, rework cost, disposal cost and shortage cost.

The flow of items in the system may be represented diagrammatically in Figure 1.

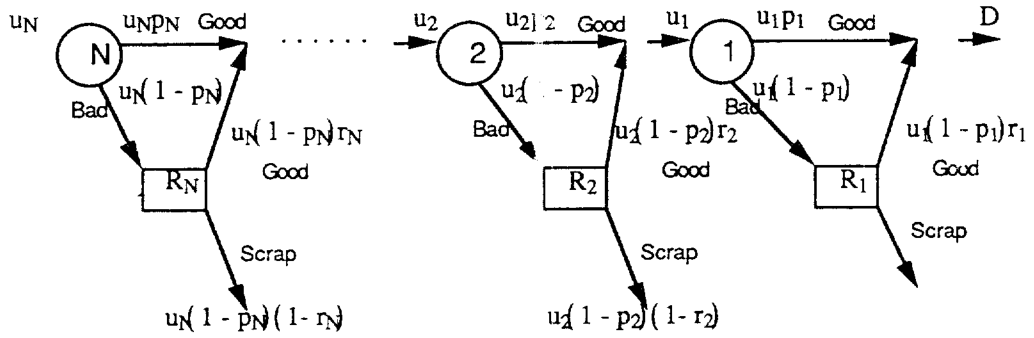

Figure 1. Serial Production System with Rework

The stages are numbered so that the stage of production to be performed first is designated as stage N, while the final stage of production is referred to as stage 1. The decision at atage n,n=1 to N, is the input quantity at stage n denoted as u_n. The input to stage n is obviously constrained by the output of stage n+1, which, in turn, depends on the input at stage n+2, etc. The effective output of stage n, including that from rework, is $y_n^E = u_n [p_n + (1-p_n) r_n]$, where p_n denotes the actual yield fraction at stage n and r_n denotes the fraction good from rework. There is effective yield loss of $(1-p_n)(1-r_n) u_n$ and overage quantity of $(y_{n+1}^E - u_n)^+$, where $(\ )^+$ indicates the positive part. At the final stage of production, the shortage quantity is $(D - y_1^E)^+$. We assume that the initial inventory of all semi-finished items is zero.

Costs to consider are : (1) w_n = production cost per unit of input at stage n, (2) h_n = net cost of disposing a unit produced at stage n but not used at stage n-1, (3) s_n = rework cost per unit reworked, and (4) π = shortage cost for each unit of unsatisfied demand. Demand at final stage is assumed to have the probability distribution function of f(D).

We shall now explain the solution approach.

Let y_{N+1} denote the available input (raw material) for the production stage N. We formulate the problem as

$$\underset{u_1,u_2,\cdots,u_N}{\text{minimize}}\ E \left\{ \sum_{n=1}^{N} [w_n u_n + (1-p_n) u_n s_n + h_{n+1} (y_{n+1}^E - u_n)^+] + h_1 (y_1^E - D)^+ + \pi (D - y_1^E)^+ \right\}$$

$$\text{subject to } 0 \le u_n \le y_{n+1}^E,\ n=1,\cdots,N,$$

$$y_n^E = u_n [p_n + r_n(1-p_n)],\ n=1,\cdots,N+1,$$

Here, D possesses the density function of f(D).

This formulation requires that the decision variables, u_n, be selected before the y_{n+1}^E values are known. In reality, one does not need to specify u_n until y^E_{n+1} is known. We develop the dynamic programming formulation as follows to reflect the dynamic nature of the problem.

Let $C_n(y^E_{n+1})$ represent the expected cost of operating the system optimally from stage n through stage 1, given the effictive output from stage n+1 is y^E_{n+1}, n=1 to N. Then $C_N(y^E_{N+1})$ represents the minimum total cost to operate the whole system.

Obviously, we must have $u_n \le y^E_{n+1}$ (i.e., the input cannot exceed the output of the preceding stage) at each stage. Therefore, the dynamic programming recursion relationships are

$$C_n(y^E_{n+1}) = \min_{0 \le u_n \le y^E_{n+1}} \{ [w_n u_n + (1-p_n)\, u_n s_n + h_{n+1}(y^E_{n+1} - u_n) + E[C_{n-1}(y^E_n)]] \},\ n=2,\cdots,N \qquad (*)$$

and

$$C_n(y^E_2) = \min_{0 \le u_1 \le y^E_2} \{ w_1 u_1 + (1-p_1)\, u_1 s_1 + h_2(y^E_2 - u_1) + h_1 E(y^E_1 - D)^{+} + \pi E(D - y^E_1)^{+} \} \qquad (**)$$

where, $y^E_n = u_n\, [p_n + r_n(1-p_n)]$, n=1,⋯,N+1.

Let u^*_n represent the optimal value of u_n. We, thus, determine u^*_1 from Equation (**), and successively determine u^*_n in the sequence given y^E_{n+1} for n=2,⋯,N form Equation (*).

We have a unique set of finite values $S_1, S_2, \cdots, S_N$ such that $S_N \ge S_{N-1} \ge \cdots \ge S_1$, and also have the optimal production decisions as

$$u^*_n = \begin{cases} y^E_{n+1} & \text{if } y^E_{n+1} < S_n \\ S_n & \text{otherwise} \end{cases}$$

if the following conditions hold.

Condition 1 is $w'_n + h_n\, p'_n > h_{n+1}$ where $w'_n = w_n + (1-p_n)s_n$ and $p'_n = p_n + r_n(1-p_n)$.

Condition 2 is $-h_{n+1} + \sum_{j=0}^{n-1} [w'_{n-j} \prod_{i=1}^{j} p'_{n-i+1}] \le \pi \prod_{i=1}^{n} p'_i$.

In other words, the optimal policy has a single critical number for each stage (S_n), which is independent of the output of the preceding stages. S_n is, of course, the value of u^*_n that satisfies the first-order condition.

3. Example

We consider data in table 1 for the two stage production line.

The demand of final stage is assumed to have probability distribution function as

$$f(D) = \frac{1}{7000} e^{-\frac{1}{7000}D},\ D>0$$

Table 1. Example data

Costs \ Stages	Stage 2	Stage 1
Production Cost / unit	0.63	0.82
Rework Cost / unit	0.35	0.50
Disposal Cost / unit	0.10	0.20
Shortage Cost / unit	N / A	2.50
Yield Rate	0.82	0.91
Rework Success Rate	0.75	0.80

We have expected cost at stage 1 as

$$C_1(y_2^E) = \min_{0\le u_1 \le y_2^E} [w_1u_1 + (1-p_1)\ u_1s_1 + h_2(y_2^E - u_1) + h_1E(y_1^E - D) + \pi E(D-y_1^E)]$$

where

$$E(y_1^E - D) = \int_0^{y_1^E} (y_1^E - D)\ f(D)\ dD \text{ and } E(D-y_1^E) = \int_{y_1^E}^{\infty} (D-y_1^E)\ f(D)\ dD$$

After differentiating $C_1(y_1^E)$ with respect to u_1 and setting this first derivative to zero, we achieve, with some calculation,

$$F(y_1^E) = \int_0^{y_1^E} f(D)dD = \frac{h_2 - w_1 - (1-p_1)s_1 + \pi[p_1 + r_1(1-p_1)]}{(h_1+\pi)(p_1+r_1(1-p_1))}$$

By substituting the values from the table, we get the values of (y_1^E) as 7140. Therefore, we

have $u_1^* = \dfrac{y_1^E}{p_1+r_1(1-p_1)} = 7270.88$

We proceed to get the values of u_2^* in the same way.

4. Summary

We have developed an approach for the production control in a serial production system in which the yield at each stage may be random and may need rework. We incorporate realistic factors such as demand uncertainty and rework into Lee and Yano's [4] model. A procedure using the dynamic programming formulation is developed and provides optimal solutions for an N-stage system. We also present an example to illustrate how the procedure can be applied.

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