Journal of the Korean
OR/ MS Society
Vol. 18, No. 1, April 1993

Grid Search Based Production Switching Heuristic for Aggregate Production Planning

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Abstract

The Production Switching Heuristic (PSH) developed by Mellichamp and Love (1978) has been suggested as a more realistic, practical and intuitively appealing approach to aggregate production planning(APP). In this research, PSH has been modified to present a more sophisticated open grid search procedure for solving the APP problem. The effectiveness of this approach has been demonstrated by determining a better near-optimal solution to the classic paint factory problem. The performance of the modified production switching heuristic is then compared in the context of the paint factory problem with results obtained by other prominent APP models including LDR, PPP, and PSH to conclude that the modified PSH offers a better minimum cost solution than the original PSH model.

1. Introduction

Aggregate production planning(APP) involves the simultaneous determination of a company's production, inventory and employment levels over a finite time horizon. Its objective is to minimize the total relevant costs while meeting non-constant, time varying demand, assuming fixed sales and production capacity[9, 12]. Since the early 1950's approaches for APP have varied from simplistic, graphical methods to more sophisticated optimizing, search, parametric, and dynamic methods. These fall into two broad categories—those which guarantee a mathematically optimal solution with respect to the model and those that do not[1, 4].

Near-optimal approaches, including Search decision rule(SDR) [10], Management coefficient model(MCM) [2], Parametric production planning(PPP) [6], overcome some of the problems associated with optimal approaches. Complex cost functions which accurately describe actual

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costs may be embodied in most near optimal models. An analysis of the impact of forecast errors on strategy development may also be performed by incorporating stochastic demand characteristics in near optimal models[7]. Despite these improvements, however, these models suffer from a limitation that also applies to optimal models. That is, most of these models produces a different set of values for the decision variables—production rate (P_t), work force level (W_t), and inventory level (I_t)—for each period in the planning horizon. This probably is the single most important factor that has contributed to limiting the application of all aggregate planning models. In other words, a majority of APP approaches incorporates continuous decision variables that require frequent adjustments to both production and work force settings to achieve a minimum cost solution.

A large set of decision variable values which frequently adjust the production and workforce level on a planning period by period basis has been observed as being inconsistent with management practices in industry. Mellichamp and Love [7] developed the Production Switching Heuristic (PSH) to address this inconsistency with the belief, thus, of having more appeal to practicing managers.

In this research, PSH has been modified to present a more elaborated grid search model that conducts all parameters to minimize the total cost over the planning horizon. To evaluate and validate the modified approach offered, the paint factory problem first described by Holt et al. [5] has been used.

2. Production Switching Heuristic (PSH)

This heuristic approach is based on the observation that managers seem to favor one large change in work force over a series of smaller and more frequent changes over the planning horizon. Thus, as long as demand is being met, i.e., stockouts do not occur too frequently and inventory levels do not increase drastically, managers are often inclined to maintain the same production and work force levels, making minor adjustments when necessary.

Furthermore, a policy that requires frequent hiring and firing of personnel might be impractical because of prior contract agreements, or undesirable due to the potential negative effects on the firm's public image[8]. If production is confined to a relatively small number of prescribed levels (so that adjustment in production is achieved by given discrete steps), experience of performance and scheduled activities at each level provide good opportunities for controlling costs and minimizing the effects of change[3].

From these and other arguments Mellichamp and Love reasoned that an aggregate production planning methodology which utilizes near-optimal solution techniques to select a small number of decision variable values that are efficient over most levels of demand would have much potential for industry applications. This approach is directed to situations where three-production levels (high, normal, and low) can only be changed in discrete increments or decrements, such as adding or removing a production shift.

In their approach they also described switching algorithms for desirable fixed production levels by analyzing alternative values of various control parameters which provided a set of production, work force, and inventory decisions which were directly related to cost performance over a planning horizon.

3. Modified Production Switching Heuristic(MPSH)

The problem, therefore, was to find the best set of the control parameters. The Production switching heuristic of Mellichamp and Love, however, limited grid search options in analyzing all sets of control parameters—in effect hindering their approach from determining a better solution.

In this research, the production switching heuristic by Mellichamp and Love [7] is modified by using a more elaborate grid search method which exhausts reasonable incremental values over the entire cost surface. This search method widely opens all grid options to evaluate a broader set of alternative parameters than the original PSH approach.

Two different schemes have been proposed as options of the grid search with this alternative approach for evaluating the productivity function used in PSH and then to determine the optimum combination between production and work force sizes. The productivity function, developed in PSH, has been modified in the proposed approach to provide for a better balance between regular work force and overtime rates than that in PSH. Furthermore, it has been demonstrated that the modified productivity function yields better results for reducing regular payroll costs.

To evaluate and validate the modified approach offered, the paint factory problem[5] has been used. Based on the two search schemes, which are labeled as MPSH1, and MPSH2, the paint factory problem is solved. The results are compared with those obtained from the Parametric Production Planning (PPP), PSH, and Linear Decision Rule (LDR)—(i.e both other near-optimal and optimal solutions) reported in Mellichamp and Love to demonstrate a better performance of the modified PSH.

3.1 Determination of Production Levels

The production switching algorithm is used for the assignment of production levels to each planning period using a reasonable control mechanism. This research uses three production levels which are H(high), L(low), and N(normal) as follows :

$$P_t = \begin{cases} L & \text{if } F_t - I_{t-1} < L - C \\ H & \text{if } F_t - I_{t-1} > H - A \\ N & \text{otherwise} \end{cases}$$

where F_t=forecasted demand for period t, I_{t-1}=ending inventory for period t−1, A=Minimum acceptable target inventory, and C=Maximum acceptable target inventory.

The heuristic suggests that if the net production required after taking into account on-hand and target inventories is less than the low level of production, produce at the low level. If the net production required is greater than the high level of production, produce at the high level. Finally, if required net production is between the low and the high levels of production, produce at the normal level.

3.2. Determination of Work Force Rates

Mellichamp and Love[7] presented this issue by introducing a productivity function, that is given below, to determine an appropriate level of regular payroll versus overtime labor.

$$W_t = f(P_t, G)$$

where G is the percent increase or decrease in the work force required to achieve high or low levels of production.

In order to determine the optimum combination between the production and the work force sizes, this research suggests the following two schemes for the operation of productivity function. The proposed schemes are labeled MPSH1 (Modified PSH1) for scheme 1, and MPSH (Modified PSH2) for scheme 2, and each of these can run separately :

Scheme 1

$$W_t \begin{cases} = N/k & \text{if } P_t = N \\ = (H/k) * G & \text{if } P_t = H \\ = (L/k) * G & \text{if } P_t = L \end{cases}$$

Scheme 2

$$W_t \begin{cases} = N/k & \text{if } P_t = N \\ = N/k + (E/k) * G & \text{if } P_t = H \\ = N/k - (E/k) * G & \text{if } P_t = L \end{cases}$$

where k is the productivity factor such that N/k equals the number of workers necessary to produce N units in regular time without incurring any overtime or undertime.

The other factor, G, as a option of grid search, controls the proportion of hiring (firing) and overtime in adjusting the work force size when production is switched from normal to high (low) levels. G ranges from 1 to 0 (decreased in steps of 0.1).

3.3 Application

In order to demonstrate the modified production switching heuristics (MPSH) described and to evaluate its performance relative to other aggregate production planning approaches, the MPSH proposed in this research is applied to the paint factory problem. The paint factory problem has been used in the context of introducing most new or modified APP models proposed by various authors to evaluate whether the newer model can perform as well as the LDR. Competing models are generally judged by evaluating the method which minimizes the total costs using LDR-type quadratic cost functions.

The cost relationships used for the paint factory were :

$C_{1t} = 340 * W_t$ (Regular Payroll)

$C_{2t} = 64.3 * (W_t - W_{t-1})^2$ (Hiring and Layoffs)

$C_{3t} = 0.2 * (P_t - 5.67 * W_t)^2 + 51.2 * P_t - 281 * W_t$ (overtime)

$C_{4t} = 0.0825 * (I_t - 320)^2$ (Inventory)

where,

P_t = production level during the period t

W_t = work force size during the period t

I_t = ending inventory for period t

$t = 1, 2, \cdots, 12.$

The objective is to minimize total costs :

$$TC = C_{1t} + C_{2t} + C_{3t} + C_{4t}.$$

Other information given for the Paint Factory are :

beginning inventory(I_0)=263

beginning work force(W_0)=81.

Especially, 5.67, the coefficient in the overtime cost component, is the average worker productivity which, is defined as k in the productivity function. The overtime cost component yields negative overtime costs for certain values of P_t and W_t. Whenever this occurred in the calculations, C_{ot} was set to zero. In the paint factory problem, when back orders occur they show up in the results as minus inventory.

4. Program Summary

The program seeks to minimize total production costs by searching at the best combinations of inventory, work force and overtime costs. It incorporates 5 parameters(N, E, B, D and G—we note that G links with productivity function) using the relationships specified below :

H=N+E,
L=N−E,
A=B−D,
C=B+D.

A fundamental assumption made is that productivity per worker is a constant. Starting with the forecasts, an initial production rate(N) is arbitrarily determined by looking at the demand over the time horizon and picking an initial value for N that falls somewhere between the high and low demand value. An estimation of the target inventory level (B) is made by looking at the current level of inventory, I_0, then picking an inventory level small enough to include the initial inventory level in a grid search (in the paint factory problem 240 was selected). With initial values for N and B the grid search can be iterated by fixed increments to determine the best values of N and B to minimize the cost function.

The level of production needs to be assigned as either High, Normal or Low by using the appropriate switching algorithms. The differences between Normal and High(Low) production rates and inventory levels are labeled E and D, respectively. These differences are also increased by fixed increments after each iteration

At each iteration, only one parameter is exhausted according to the DO LOOP in a computer program. The search method is carried out in three steps with reasonable incremental values to reduce CPU time and systematically search over the entire cost surface for the minimum cost point.

Given the demand forecasts for the next twelve months in the paint factory problem, the PSH determines the best control parameters available. These parameters determine shift settings, overtime levels, production levels, and inventory levels that minimize aggregate costs.

5. Results and Analysis

The computer search routine was conducted to determine the best values to use for control parameters with the either MSPH scheme. Table 1, 2, and 3 provide the period by period production plans resulting from using LDR, PPP, PSH with the paint factory problem, respectively, when perfect forecasts (no errors between forecasted and actual sales) are available to the firm[7]. At this point, it should be recognized that LDR gives an optimal solution to the paint factory problem against which all other approaches should be compared. The PPP and PSH results are both near optimal.

The search routines for MPSH's result in a set of parameters, respectively, that minimizes the total cost. These provide the results for the period by period production plans using MPSH1 and MPSH2 as shown in table 4.

Table 1. Linear Decision Rule (LDR) Optimal Aggregate Plan

Period	Forecast (gallons)	Production (gallons)	Work force (people)	Inventory (gallons)
0	–	-	81.00	263.00
1	430	467.72	78.63	292.72
2	447	441.32	75.32	289.08
3	440	414.88	72.24	263.92
4	316	379.83	69.55	328.75
5	397	375.28	67.21	309.03
6	375	367.09	66.29	301.12
7	292	358.51	65.66	369.64
8	458	380.57	65.87	295.21
9	400	376.80	66.49	270.01
10	350	366.70	67.68	283.71
11	284	366.59	69.67	365.30
12	400	405.95	72.62	366.24

Table 2. Parametric Production Planning (PPP) Aggregate Plan

Period	Forecast (gallons)	Production (gallons)	Work force (people)	Inventory (gallons)
0	–	–	81.00	263.00
1	430	461.26	78.56	286.26
2	447	440.50	75.37	281.76
3	440	417.[illegible]1	72.44	258.88
4	316	380.38	69.82	324.28
5	397	379.30	67.98	309.06
6	375	371.34	66.74	305.39
7	292	360.[illegible]	66.13	376.30
8	458	390.73	66.53	312.03
9	400	385.[illegible]	67.25	297.74
10	350	372.[illegible]4	68.39	314.88
11	284	367.[illegible]	70.17	397.19
12	400	408.72	72.93	400.91

Table 3. Production Switching Heuristic (PSH) Aggregate Plan

Period	Forecast (gallons)	Production (gallons)	Work force (people)	Inventory (gallons)
0	–	–	81.00	263.00
1	430	452.[illegible]2	70.82	285.42
2	447	452.[illegible]2	70.82	290.83
3	440	382.[illegible]2	67.45	233.25
4	316	382.[illegible]2	67.45	299.67
5	397	382.[illegible]2	67.45	285.09
6	375	382.[illegible]2	67.45	292.50
7	292	382.[illegible]2	67.45	382.92
8	458	382.[illegible]2	67.45	307.34
9	400	382.[illegible]2	67.45	289.75
10	350	382.[illegible]2	67.45	322.17
11	284	312.[illegible]2	64.07	350.59
12	400	382.[illegible]2	67.45	333.00

Table 4. Modified PSH Aggregate Plan

		MPSH1			MPSH2		
Period	Forecast	Production	Work force	Inventory	Production	Work force	Inventory
0	–	–	81.00	263	–	81.00	263
1	430	450	71.43	283	447	72.83	280
2	447	450	71.43	286	447	72.83	280
3	440	450	71.43	296	447	72.83	287
4	316	360	63.49	340	362	63.84	333
5	397	360	63.49	303	362	63.84	298
6	375	360	63.49	288	362	63.84	285
7	292	360	63.49	356	362	63.84	355
8	458	360	63.49	258	362	63.84	259
9	400	360	63.49	218	362	63.84	221
10	350	360	63.49	228	362	63.84	233
11	284	360	63.49	304	362	63.84	311
12	400	360	63.49	264	362	63.84	273

If total production is considered between the various techniques, PPP yields the largest annual production with 4735 gallons. Next is LDR with 4,700. PSH yields 4,659 while MPSH1 & 2 yields 4,590 and 4,599, respectively. This production can be compared against the total annual demand of 4,589 gallons to show that MPSH1 & 2 yield the least difference between the amount of total annual production and forecasted demand.

In the paint factory problem, the inventory cost function is represented by the inventory costs accrued from the difference between the end of the month inventory and a target inventory value of 320 (i.e. $C_{I_t}=0.0825*(I_t-320)^2$). This means that the penalty costs of holding inventory will be greatly reduced even when I_t is large, due to multiplying by a very small constant (.0825). Another way to view this would be that with regard to the total inventory costs where the holding costs C_I is some amount in the formulation :

$$IC = C_I*(P_t-F_t+I_{t-1}).$$

The best policy would be to chase production (that's why LDR production is greater than the other approaches) since doing so serves to minimize the total costs for holding inventory.

As stated previously, the work force level changes at every period with both the optimal LDR approach and the PPP near-optimal approach. PSH was developed in part to reduce these numerous production changes. The results presented in table 3 show four changes with respect to work force, to include the requirement for low level production in period 11 with PSH.

In table 4, with the MPSH1 & 2 approaches, these changes are further reduced to just two production changes over the planning horizon. This should be even more appealing to practitioners. Finally the bables show that when ending inventory balances are not restricted to same level, MPSH1 & 2 yield the least amount of ending inventory.

Table 5 gives the total costs and cost components (i.e. payroll, hiring, firing, overtime and inventory costs) for MPSH1 and MPSH2, respectively. The overall total cost was $295,178 with MPSH1 and $294,979 with MPSH2. This is reasonable since MPSH2 requires a more elaborate search and thus should produce a better cost solution. It should also be noted that MPSH1 yields a solution with less regular labor costs and more overtime costs than does MPSH2. This is further reflected in the hiring and firing costs in that, since MPSH2 chases a higher regular payroll and less overtime combination, it produces hiring and firing costs that are somewhat less than MPSH1 which chases an opposite combination of regular payroll and overtime.

Table 5. Cost Component and Total Cost for Paint Factory

Cost Component	MPSH1	MPSH2
Regular payroll	$267,142	$269,661
Overtime	15,473	13,259
Hiring / Firing	9,941	9,484
Inventory	2,658	2,538
Total cost	$295,178	$249,979

Table 6 below shows a comparison for all approaches with respect to their total variable costs. It should be noted that, as in previous comparisons of APP approaches, the regular payroll costs are considered in analyzing the performance of the approaches.

Table 6. Cost Comparison Results

Cost Component	MPSH1	MPSH2	PSH	PPP	LDR
Regular payroll	$267,142	$269,661	$267,338	$285,141	$282,642
Overtime	15,437	13,295	13,200	7,810	8,518
Hiring / Firing	9,941	9,484	8,863	3,229	3,514
Inventory	2,658	2,538	1,494	1,865	1,362
Total cost	295,178	294,979	299,895	298,045	296,036
Adjusted cost	$301,294*	$300,555	$301,873		

* $301,294=295,178−340*(120gallons) / (5.67gallons / man month).

Vergin[11] has pointed out that these payroll costs should be treated as essentially fixed or the various models should require some similar ending conditions if any comparative analysis is to be made. The total cost values of $295,178 for MPSH1, and $294,979 for MPSH2 are excellent in comparison to PSH and the other models. However, noting Vergin's notes, no direct comparison of the models can be made since the figures in table 6 include neither only "relevant" costs nor similar ending conditions.

To develope similar ending conditions (i.e. inventories) it is known that MPSH1, and MPSH2 resulted in an ending inventory difference from the optimal LDR balance of 102 gallons (366−264) and 93 gallons (366−273), respectively. In order to make MPSH1 and MPSH2 costs comparable we must consider the cost without overtime to make their ending inventories equal to that in the optimal LDR. The regular payroll cost associated with producing 102 and 93 gallons is determined from the cost function $C_1 = 340*W_t$. This means that, at most, we must incur $6116 (=102*340/5.67) and $5576(=93*340/5.67), respectively. These amounts are added to the total costs in table 6 to arrive at the adjusted MPSH1, MPSH2 cost figures.

Now comparing these results based on similar ending conditions we find that the modified PSH's−both MPSH1 and MPSH2, perform better than PSH by $579 and $1,318, respectively. The total cost value of $300,555($301,294) obtained with MPSH2(MPSH1) is only 1.52(1.77) percent greater than the optimum value of $296,036 generated by LDR. This coupled with the less frequent production changes should make both MPSH models more appealing to practitioners than the PSH approach.

6. Conclusions

The production switching approach described in this research offers several clear advantages over other approaches for handling the aggregate production planning (APP) problem. The principal advantage is that it produces production, work force, and inventory decisions which require a minimum amount of period to period adjustment−a characteristic that should be appealing to the natural inclinations of practicing managers in industry.

In this research, the PSH has been modified with an improved search method, which exhaustively searches over the entire cost surface. These modifications are accomplished using two schemes which are labeled MPSH1 and MPSH2 for convenience. The computational requirements of either schemes are quite large and time−consuming, however, the search procedure is relatively straightforward.

The result of modified PSH, applied to the paint factory problem has shown that the total cost performance is improved comparing with the original PSH. The modified production switching approach also offers benefits in simplicity and flexibility for a minimal sacrifice in cost-around two percent of optimal. This feature may appeal to decision makers in industries who are not pursuing an optimal scheduling policy.

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