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COMMON FIXED POINT THEOREMS IN PROBABILISTIC METRIC SPACES

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1. INTRODUCTION

K. Menger ([22]) introduced the concept of probabilistic metric spaces , which is a generalization of metric spaces. Since K. Menger, a number of authors, especially, B. Schweizer and A. Sklar ([24], [25]), A.N. Serstnev ([27]) and H. Sherwood ([29]), etc., have extensively developed these spaces. For the detailed discussions of these spaces and their applications, refer to [5], [11], [35] and [39].

Recently, since V.M. Sehgal and A.T. Bharucha–Reid ([26]) have shown the existence and uniqueness of fixed points of contraction mappings on PM–spaces, a number of generalizations of the results of V.M. Sehgal and A.T. Bharucha–Reid were obtained in PM–spaces ([2]–[7], [9], [10], [12]–[16], [23], [32]–[34], [36]–[38]). On the other hand, G. Jungck ([18]) introduced the concept of conpatible mappings on metric spaces and proved some fixed point theorems for such mappings. Of course, commuting and weakly commuting mappings are compatible, but the converse is not true ([20], [25], [30]). Recently, in [6], Y.J. Cho, P.P. Murthy and M. Stojakovic newly introduced the concept of compatible mappings of type (A) on PM–spaces, which extends the concept of compatible mappings of type (A) on metric spaces ([20]), and proved that the concepts of compatible mappings and compatible mappings of type (A) on PM–spaces are equivalent under some conditions.

In this paper, we prove some fixed point theorems for compatible mappings of type (A) on PM–spaces and use our results to prove some fixed point theorems in metrice spaces and uniform spaces. Our results also extend, generalize and improve a number of fixed point theorems in metric spaces and PM–spaces.

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2. PRELIMINARIES

Let R denote the set of real numbers and R^+ the non-negative reals. A mapping $\mathcal{F} : R \to R^+$ is called a *distribution function* if it is nondecreasing left continuous with $\inf \mathcal{F} = 0$ and $\sup \mathcal{F} = 1$. We will denote $\mathcal{L}$ by the set of all distribution functions.

A *probabilistic metric space* (beriefly, a PM–space) is a pair $(X, \mathcal{F})$, where X is a nonempty set and $\mathcal{F}$ is a mapping from $X \times X$ to $\mathcal{L}$. For $(u, v) \in X \times X$ the distribution function $\mathcal{F}(u, v)$ is denoted by $F_{u,v}$. The functions $F_{u,v}$ are assumed to satisfy the following conditions:

(P_1) $F_{u,v}(x) = 1$ for all $x > 0$ if and only if $u = v$,

(P_2) $F_{u,v}(0) = 0$ for all $u, v \in X$,

(P_3) $F_{u,v}(x) = F_{v,u}(x)$ for all $u, v \in X$,

(P_4) If $F_{u,v}(x) = 1$ and $F_{v,w}(y) = 1$, then $F_{u,w}(x + y) = 1$ for all $u, v, w \in X$.

In a metric space (X, d), the metric d induces a mapping $\mathcal{F} : X \times X \to \mathcal{L}$ such that $\mathcal{F}(u, v)(x) = F_{u,v}(x) = H(x - d(u, v))$, where H is a specific distribution function defined by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases}$$

A function $t : [0, 1] \times [0, 1] \to [0, 1]$ is called a T–*norm* if it satisfies the following conditions:

(T_1) $t(a, 1) = a$ for every $a \in [0, 1]$ and $t(0, 0) = 0$,

(T_2) $t(a, b) = t(b, a)$ for every $a, b \in [0, 1]$,

(T_3) If $a \geq c$ and $b \geq d$, then $t(a, b) \geq t(c, d)$,

(T_4) $t(t(a, b), c) = t(a, t(b, c))$ for every $a, b, c \in [0, 1]$.

A *Menger space* is a triplet $(X, \mathcal{F}, t)$, where $(X, \mathcal{F})$ is a PM–space and t is a T–norm with the following condition:

(P_5) $F_{v,w}(x + y) \geq t(F_{u,v}(x), F_{v,w}(y))$ for all $u, v, w \in X$ and $x, y \in R^+$.

The concept of neighbourhoods in PM–spaces was introduced by B. Schweizer and A. Sklar ([24]). If $u \in X, \epsilon > 0$ and $\lambda \in (0, 1)$, then an (ϵ, λ)–neighbourhood of u, $U_u(\epsilon, \lambda)$, is defined by

$$U_u(\epsilon, \lambda) = \{v \in X : F_{u,v}(\epsilon) > 1 - \lambda\}.$$

If $(X, \mathcal{F}, t)$ is a Menger space with a continuous T–norm t, then the family

$$\{U_u(\epsilon, \lambda) : u \in X, \epsilon > 0, \lambda \in (0,1)\}$$

of neighbourhoods induces a Hausdorff topology τ on X.

The following definitions and theorems are well–known:

DEFINITION 2.1. Let $(X, \mathcal{F}, t)$ be a Menger space. A mapping S from X into itself is said to be *continuous at a point* $p \in X$ if for every $\epsilon > 0$ and $\lambda > 0$ there exist $\epsilon_1 > 0$ and $\lambda_1 > 0$ such that if $q \in U_p(\epsilon_1, \lambda_1)$, then $Sq \in U_{Sp}(\epsilon, \lambda)$, that is, if $F_{p,q}(\epsilon_1) > 1 - \lambda_1$, then $F_{Sp,Sq}(\epsilon) > 1 - \lambda$.

DEFINITION 2.2. Let $(X, \mathcal{F}, t)$ be a Menger space with the continuous T–norm t. A sequence $\{p_n\}$ in X is said to be *convergent* to a point $p \in X$ if for every $\epsilon > 0$ and $\lambda > 0$, there exists an integer $N = N(\epsilon, \lambda)$ such that $p_n \in U_p(\epsilon, \lambda)$ for all $n \geq N$ or equivalently,

$$F_{p,p_n}(\epsilon) > 1 - \lambda$$

for all $n \geq N$. We write $p_n \to p$ as $n \to \infty$ or $\lim_{n\to\infty} p_n = p$.

Since the (ϵ, λ)–topology τ on a Menger space $(X, \mathcal{F}, t)$ satisfies the first axiom of the countability, we have the following:

THEOREM 2.1. *Let $(X, \mathcal{F}, t)$ be a Menger space with the continuous T–norm t and S be a mapping from X into itself. Then S is continuous at a point p if and only if for every sequence $\{p_n\}$ in X converging to p, the sequence $\{Sp_n\}$ converges to the point Sp.*

THEOREM 2.2. *Let $(X, \mathcal{F}, t)$ be a Menger space with the continuous T–norm t. Then F is a lower semi–continuous function on X, that is, for every fixed $x \in R^+$, if $q_n \to q$ and $p_n \to p$ as $n \to \infty$, then*

$$\lim_{n\to\infty} \inf F_{p_n,q_n}(x) = F_{p,q}(x).$$

DEFINITION 2.3. Let $(X, \mathcal{F}, t)$ be a Menger space with the continuous T–norm t. A sequence $\{p_n\}$ in X is said to be a *Cauchy sequence* if for every $\epsilon > 0$ and $\lambda > 0$ there exists an integer $N = N(\epsilon, \lambda) > 0$ such that

$$F_{p_n,p_m}(\epsilon) > 1 - \lambda$$

for all $m, n \geq N$.

DEFINITION 2.4. A Menger space $(X, \mathcal{F}, t)$ with the continuous T–norm t is said to be *complete* if every Cauchy sequence in X converges to a point in X.

The following theorems establish the relations between a metric space and a Menger space. Recall that the Menger space $(X, \mathcal{F}, t)$ induced by the metric d in a metric space (X, d) is called an *induced Menger space*.

THEOREM 2.3. *Let t be a T–norm defined by $t(a, b) = \min\{a, b\}$. Then an induced Menger space $(X, \mathcal{F}, t)$ is complete if a metric space (X, d) is complete.*

THEOREM 2.4. *Let $(X, \mathcal{F}, t)$ be an induced Menger space by the metric d. Let $\{p_n\}$ be a sequence in X and S be a mapping from X into itself. Then for every $\epsilon > 0$ and $\lambda > 0, F_{p_n,p}(\epsilon) > 1 - \lambda$ if and only if there exists an integer N such that $d(p_n, p) < \epsilon$ for all $n \geq N$, and S is continuous at p in the sense of the Menger space if and only if S is continuous at p in the sense of the metric space.*

3. COMPATIBLE MAPPINGS OF TYPE (A)

In this section, motivated with the concepts of compatible mappings of type (A) in metric spaces ([6]) and compatible mappings in PM–spaces ([23]), we introduce the concept of compatible mappings of type (A) in Menger spaces and some properties of these mapping ([6]).

DEFINITION 3.1. Let $(X, \mathcal{F}, t)$ be a Menger space with the continuous T–norm t and let S, T be mappings from X into itself. S and T are said to be *compatible* if

$$\lim_{n \to \infty} F_{STx_n, TSx_n}(x) = 1$$

for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

DEFINITION 3.2. Let $(X, \mathcal{F}, t)$ be a Menger space with the continuous T–norm t and let S, T be mappings from X into itself. S

and T are said to be *compatible of type (A)* if

$$\lim_{n\to\infty} F_{TSx_n,STx_n} = 1 \ \ and \ \ \lim_{n\to\infty} F_{STx_n,TSx_n} = 1$$

for all $x > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ *for some* $z \in X$.

The following Proposition 3.1 and 3.2 show that Definitions 3.1 and 3.2 are equivalent under some conditions ([6]):

PROPOSITION 3.1. *Let* $(X, \mathcal{F}, t)$ *be a Menger space with the continuous T–norm* t *and* $t(x, x) \geq x$ *for all* $x \in [0, 1]$, *and* $S, T : X \to X$ *be continuous mappings. If* S *and* T *are compatible, then they are compatible of type (A).*

PROPOSITION 3.2. *Let* $(X, \mathcal{F}, t)$ *be a Menger space with the continuous T–norm* t *and* $t(x, x) \geq x$ *for all* $x \in [0, 1]$ *and let* $S, T : X \to X$ *be compatible mappings of type (A). If one of* S *and* T *is continuous, then* S *and* T *are compatible*

The following is a direct consequence of Proposition 3.1 and 3.2:

PROPOSITION 3.3. *Let* $(X, \mathcal{F}, t)$ *be a Menger space with the continuous T–norm* t *and* $t(x, x) \geq x$ *for all* $x \in [0, 1]$, *and* $S, T : X \to X$ *be continuous mappings. Then* S *and* T *are compatible if and only if they are compatible of type (A).*

REMARK 1. In [19], we can find two examples that Proposotion 3.3 is not true if S and T are not continuous on X.

Next, we give properties of compatible mappings of type (A) on a Menger space for our main theorems ([6]):

PROPOSITION 3.4. *Let* $(X, \mathcal{F}, t)$ *be a Menger space with the continuous T–norm* t *and* $t(x, x) \geq x$ *for all* $x \in [0, 1]$, *and* $S, T : X \to X$ *be mappings If* S *and* T *are compatible of type (A) and* $Sz = Tz$ *for some* $z \in X$, *then* $STz = TTz = TSz = SSz$

PROPOSITION 3.5. *Let* $(X, \mathcal{F}, t)$ *be a Menger space with the continuous T–norm* t *and* $t(x, x) \geq x$ *for all* $x \in [0, 1]$, *and* $S, T : X \to X$ *be mappings Let* S *and* T *be compatible mappings of type (A) and* $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ *for some* $z \in X$. *Then we have the following:*

(1) $\lim_{n\to\infty} TSx_n = Sz$ *if* S *is continuous at* z.

(2) $STz = TSz$ *and* $Sz = Tz$ *if* S *and* T *are continuous at* z.

4. COMMON FIXED POINT THEOREMS

In this section, we give a common fixed point theorem for compatible mappings of type (A) in Menger spaces.

Let A, B, S and T be mappings from a PM–space $(X, \mathcal{F})$ into itself. If there exists a point u_o in X and a sequence $\{u_n\}$ in X such that

$$ATu_{2n} = TSu_{2n+1} \text{ and } BSu_{2n+1} = TSu_{2n+2}$$

for $n = 0, 1, 2, \ldots$, then the space X is said to be $(A, B; ST(u_o))$–*orbitally complete* if the closure of $\{STu_n : n = 1, 2, \ldots\}$ is complete. A mapping on X is said to be $(A, B; ST(u_o))$–*orbitally continuous* if the restriction of the mapping on the closure of $\{STu_n : n = 1, 2, \ldots\}$ is continuous.

We need the following lemma for our main theorem.

LEMMA 4.1([32]). *Let* $\{y_n\}$ *be a sequence in a Menger space* $(X, \mathcal{F}, t)$ *with the continuous T–norm* t *and* $t(x, x) \geq x$ *for every* $x \in [0, 1]$. *If there exists a constant* $h \in (0, 1)$ *such that*

$$F_{y_n, y_{n+1}}(hx) \geq F_{y_{n-1}, y_n}(x)$$

for all $x > 0$ *and* $n = 1, 2 \ldots$, *then* $\{y_n\}$ *is a Cauchy sequence in* X.

THEOREM 4.2. *Let* $(X, \mathcal{F}, t)$ *be a Menger space with the continuous T–norm* t *and* $t(x, x) \geq x$ *for all* $x \in [0, 1]$ *and let* A, B, S *and* T *be mappings from* X *into itself such that*

(4.1) $ST = TS$,

(4.2) the pairs A, S *and* B, T *are compatible mappings of type (A),*

(4.3) there exists a constant $h \in (0, 1)$ *such that*

$$F_{Au,Bv}(hx) \geq min\{F_{Su,Tv}(x), F_{Au,Su}(x), F_{Tv,Bv}(x), F_{Su,Bv}(2x), F_{Au,Tv}(2x)\}$$

for all $x > 0$ *and* $u, v \in X$

(4.4) there exists a point $u_o \in X$ such that X is $(A, B; ST(u_o))$-orbitally complete and S, T are $(A, B; ST(u_o))$-orbitally continuous Then A, B, S and T have a unique common fixed point z in X and the sequence $\{TS_{u_n}\}$ converges to the common fixed point z.

Proof. By (4.3), we have

$$\begin{aligned} F_{TSu_{2n+1},TSu_{2n+2}}(hx) &= F_{ATu_{2n},BSu_{2n+1}}(hx) \\ &\geq \min\{F_{TSu_{2n},TSu_{2n+1}}(x), F_{ATu_{2n},STu_{2n}}(x), \\ &\quad F_{TSu_{2n+1},BSu_{2n+1}}(x), F_{STu_{2n},BSu_{2n+1}}(2x), \\ &\quad F_{ATu_{2n},TSu_{2n+1}}(2x)\} \\ &= \min\{F_{TSu_{2n},TSu_{2n+1}}(x), F_{TSu_{2n+1},TSu_{2n}}(x), \\ &\quad F_{TSu_{2n+1},TSu_{2n+2}}(x), F_{TSu_{2n},TSu_{2n+2}}(2x), \\ &\quad F_{TSu_{2n+1},TSu_{2n+1}}(2x)\}, \end{aligned}$$

which gives

$$F_{TSu_{2n+1},TSu_{2n+2}}(hx) \geq F_{TSu_{2n},TSu_{2n+1}}(x)$$

since we have

$$F_{TSu_{2n+2},TSu_{2n}}(2x) \geq \min\{F_{TSu_{2n+2},TSu_{2n+1}}(x), F_{TSu_{2n+1},TSu_{2n}}(x)\}.$$

Similarly, we have $F_{TSu_{2n+2},TSu_{2n+3}}(hx) \geq F_{TSu_{2n+1},TSu_{2n+2}}(x)$. Thus, in geneal, we have

$$F_{TSu_{n+1},TSu_{n+2}}(hx) \geq F_{TSu_n,TSu_{n+1}}(x)$$

for $n = 0, 1, 2, \ldots$. So, by Lemma 4.1, $\{TSu_n\}$ is a Cauchy sequence in X. Since X is $(A, B; ST(u_o))$-orbitally complete, $\{TSu_n\}$ converges is a point z in X and hence the subsequences $\{ATu_{2n}\}$, $\{TSu_{2n}\}$, $\{TSu_{2n+1}\}$ and $\{BSu_{2n+1}\}$ of $\{TSu_n\}$ also converges to the same point z.

First, we shall prove that $Tz = z$. Suppose that T is $(A, B; ST(u_o))$-orbitally continuous. Then $\{TTSu_{2n+1}\}$ converges to Tz and, since B and T are compatible mappings of type (A), by Proposition 3.5,

$\{BTSu_{2n+1}\}$ also converges to Tz. Putting $u = Tu_{2n}$ and $v = TSu_{2n+1}$ in (4.3), we have

$$\begin{aligned} &F_{ATu_{2n},BTSu_{2n+1}}(hx) \\ (4.5) \qquad &\geq \min\{F_{STu_{2n},TTSu_{2n+1}}(x), F_{ATu_{2n},STu_{2n}}(x), \\ &F_{TTSu_{2n+1},BTSu_{2n+1}}(x), F_{STu_{2n},BTSu_{2n+1}}(2x), \\ &F_{ATu_{2n},TTSu_{2n+1}}(2x)\}. \end{aligned}$$

Letting $n \to \infty$ in 4.5, we have

$$F_{z,Tz}(hx) \geq \min\{F_{z,Tz}(x), F_{z,z}(x), F_{Tz,z}(x)F_{z,Tz}(2x), F_{z,Tz}(2x)\},$$

which implies that $z = Tz$. Again, replacing u and v by Tu_{2n} and z in (4.3), respectively, we have

$$\begin{aligned} &F_{ATu_{2n},Bz}(hx) \\ (4.6) \qquad &\geq \min\{F_{STu_{2n},Tz}(x), F_{ATu_{2n},STu_{2n}}(x), F_{Tz,Bz}(x), \\ &\qquad F_{STu_{2n},Bz}(2x), F_{ATu_{2n},Tz}(2x)\}. \end{aligned}$$

Thus, letting $n \to \infty$ in (4.6), we have also

$$F_{z,Bz}(hx) \geq \min\{F_{z,Tz}(x), F_{z,z}(x), F_{z,Bz}(x), F_{z,Bz}(2x), F_{z,Tz}(2x)\},$$

which also implies that $z = Bz$. Therefore, we have $Bz = Tz = z$. Similarly, in case S is $(A, B; ST(u_o))$–orbitally continuous, since A and S are compatible mappings of type (A), by (4.3), we have $Az = Sz = z$. Secondly, from (4.3), we can show easily that $Az = Bz$ and so $Az = Bz = Sz = Tz = z$, that is, z is a common fixed point of A, B, S and T.

Finally, the uniquenees of the fixed point z follows easily from (4.3). This completes the proof.

REMARK 2.(1) From Proposition 3.3, the conclusions of Theorem 4.2 are still true even though the condition (4.2) is replaced by the following condition:

(4.7) the pairs A, S and B, T are compatible mappings.

(2) If $A = B$ and $S = T = i_X$(: the identity mapping on X) in (4.3), then the contraction condition introduced by Lj.B. Ciric ([7]) is obtained and Theorem 4.2 extends his theorem.

(3) If $S = T$ is in (4.3), then the contraction condition introduced by S.L. Singh and B.D. Part ([33]) is obtained and so Theorem 1 in [33] is a special case of Theorem 4.2.

(4) If $A = B$ in (4.3), then the contraction condition introduced by S.L. Singh and B.D. Part ([32]) is obtained and so Theorem 1 of S.L. Singh and B.D. Part is contained in Theorem 4.2 as a special case.

(5) Theorem 4.2 extends also Theorem 1 of S.L. Singh, S.N. Mishra and B.D. Pant ([34]).

Let (X, d) be a metric space. Then the metric d induces a mapping $\mathcal{F} : X \times X \to \mathcal{L}$ defined by $\mathcal{F}(p,q) = H(x - d(p,q))$ for $p, q \in X$ and $x \in R$. By Theorem 2.3, if $t(a,b) = \min\{a,b\}$, then $(X, \mathcal{F}, t)$ is a Menger space. Further, $(X, \mathcal{F}, t)$ is complete if (X, d) is complete.

Hence, by using Theorem 4.2, we have the following:

THEOREM 4.3. *Let (X, d) be a metric space and let A, B, S and T be mappings from X into itself such that*

(4.8) X is $(A, B; ST(u_o))$–orbitally complete,

(4.9) S and T are $(A, B; ST(u_o))$–orbitally continuous,

(4.10) the pairs A, S and B, T are compatible mappings of type (A),

(4.11) $ST = TS$,

(4.12) there exists a constant $h \in (0,1)$ such that

$$d(Au, Bv) \le \max\{d(Su, Tv), d(Au, Su), d(Tv, Bv), \frac{1}{2}d(Su, Bv), \frac{1}{2}d(Au, Tv)\}$$

for all $u, v \in X$.

Then A, B, S and T have a unique common fixed point z in X and $\{TS_{un}\}$ converges to the common fixed point z.

REMARK 3. *(1) The concept of compatible mappings of type (A) in metric spaces and some fixed point theorems for such mappings are given in [6]. From Theorem 3.3 in [6], the conclusions of Theorem 4.3 are still true even though the condition (4.10) of Theorem 4.3 is replaced by (4.7).*

(2) A number of fixed point theorems in metric spaces may be obtained as spacial cases of Theorem 4.3. For example, Theorem 4.3 extends, generalizes and improves some theorems of Lj.B. Ciric ([7]), K.M. Das and K.V. Naik ([8]), G. Jungck ([17]) and S.L. Singh ([30],[31]) for a pair of commuting mappings.

5. EXTENSION TO UNIFORM SPACES

Throughout this section, X *is assumed to be a set and* $D = \{d_\alpha\}$ *is a nonempty collection of pseudo–metrics on* X. *It is well-known that the uniformity* $\mathcal{U}$ *generated by* D *is obtained by taking as a subbase of all sets of the form*

$$U_{\alpha,\epsilon} = \{(x,y) \in X \times X : d_\alpha(x,y) < \epsilon\},$$

where $d_\alpha \in D$ *and* $\epsilon > 0$. *In fact, the topology* τ *determined by the uniformity* $\mathcal{U}$ *has all* d_α*–spheres as a subbase. In [3], G.L. Cain and R.H. Kasriel have shown that a collection of pseudo–metrics* $\{d_\alpha\}$ *can be defined which generates the usual structures for Menger spaces.*

Hence, the following theorem is a direct consequence of Theorem 4.2.

THEOREM 5.1. *Let* X *be a sequentially complete Hausdorff space and let* $A, B, S,$ *and* T *be mappings from* X *into itself such that*

(5.1) $AT(X) \cup BS(X) \subset TS(X)$,

(5.2) $ST = TS$,

(5.3) *the pairs* A, S *and* B, T *are compatible mappings of type (A),*

(5.4) S *and* T *are continuous,*

(5.5) *for every* $d_\alpha \in D$ *there exists a constant* $h \in (0,1)$ *such that*

$$d_\alpha(Au,Bv) \le h \max\{d_\alpha(Su,Tv), d_\alpha(Au,Su), d_\alpha(Tv,Bv), \frac{1}{2}d_\alpha(Su,Bv), \frac{1}{2}d_\alpha(Bu,Av)\}$$

for all $u, v \in X$.

Then A, B, S *and* T *have a unique common fixed point in* X.

REMARK 4. *Theorem 5.1 includes a number of fixed point theorems in metric spaces, Menger spaces and uniform spaces, which may be obtained by choosing* A, B, S *and* T *suitably ([20], [31]–[33]).*

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