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CONTINUATION AND VANISHING THEOREM FOR COHOMOLOGY OF INFINITE DIMENSIONAL SPACES

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1. Introduction

The aim of the present paper is to extend S.Dineen[7]'s cohomology vanishing theorem $H^1(D, \mathcal{O}) = 0$ obtained for the cohomology of degree 1 of pseudoconvex open sets in infinite dimensional vector spaces equipped with the finite open topology to the cohomology vanishing theorem $H^p(D, \mathcal{O}) = 0$ for cohomology of higher degree p and to extend Scheja[21]'s Theorem $H^p(D - A, \mathcal{O}) \cong H^p(D, \mathcal{O})$ of continuation of cohomology classes to complex spaces of dimension infinite equipped with the finite open topology. We join the latter results with the former and obtain the infinite dimensional cohomology vanishing theorem $H^p(D - A, \mathcal{O}) = 0$ for cohomology of degree $p \leq codim\ A - 2$ of the complement $D - A$ of analytic set A with respect to pseudoconvex open sets D of vector space E with the finite open topology.

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2. Finite open topology

Let E be a complex vector space with a locally connected Hausdorff topology T and Λ be the set of $\mathbb{C}$-linear subspaces of E. A complex valued function f on an open subset D of (E, T) is said to be *Gâteaux*

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holomorphic if, for any $L \in \Lambda$, the restriction of f to $D \cap L$ is holomorphic on the open subset $D \cap L$ of dimension finite. A complex valued Gâteaux holomorphic function f on an open subset D of (E, T) is said to be ***holomorphic*** if f is continuous on D. The sheaf $\mathcal{O}$ of germs of holomorphic functions over E is called the ***structural sheaf*** of (E, T). An open subset D of (E, T) is said to be ***finitely pseudoconvex*** if, for any $L \in \Lambda$, the intersection $D \cap L$ is a pseudoconvex domain of the $\mathbb{C}$-linear space L of dimension finite for any $L \in \Lambda$. The topology T_0 on E is said to be ***finite open*** if the family of open sets consists of subsets O of E such that, for any $L \in \Lambda$, the intersections $O \cap L$ are open in the finite dimensional Hausdorff $\mathbb{C}$-linear spaces L. A Gâteaux holomorphic function on an open subset of (E, T_0) is unconditionally holomorphic.

The finite open topology is the strongest and the product topology is the weakest among topologies with which we can do Function Theory.

3. Work's of Scheja and Works' of Dineen and Gruman

Let X be a complex space of dimension finite, $\mathcal{O}$ be its structural sheaf and A be an analytic subset of X. Then G.Scheja[21] proved that the canonical homomorphosim $\Psi^p : H^p(D, \mathcal{O}) \to H^p(D - A, \mathcal{O})$ is isomorphic if $p \leq codim\ A - 2$.

About twenty years ago and when the first author begun to investigate the theory of functions of infinite complex variables under Professor Lelong in Paris, S.Dineen[7] proved the vanishing theorem $H^1(D, \mathcal{O}) = 0$ of the cohomology of degree 1 of a pseudoconvex domain D with coefficients in the structural sheaf $\mathcal{O}$ of the $\mathbb{C}$-linear space (E, T_0) equipped with the finite open topology and L.Gruman[10] solved the Levi problem proving that any finitely pseudoconvex D of the space (E, T_0) is the existence domain of a holomorphic function on D.

4. Cohomology vanishing theorem

PROPOSITION$_p$. *Let $E := \{z = (z_1, z_2, \cdots, z_n) \in \mathbb{C}^n; z_j \in \mathbb{C}(j \in \{1, 2, \ldots, n\})\}$ be the finite n dimensional $\mathbb{C}$-linear space $\mathbb{C}^n$ equipped with the Hausdorff topology and F be the reducible analytic space which is 1 codimensional reducible analytic subset of E defined by*

$$F := \{z = (z_1, z_2, \ldots, z_n) \in E; z_1 z_2 \cdots z_n = 0\} \tag{1}$$

equipped with the usual structural sheaf $\mathcal{O}_F := \mathcal{O}/z_1z_2\cdots z_n\mathcal{O}$. Let D be its pseudoconvex open set containing the origin 0, $\mathcal{O}$ be the structural sheaf over D, $\mathcal{U} = \{U_\iota; \iota \in I\}$ be a Stein covering of D, i.e. all open set U_ι is a domain of holomorphic for $\iota \in I$, such that the origin 0 is contained in one and only one open set of $\mathcal{U}$. Let D_F be the trace $D \cap F$ of D to F, reducible analytic set of D with the restriction of $\mathcal{O}_F$ to D_F which is denoted by the same symbol $\mathcal{O}_F$. Let $\mathcal{U}_F := \{U_\iota \cap F; \iota \in I\}$ be the trace of the covering $\mathcal{U}$ of the open set D to the analytic set D_F of D. Let $\mathcal{Z}_F$ be a p-cocycle of the covering $\mathcal{U}_F$ with coefficients in the structural sheaf $\mathcal{O}_F$ of the reducible analytic set D_F of D. Then, there exists a $(p-1)$-cochain $\mathcal{C} = \{c_\beta; \beta \in I^{p-1}\}$ of the covering $\mathcal{U}$ with coefficients in the sheaf $\mathcal{O}$ such that the trace of the coboundary $\delta(\mathcal{C})$ of $\mathcal{C}$ to D_F coincides with the p-cocycle $\mathcal{Z}_F$.

Proof. In case $p = 1$, Proposition$_1$ was proven as Lemma 2.2 in L.Gruman[10]. So we may assume that $p \geq 2$.

Since the sheaf $\mathcal{O}/z_1z_2\cdots z_n\mathcal{O}$ over E is an analytic coherent sheaf over E and it is supported by the reducible analytic set F of E. Hence its restriction $\mathcal{O}_F$ to the reducible Stein space D_F is also an analytic coherent sheaf over the space D_F. Since $\mathcal{U}_F$ is a Stein covering of the Stein space D_F, any cohomology with positive degree of the open set, which is a support of a simplex of I, vanishes, by Lemma L_p of G.Scheja[21], any cohomology with positive degree of the open set D_F with values in the analytic coherent sheaf $\mathcal{O}_F$ coincides with that of the covering $\mathcal{U}_F$. Since the former vanishes by the theorem of Oka-Cartan-Serre, any p-cocycle of $\mathcal{U}_F$ with values in the sheaf $\mathcal{O}_F$ is a coboundary of a $(p-1)$-cochain $\mathcal{C}_F = \{c_{F\beta}; \beta \in I^{p-1}\}$ of the covering $\mathcal{U}_F$ with coefficients in the sheaf $\mathcal{O}_F$. Here $(p-1)$-simplex β of I is an element $\beta = (\beta_1, \beta_2, \ldots, \beta_p)$ of I^{p-1} and its support U_β is the open set $U_\beta := U_{\beta_1} \cap U_{\beta_2} \cap \cdots \cap U_{\beta_p}$.

We have the short exact sequence

$$(2) \qquad 0 \to z_1z_2\cdots z_n\mathcal{O} \to \mathcal{O} \to \mathcal{O}/z_1z_2\cdots z_n\mathcal{O} \to 0$$

of sheaves over D. For any $(p-1)$ simplex β of I and for its support U_β, we have the long exact sequence of cohomology modules

$$(3) \qquad 0 \to H^0(U_\beta, z_1z_2\cdots z_n\mathcal{O}) \to H^0(U_\beta, \mathcal{O}) \to$$

$$H^0(U_\beta, \mathcal{O}/z_1z_2\cdots z_n\mathcal{O}) \to H^1(U_\beta, z_1z_2\cdots z_n\mathcal{O}) \to \cdots$$

of the open set U_β of D. Since the above cohomology of the Stein space U_β of the analytic coherent sheaf $z_1z_2\cdots z_n\mathcal{O}$ vanishes by the theorem Oka-Cartan-Serre, the canonical mapping

$$\varphi_\beta : H^0(U_\beta, \mathcal{O}) \to H^0(U_\beta, \mathcal{O}/z_1z_2\cdots z_n\mathcal{O}) \tag{4}$$

is surjective. Since our $c_{F\beta}$ can be regarded as an element of the group $H^0(U_\beta, \mathcal{O}/z_1z_2\cdots z_n\mathcal{O})$, there exists an element c_β of $H^0(U_\beta, \mathcal{O})$ with $\varphi_\beta c_\beta = c_{F\beta}$. Then the restriction to the reducible analytic set D_F of the coboundary of the $(p-1)$-cochain $\mathcal{C} := \{c_\beta; \beta \in I^{p-1}\}$ of the covering $\mathcal{U}$ with coefficients in the structural sheaf $\mathcal{O}$ of the pseudoconvex open set D coincides with the said p-cocycle of the covering $\mathcal{U}_F$ with coefficients in the structural sheaf $\mathcal{O}_F$ of the reducible analytic set D_F of D.

THEOREM 1. *Let E be a $\mathbb{C}$-vector space with a locally connected Hausdorff and finite open topology, $\mathcal{O}$ be its structural sheaf and D be a finitely pseudoconvex open set of E. Then, for any positive integer p, we have $H^P(D, \mathcal{O}) = 0$.*

Proof. In case that $p = 1$,the theorem is proved by S.Dineen[7]. So we may assume that $p \geq 2$. and the origin 0 is contained in D.

Let $\mathcal{U} = \{U_\imath; \imath \in I\}$ be a Stein covering of D and $\mathcal{Z} = \{z_\alpha; \alpha \in I^p\}$ be a p-cocycle of the covering $\mathcal{U}$ with coefficients in the structural sheaf $\mathcal{O}$ of D. We prove the proposition using Zorn's Lemma which is equivalent to the transfinite induction used by S.Dineen[7] and L.Gruman[10] that there exists a $(p-1)$-cochain $\mathcal{C} := \{c_\beta; \beta \in I^p\}$ of the covering $\mathcal{U}$ with coefficients in the sheaf $\mathcal{O}$ such that the coboundary $\delta(\mathcal{C})$ of $\mathcal{C}$ coincides with the p-cochain $\mathcal{Z}$. Let Σ be the set of pairs $(S, \mathcal{C}_S)$ of $\mathbb{C}$-linear subspaces S of E and $(p-1)$-cochains $\mathcal{C}_S = \{c_{S\beta}; \beta \in I^{p-1}\}$ whose cboundary is the rstriction $\mathcal{Z}_S$ of the $\mathcal{Z}$ to $D \cap S$. We induce a partial order in Σ so that two pairs $(S, \mathcal{C}_S)$ and $(T, \mathcal{C}_T)$ of $\mathbb{C}$-linear subspace S and T of E and $(p-1)$-cochains $\mathcal{C}_S = \{c_{S\beta}; \beta \in I^{p-1}\}$ and $\mathcal{C}_T = \{c_{T\beta}; \beta \in I^{p-1}\}$ satisfy $(S, \mathcal{C}_S) \subseteq (T, \mathcal{C}_T)$ if and only if $S \subset T$ and the restriction of $\mathcal{C}_T$ to $D \cap S$ coincides with $\mathcal{C}_S$. The partially ordered set Σ is not totally ordered. Let $\mathcal{T} = \{(T, \mathcal{C}_T); T \in \Xi\}$ be a totally ordered subset of the partially ordered set Σ. We consider two elements $(S, \mathcal{C}_S)$ and $(T, \mathcal{C}_T)$ with $(S, \mathcal{C}_S) \subsetneq (T, \mathcal{C}_T)$. Then, for any

$\beta \in I^{p-1}$ the restriction of $c_{T\beta}$ to $U_\beta \cap S$ coincides with $c_{S\beta}$ on $U_\beta \cap S$. Let T_{sup} be the $\mathbb{C}$-linear span of the family Ξ of $\mathbb{C}$-linear subspaces of E. Since the set T is totally ordered, the set Ξ is also totally ordered by the usual inclusion $\subset$. So the linear span T_{sup} coincides with the union of the family Ξ. We put, for any $\beta \in I^{p-1}, c_{T_{sup}\beta} = c_{T\beta}$ on $U_\beta \cap T$. Then, for any $\beta \in I^{p-1}$, the mapping $c_{T_{sup}\beta} : U_\beta \cap T_{sup} \to \mathcal{O}$ is well-defined. Since its restriction on any finite dimensional linear subspace of T_{sup} is continuous, by the definition of the finite open topology, it is continuous on $U_\beta \cap T_{sup}$ too. The collection $c_{T_{sup}} = \{c_{T_{sup}\beta}; \beta \in I^{p-1}\}$ defines a $(p-1)$-cochain of the covering $\mathcal{U}_{T_{sup}} = \{U_\imath \cap T_{sup}; \imath \in I\}$ with coefficients in $\mathcal{O}$. And its coboundary is the restriction $\mathcal{Z}_{T_{sup}}$ to of the said p-cocycle $\mathcal{Z}$. Thus we have proved that any totally ordered subset T of the partially ordered set Σ has a supremum. Hence, by Zorn's Lemma, the partially ordered set Σ has a maximal element $(T_{max}, \mathcal{C}_{T_{max}})$. We put $\mathcal{U}_{T_{max}} = \{U_\imath \cap T_{max}; \imath \in I\}$ and $\mathcal{C}_{T_{max}} = \{c_{T_{max}\beta}; \beta \in I^{p-1}\}$.

If T_{max} were a proper $\mathbb{C}$-linear subspace of the said space E, there would exist an element z_∞ of the complement $E - T_{max}$ of T_{max}. We may assume that one and only one set of $\mathcal{U}$ contains the point z_∞. We denote $\mathcal{C}_{T_{max}}$ more precisely by $\{c_{T_{max}\beta}; \beta \in I^{p-1}\}$ We denote by $[z_\infty]$ the 1-dimensional $\mathbb{C}$-linear subspace $\{tz_\infty; t \in \mathbb{C}\}$. We denote by $T_{max\infty}$ the linear subspace of E spanned by T_{max} and $[z_\infty]$.

Let $B = \{z_\theta; \theta \in \Theta\}$ be a Hamel basis for the $\mathbb{C}$-linear space T_{max}. As in the proof of Theorem 2.3 of L.Gruman[10], we prove the following proposition by induction with respect to a non negative integer n:

For any $(n+1)$ dimensional $\mathbb{C}$-linear subspace L_∞ of E generated by the vector z_∞ and n elements of B of T_{max}, we put $\mathcal{U}_{T_{max\infty}} = \{U_\imath \cap T_{max\infty}; \imath \in I\}$, $L := L_\infty \cap T_{max}$. Then the restriction to $\mathcal{U}_{T_{max\infty}}$ of the $(p-1)$-cochain $\mathcal{C}_{T_{max}}$ of the covering $\mathcal{U}_{T_{max}}$ can be continued to a $(p-1)$-cochain $C_{L_\infty} = \{c_{L_\infty\beta}; \beta \in I^{p-1}\}$ of the covering $\mathcal{U}_{T_{max\infty}}$. These continuations of the $(p-1)$-cochains of the coverings of the n-dimensional linear $\mathbb{C}$-subspaces L's to the n-dimensional linear $\mathbb{C}$-subspaces L_∞ are compatible in the following sense : For two these $(n+1)$ dimensional $\mathbb{C}$-Linear subspaces L_∞ and M_∞ of E, and for these continuations $C_{L_\infty} = \{c_{L_\infty\beta}; \beta \in I^{p-1}\}$ and $C_{M_\infty} = \{c_{M_\infty\beta}; \beta \in I^{p-1}\}$ of the restrictions to $\mathcal{U}_L$ and $\mathcal{U}_M$ of the $(p-1)$-cochain $\mathcal{C}_{T_{max}} := \{c_{T_{max}\beta}; \beta \in I^{p-1}\}$ of the covering $\mathcal{U}_{T_{sup}}$, there holds the condition of compatibility $c_{L_\infty \beta} = c_{M_\infty \beta}$ on $U \cap L_\infty \cap M_\infty$ for any $\beta \in I^{p-1}$.

In case that $n = 0$, the target space is only the one dimensional $\mathbb{C}$-linear space $[z_\infty]$ whose trace $D \cap [z_\infty]$ to D is a Stein manifold. Hence the restriction to $\mathcal{U}_{[z_\infty]}$ of the p-cocycle $\mathcal{Z} = \{z_\alpha; \alpha \in I^p\}$ of the covering $\mathcal{U}$ with coefficients in the structural sheaf $\mathcal{O}$ of D is a coboundary of a $(p-1)$-cochain of the covering $\mathcal{U}_{[z_\infty]}$ with coefficients in the structural sheaf of $[z_\infty]$.

Now assume that the above proposition is valid for a non negative integer $(n-1)$. Then, using Proposition$_n$ by the arguments in the proof of Theorem 2.3 of L.Gruman[10], we can prove the validity of above proposition for the integer n too.Thus, by induction with respect to non negative integers n, we have proved the validity of the above proposition. Let β be any $(p-1)$-simplex belonging to I^{p-1} and ζ be any point of the support $U_{T_{max}\infty \beta}$ of the simplex β. Since $B = \{z_\theta; \theta \in \Theta\}$ is a Hamel basis for the $\mathbb{C}$-linearspace T_{max}, there exists a positive integer n and an n-dimensional subspace L of E such that L is spanned by n elements of B. We put $c_{T_{max}\infty \beta} = c_{L_\infty \beta}$ in $U_{L_\infty \beta}$. Then, $c_{T_{max}\infty \beta}$ is a well defined Gâteaux holomorphic function on $U_{T_{max}\infty \beta}$ which is continuous by the cause of the finite open topology of the space on $T_{max}\infty$ and therefore is holomorphic on $U_{T_{max}\infty \beta}$. In this way, we established the continuation $\mathcal{C}_{T_{max}\infty} = \{c_{T_{max}\infty \beta}; \beta \in I^{p-1}\}$ of the p-cochain $\mathcal{C}_{T_{max}} = \{c_{T_{max}\infty \beta}; \beta \in I^{p-1}\}$ to the space $T_{max}\infty$ so that the coboundary of $\mathcal{C}_{T_{max}\infty}$ is the restriction of the said p-cocycle $\mathcal{Z}$. Hence the pair $(T_{max}\infty, \mathcal{C}_{T_{max}\infty})$ is strictly larger than the maximal pair $(T_{max}, \mathcal{C}_{T_{max}})$. This is a contradiction. Thus we have proved $T_{max}\infty = E$ what was to be proved.

5. Continuation theorem

Let E be a $\mathbb{C}$-vector space and D be an open set of E, A be a closed subset of D and p be a positive integer. We write *codim* $A \geq p$, if, for any finite dimensional $\mathbb{C}$-linear subspace L of E, there exists a finite dimensional $\mathbb{C}$-linear subspace M of E such that M is a subspace of L and that the codimension of the analytic set $A \cap M$ in $D \cap M$ is not smaller than p at each point of $A \cap M$.

THEOREM 2. *Let E be a $\mathbb{C}$-vector space with a locally connected Hausdorff and finite open topology, $\mathcal{O}$ be its structural sheaf and D*

be an open set of E, A be an analytic subset of D and p be a positive integer. If $p \leq codim\ A - 2$, then the canonical homomorphism Ψ^p : $H^p(D, \mathcal{O}) \to H^p(D - A, \mathcal{O})$ isomorphic.

Proof. Let $\mathcal{U} = \{U_i; i \in I\}$ be a Stein covering of D. Then $\mathcal{U} - A :=$ $\{U_i - A; i \in I\}$ is an open covering of the open set $D - A$. Those coverings $\mathcal{U} - A$ are cofinal in a collection of open coverings of $D - A$. Let $\mathcal{Z} = \{z_\alpha; \alpha \in I^p\}$ be a p-cocycle of the covering $\mathcal{U} - A$ with coefficients in the structural sheaf $\mathcal{O}$ of $D - A$. For any $\alpha \in I^p$, z_α is a holomorphic function on the support $U_\alpha - A$ of the p-simplex α. Let L be any finite dimensional $\mathbb{C}$-linear subspaces of E. By the definition of codimension of A, there exists a finite dimensional $\mathbb{C}$-linear subspace M of E such that M is a superspace of L and that the analytic set $A \cap M$ is an analytic set in $D \cap M$ the codimension of which is at least *codim* A, for any $\alpha \in I^p$, the restriction z_{M^α} of the holomorphic function z_α to the trace $U_\alpha \cap M - A$ of D to $M - A$ can be continued to the analytic set $A \cap U_\alpha \cap M$ whose *codimension* $\geq p + 2 \geq 2$. By the finite dimensional Riemann's theorem concerning removable singularities, z_{M^α} can be continued to a holomorphic function $z_{M^{\alpha *}}$ on the open set $U_\alpha \cap M$. We denote by z_{L^α} the restriction of z_{M^α} to $U_\alpha \cap L$ Then, z_{L^α} is independent of the special choice of a superspace M of L. Quite similarly as in the proof of Theorem 1, we can establish a continuation of the holomorphic function z_α on $D - A$ to a holomorphic function z_{α^*} on D. By the principle of uniqueness of analytic prolongations, the collection $\mathcal{Z}^* = \{z_{\alpha^*}; \alpha \in I^p\}$ is a cocycle of the open covering $\mathcal{U} - A$.

THEOREM 3. *Let E be a $\mathbb{C}$-vector space with a locally connected Hausdorff and finite open topology, $\mathcal{O}$ be its structural sheaf and D be a finitely pseudoconvex open set of E, A be an analytic subset of D and p be a positive integer. If $p \leq codim\ A - 2$, then we have $H^p(D - A, \mathcal{O}) = 0$.*

Proof. By Theorems 2 and 1, we have $H^p(D - A, \mathcal{O}) \cong H^p(D, \mathcal{O}) =$ 0.

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