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PULSATIONAL CHARACTERISTICS OF V1719 CYGNI WITH PECULIAR LIGHT CURVE*

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ABSTRACT

The light curve and radial velocity curve of multiperiodic dwarf cepheid V1719 Cyg (HD200925) with peculiar light curve have been reanalyzed in order to identify the oscillation modes to confirm the helium settling within the envelope. To do these, through the period search for the photometric and radial velocity data from the literature, two different periods were determined and the oscillation modes corresponding to the first and second periods were identified as the fundamental and first radial overtones. Hence the helium settling within the envelope was confirmed from the period ratio. The color excess, metallicity, effective temperature, and surface gravity corresponding to two different modes were determined and it was found that these parameters almost do not depend upon different oscillation mode. By utilizing the surface brightness method, we investigated the variation of angular diameter and radial displacement and it was found that the angular variation is very peculiar. Also by referring to the stellar models, mass and age were determined as 2.7 $M_{\odot}$ and 0.42 Gyr respectively which make this variable star heavier and younger than other multimode dwarf cepheids. Preliminary spectroscopic CCD observations were carried out and it was found that Mg in V1719 Cygni is nearly solar abundent according to the analysis of 5172.68ÅMgI line which is inconsistent with the photometric result. It was suggested that V1719 Cyg may be classified as a ρ Pup stars according to the photometric characteristics.

Key Words : pulsating star, dwarf cepheid, metallicity.

I. INTRODUCTION

V1719 Cyg(HD200925) is classified in the Catalogue of Variable Star (GCVS) as a δ Scuti variable star. It was first noticed by Joner and Johnson (1985) that the light curve of V1719 Cyg is very peculiar. The main feature is an asymmetric light curve with the descending branch steeper than the ascending branch. Such an asymmetry has so far never been encountered in other regularly pulsating stars except only two other variables of V798 Cyg and V974 Oph.

They also found that V1719 Cyg shows a definite variation of m_1 with temperature, but this variations is opposite the variation predicted in Crawford (1979). On the other hand by using the one zone model proposed by Stellingwerf (1972), Antonello (1991) obtained an interesting result concerning the inverse variation of m_1-index and he argued that the cool stars with mild metal rich atmospheres as in V1719 Cyg can show this kind of peculiarity.

Also another peculiarity of V1719 Cyg was found by Antonello, et al. (1986) through the investigation of Fourier

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decomposition which is very useful for the description of light curve of variable star. The amplitude ratios and phase differences are introduced as followed; $R_{jk} = R_j/R_k$, $\phi_{jk} = k\phi_j - j\phi_k(+2\pi n)$, j>k, which are a generalization of the original definitions given by Simon and Lee (1981). The amplitude ratio and phase difference of V1719 Cyg are R_{21}=0.204, R_{31}=0.080, and ϕ_{21}=2.47, ϕ_{31}=6.16 and these values are very different from ϕ_{21} =3.6-4.8 and ϕ_{31}=1.0-3.0 for other δ Scuti stars.

More than anything else, it must be noted that in the case of V1719 cyg, the period ratio of $f_1/f_2 = 0.800$. Cox et al. (1984) showed that this ratio is possible for the fundamental and first overtone radial modes in relation with helium settling within the envelope of δ Scuti star. In fact, it has been known that high metallicity and helium settling can be accounted by the process of diffusion in Am/Fm stars. With regard this, Poretti and Antonello (1988, hereafter PA88) argued the possibility of the fundamental and first overtone modes indirectly by considering the colour index and luminosity which place this star near the cool edge of the instability strip. Cox et al. (1979) already proposed that a star near the cool edge of the instability strip may pulsate in spite of a low helium content in its envelope and a high metallicity. PA88 also pointed out that the period of P_1=0.267 d is rather long as δ Scuti star, and this period can not correspond to the first overtone mode.

To investigate the peculiar properties of variables in detailed way, it is absolutely required to determine the atmospheric parameters accurately as possible as we can. This fact is more critical for multiperiodic variables like V1719 Cyg due to the superposition of two or more different pulsating modes. We believe, for multimode variables, that atmospheric parameters should be determined by using the indices corresponding to each different oscillation mode separately. If the normal point of multimode pulsators is taken simply by averaging the light curves whose width reaches to nearly 0.1 or more magnitude for most of all phase, the atmospheric parameters would inevitably lead the large uncertainity. Also it would be difficult to investigate the difference between single-mode and multi-mode pulsator if we do not distinguish their oscillation mode in the analysis of light curve.

The purpose of this paper is to identify the oscillation mode in order to confirm helium settling within the envelope and to estimate the metallicity of V1719 Cyg through specroscopic study. Also by adopting the most recent model atmosphere grid by Kurucz (1992), we determined the atmospheric parameters of effective temperature and surface gravity for each different pulsation mode, and to investigate the pulsation properties for V1719 Cyg. Then the evolutionary status of this variable is investigated. About the period search method and the determination of reddening, metallicity and other atmospheric parameters are discussed in section two. In section three, spectroscopic observation and analysis for the determination of magnesium abundance are presented. Pulsation mode for the first and second period is identified in section four, and the visual surface brightness method related with angular diameter variation is discussed in section five. In section six, evolutionary status of V1719 Cyg is discussed.

II. LIGHT CURVE ANALYSIS

By applying the traditional method of Fourier transformation and least square fitting for period search (for detailed procedure, see Kurtz and Cropper (1987)) we redetermined the first and second frequencies of f_1=3.74116c/d and f_2=4.67761c/d, and these values are consistent with f_1=3.741186c/d and f_2=4.677530c/d obtained by PA88 by means of the different method. The window function and the power spectrums are presented in Figure 1. By examining other pannels it is possible to identify the following terms : $2f_1$, $3f_1$, $2f_2$, f_1+f_2, f_2-f_1. Table 1 summarizes the parameters of the interpolating curve

$$m(t) = a_0 + a_j cos[2\pi f_j(t - to) + \phi_j], to = 2440000.(H.J.D.) \qquad (1)$$

where a_0 is the mean value and a_j, ϕ_j are amplitudes and phase for each frequency respectively.

These parameters are derived from the least square method. Figure 2 shows the y observations with the fitted curves according to the above equation for the first eight nights and we can note that all our light curves are well produced. Once the solution was obtained, it was then possible to separate the light curve corresponding to each period. To do this, we subtracted the component corresponding to one of the two periods then plotted the residules against the phase (shown in Figure 3 and 4).

In order to discuss the variation in the physical parameters of V1719 Cyg, we have sorted the photometric data by phase into 20 equally spaced bins around the light cycle corresponding to two periods. A smooth Fourier fitted curve

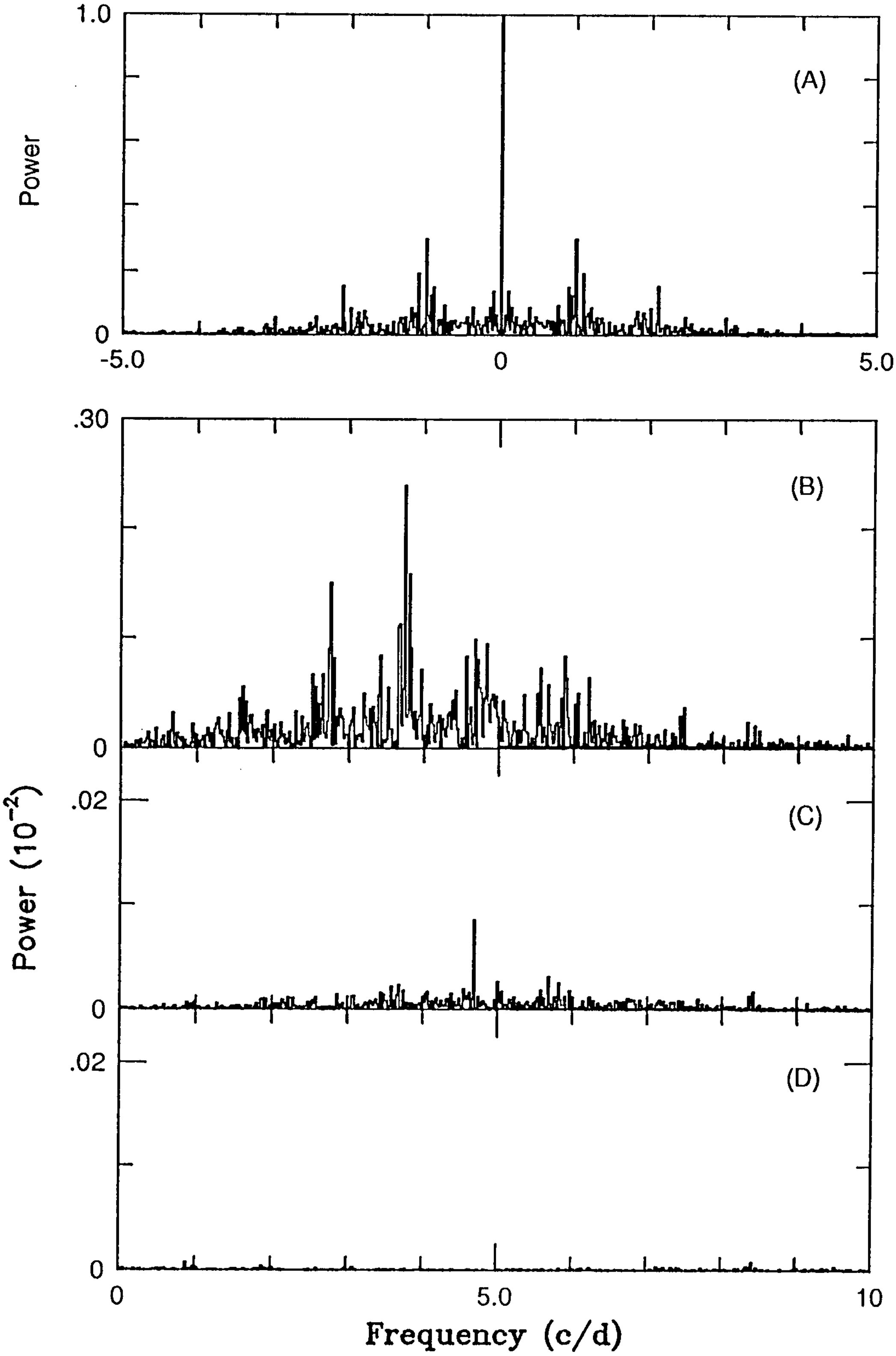

Fig. 1. Analysis of the complete y dataset by means of the method of Fourier transformation. Upper pannel shows the window function.

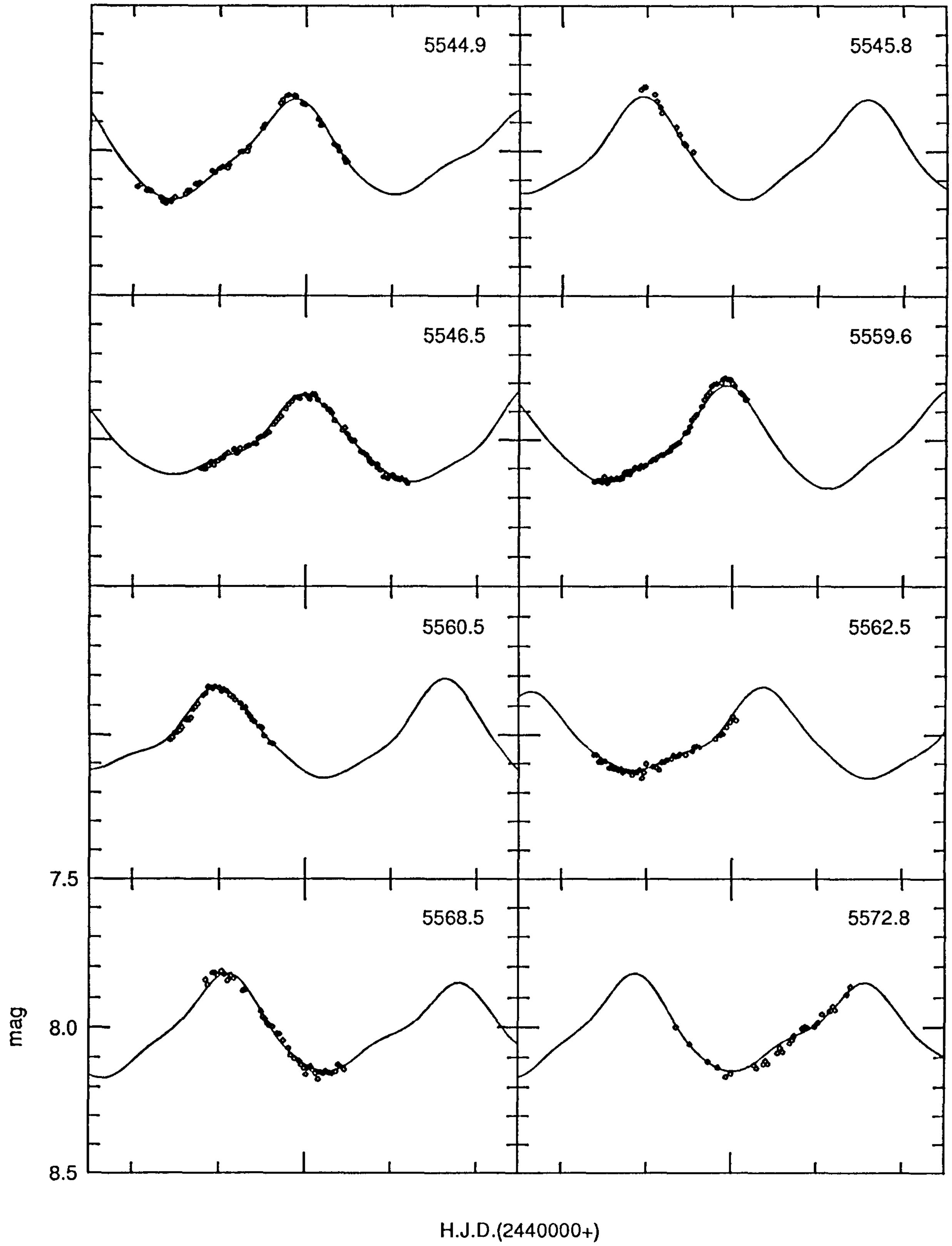

Fig. 2. y observations fitted with the computed solution.

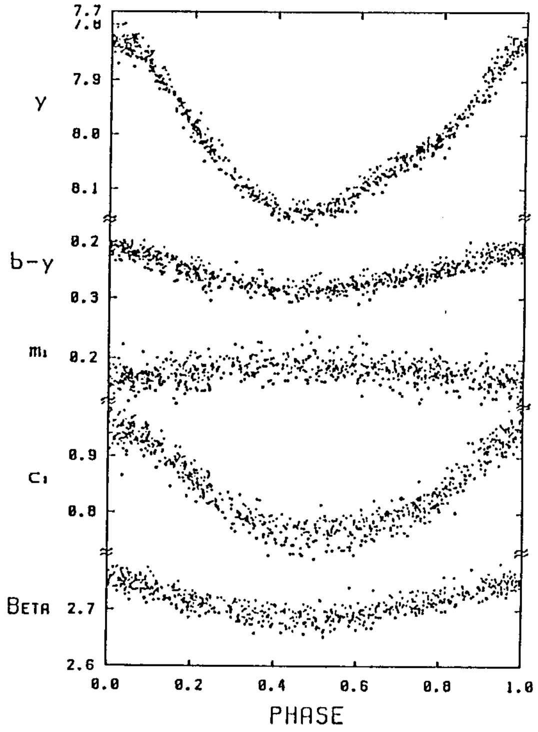

Fig. 3. Light curves for the first period.

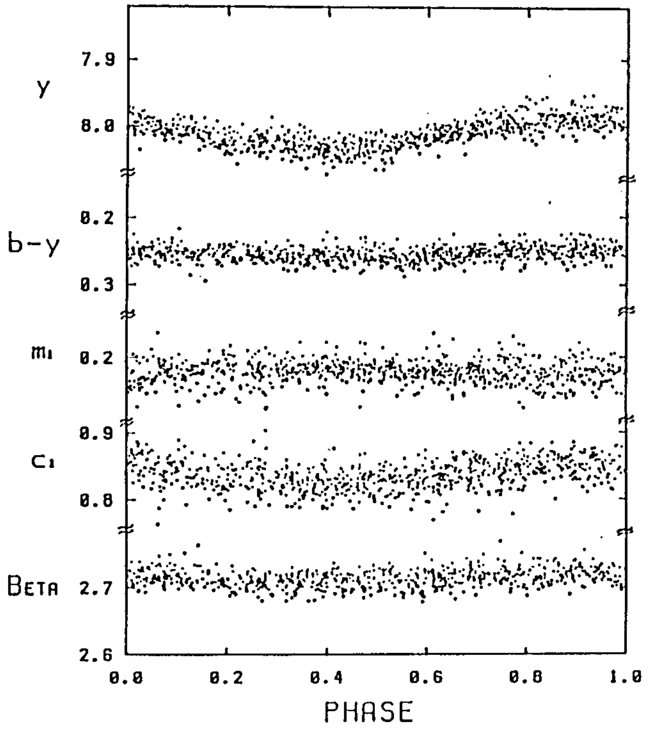

Fig. 4. Light curves for the second peiod.

Table 1. Coefficients of the synthetic light curves of V1719 Cyg in uvbyβ photometry

	$f_j(c/d)$	y		b-y		m_1		c_1		β		V_r	
		a_j	ϕ_j	a_j	ϕ_j	a_j	ϕ_j	a_j	ϕ_j	a_j	ϕ_j	a_j	ϕ_j
f_1	3.74116	0.1437	1.220	0.0333	1.309	0.0137	1.265	0.0910	4.235	0.0319	4.425	10.243	0.902
		± .0007	± .005	± .0006	± .017	± .0010	± .067	± .0010	± .011	± .0008	± .024	± .094	± .009
$2f_2$	7.48232	0.0293	4.910	0.0059	5.283	0.0023	4.931	0.0179	2.014	0.0049	2.263	0.778	2.080
		± .0007	± .024	± .0006	± .098	± .0009	± .396	± .0010	± .057	± .0008	± .155	± .091	± .115
$3f_1$	11.22348	0.0115	30542	0.0025	3.570	0.0010	4.447	0.0046	0.812	0.0022	0.154	0.400	1.614
		± .0007	± .060	± .0006	± .226	± .0009	± .918	± .0010	± .220	± .0007	± .339	± .089	± .220
f_2	4.67761	0.0234	50268	0.0053	5.023	0.0038	5.140	0.0140	1.952	0.0067	2.204	1.242	2.909
		± .0007	± .029	± .0006	± .108	± .0009	± .245	± .0010	± .072	± .0007	± .111	± .093	± .080
	a_0	8.0106		0.2539		0.1784		0.8378		2.7142		8.3258	
		± .0005		± .0004		± .0007		± .0007		± .0005		± .0657	
	s.d.	0.0129		0.0108		0.0173		0.0191		0.0130		0.7326	

drawn through these mean values (shown in Figure 5 and 6) has been used to yield the normal points tabulated in Table 2-3. The subsequent stellar parameters we derived are all based on the tabulated normals.

Intrinsic $(b-y)$ values have been calculated with the aid of the Crawford (1979) calibration of A and F stars at normal points around the light cycle. By comparing intrinsic $(b-y)$ values with the observed $(b-y)$ values in the all phase interval we find an average color excess of $< E(b-y) >=0.039 \pm 0.006$ and 0.040 ± 0.009 for the first and second period respectively and, therefore, we can see that reddening is not dependent upon different pulsation mode.

Usually color excess of pulsating variables is determined for the region of descending brench because this region is relatively more stable than ascending region. However, for V1719 Cyg, since descending brench is more steeper than ascending brench, color excess was determined for all region. The $E(b-y)=0.029$ obtained by JJ86 without separation of light curves corresponding to two different oscillation modes can be compared with our value. The value of our $E(b-y)=0.039$ looks too small with respect to the low galactic latitude ($b=2.^\circ5$) of V1719 Cyg and therefore this variable should be very close to the Sun. Actually it was found that the distance of this variable is from 240 to 300 pc according to Crawford calibration for the first period. It was also found that the spectral characteristics is closer to A-type for the first period.

According to the early F-star calibration relating $[Fe/H]$ to m_1 (McNamara an Powell 1985), Crawford and Perry (1976a) calibration for A star, and Nissen (1980) calibration for F star, $[Fe/H]$ changes greatly from 0.35 to 0.64 and 0.21 to 0.54 for the first and second period respectively for all phase interval and our result can be compared with $[Fe/H]=0.46$ to 0.66 by JJ86. Also the mean values corresponding to spectral type A and F are 0.135 and 0.538 respectively for the first period. It seems, therefore, that the big difference of metallicity is due to the problem related with calibration for different spectral class for this variable. Also, although it was found by JJ86 that V1719 cyg is abnormally metal rich star, it is difficult to determine the accurate metallicity with given metallicity calibration. These are the reasons why we tried to determine the metallicity of V1719 Cyg through spectroscopic CCD observation which is presented in the next section. We also like to point out that it was found by Kim and Joner (1994) that V798 Cyg which shows similar peculiar light curve as in V1719 Cyg is solar abundant.

By adopting $E(b-y)=0.039$ for the color excess and by utilizing $E(c_1)=0.2E(b-y)$ and $E(m_1)=-0.32E(b-y)$, we have calculated the intrinsic $(b-y)_0$, $(c_1)_0$, and $(m_1)_0$ values given in Table 4-5. By interpolating the $(b-y)_0$ and $(c_1)_0$ values in a grid of model atmospheres computed by Kurucz (1992) recently the effective temperature and the surface gravity were determined for a few different values in between above minimum and maximum value of $[Fe/H]$ for the first and second period. It was found that the resonable result can be obtained for only $[Fe/H]=0.30$ to 0.50 and we take the mean value of $[Fe/H]=0.4$ as the metallicity of V1719 Cyg.

As can be seen in Figure 7 and 8, the dependency of the effective temperature upon the metallicity is almost negligiable but the metallicity effect is larger for the surface gravity. Furthermore the variation of surface gravity is somewhat unusual i.e., the maximum gravity corresponds to the minimum radial displacement but the minimum

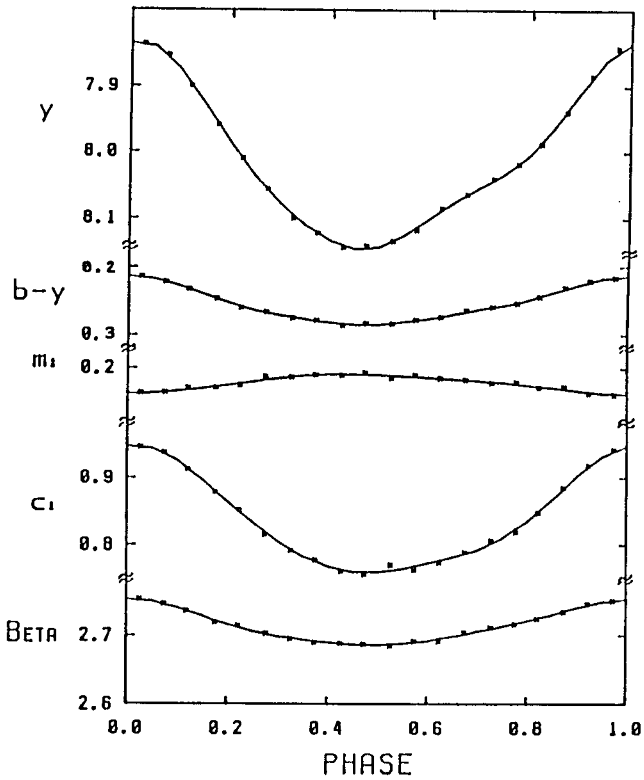

Fig. 5. Photometric normals and Fourier fitted curves for the first period.

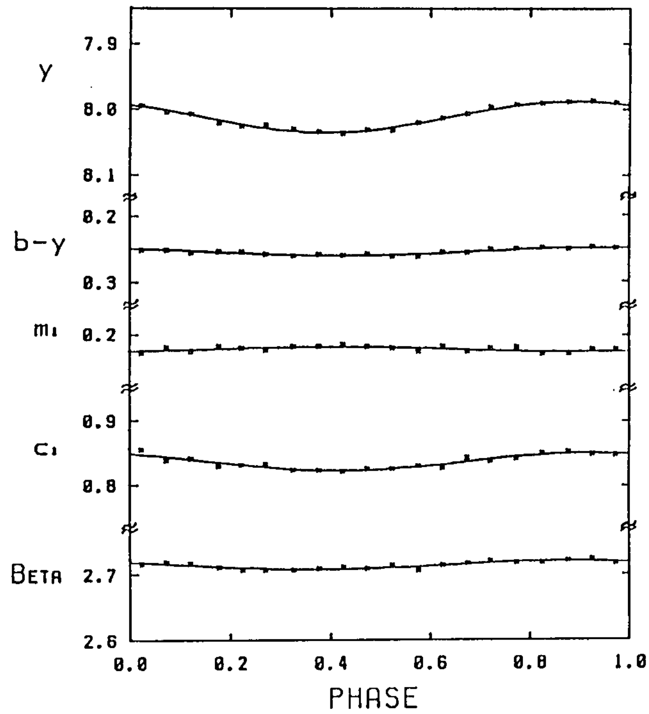

Fig. 6. Same as on Figure 5 but for the second period.

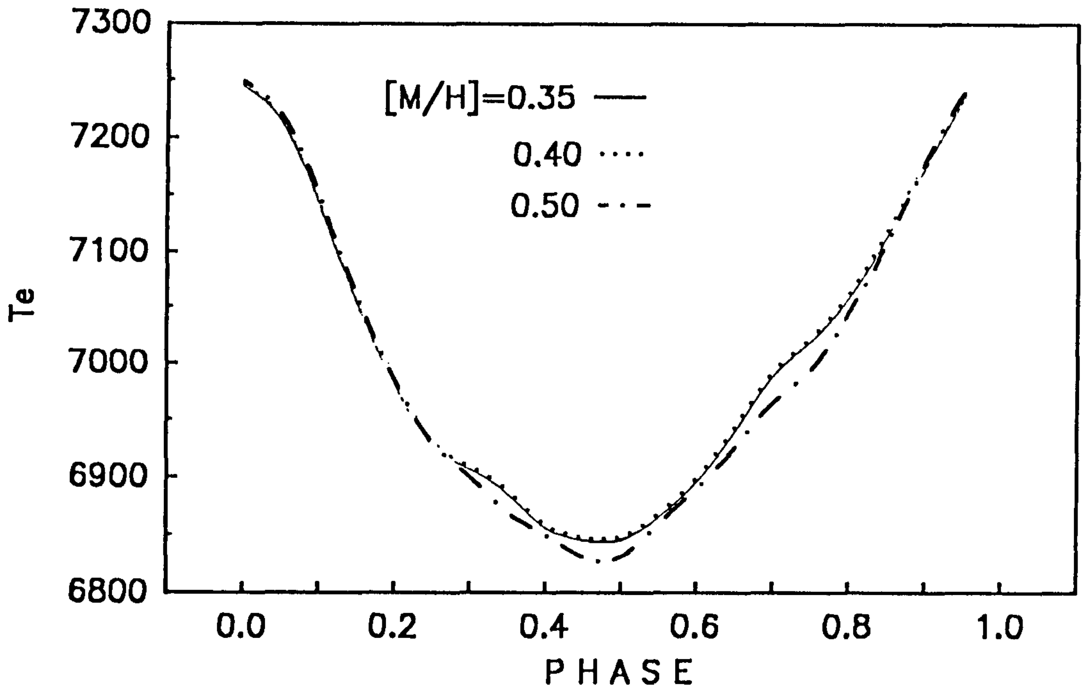

Fig. 7. Effective temperature variation along three different metallicity for the first period.

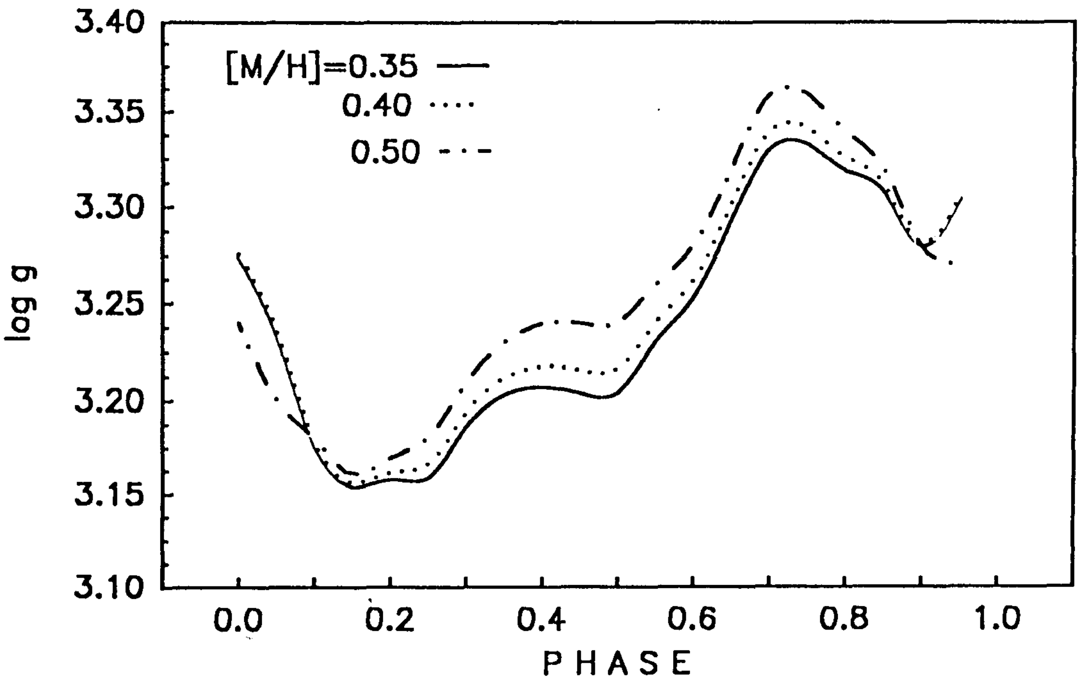

Fig. 8. Same as on Figure 7 but for the surface gravity.

Table 2. Photometric Normals of V1719 Cyg for the First Period

PHASE	y	b-y	m_1	c_1	β
.00	7.834	.213	.163	.947	2.753
.05	7.839	.217	.164	.945	2.750
.10	7.875	.227	.167	.926	2.741
.15	7.929	.240	.171	.897	2.728
.20	7.986	.252	.176	.864	2.716
.25	8.036	.263	.181	.832	2.706
.30	8.076	.270	.186	.805	2.699
.35	8.109	.276	.190	.783	2.694
.40	8.133	.281	.192	.768	2.690
.45	8.145	.284	.192	.760	2.687
.50	8.142	.284	.191	.760	2.686
.55	8.125	.280	.189	.765	2.688
.60	8.099	.275	.187	.773	2.692
.65	8.073	.269	.185	.782	2.699
.70	8.052	.262	.183	.793	2.705
.75	8.031	.256	.180	.810	2.713
.80	8.003	.248	.176	.835	2.721
.85	7.960	.237	.172	.868	2.730
.90	7.908	.226	.168	.903	2.741
.95	7.860	.216	.164	.932	2.749

Table 3. Photometric Normals of V1719 Cyg for the Second Period

PHASE	y	b-y	m_1	c_1	β
.00	7.992	.249	.175	.850	2.719
.05	7.998	.249	.176	.847	2.717
.10	8.005	.251	.177	.843	2.715
.15	8.012	.253	.178	.839	2.713
.20	8.019	.255	.179	.835	2.711
.30	8.031	.257	.181	.827	2.708
.35	8.033	.259	.182	.825	2.708
.40	8.034	.259	.182	.824	2.708
.45	8.033	.259	.182	.824	2.708
.50	8.029	.259	.182	.826	2.710
.55	8.023	.258	.181	.828	2.711
.60	8.017	.257	.180	.832	2.713
.65	8.009	.255	.178	.836	2.715
.70	8.002	.253	.177	.841	2.717
.75	7.996	.252	.176	.845	2.719
.80	7.991	.250	.175	.848	2.720
.85	7.988	.249	.175	.850	2.721
.90	7.987	.249	.175	.852	2.721
.95	7.989	.249	.175	.851	2.720

gravity is shifted about 0.15 phase with respect to the maximum expansion. These kinds of peculiarities are not so evident for other dwarf cepheids. Although the reason is uncertain, there may be a possibility that, at least near the maximum expansion, the calibration of surface gravity with Kurucz model does not work well to V1719 Cyg.

The calculated Te and $logg$ for $[Fe/H]$=0.4 are presented in the Table 4-5 for the first and second period. The effective temperatures of V1719 Cyg vary from 6850K at light minimum to 7250K at light maximum (shown in Figure 7). The surface gravity varies from 3.16 to 3.34 (shown at Figure 8). Finally, the mean values of the temperature and the surface gravity are 7010 $\pm$ 130 K, 3.25 $\pm$ 0.06 for the first period and 7010 $\pm$ 20K, 3.44 $\pm$ 0.01 for the second period. Again we can see that the different pulsation mode does not affect so much to the effective temperature and surface gravity. However the amplitude variation of these parameters along phase are reduced greatly for the second period.

It is well known that the gravities for dwarf cepheids seem to be overestimated with about 0.1 in $logg$, as the calibration of $logg$ from $uvby\beta$ photometry is based upon standard stars (ordinary rotational velocity) while the dwarf cepheids are slow rotators (McNamara and Feltz, 1978, hereafter MF; Breger, 1980; Andreasen, 1983, hereafter A83)

Table 4. Photometric Properties of V1719 Cyg for the First Period

PHASE	y_0	$(b-y)_0$	$(c_1)_0$	$(m_1)_0$	β	Sp	Te	log g	θ
.000	7.684	174	0175	.939	2.759	A	7250	3.27	14.141
.050	7.689	178	.176	.939	2.750	A	7220	3.23	14.215
.100	7.725	188	.179	.918	2.741	A	7150	3.17	14.246
.150	7.779	201	.183	.889	2.728	A	7060	3.15	14.247
.200	7.836	213	.188	.856	2.716	F	6990	3.16	14.182
.250	7.886	224	.193	.824	2.706	F	6930	3.16	14.101
.300	7.926	231	.198	.797	2.699	F	6910	3.19	13.942
.350	7.959	.237	.202	.775	2.694	F	6890	3.21	13.827
.400	7.983	.242	.204	.760	2.690	F	6860	3.21	13.803
.450	7.995	.245	.204	.752	2.687	F	6850	3.21	13.774
.500	7.992	.245	.203	.752	2.686	F	6848	3.21	13.793
.550	7.975	.241	.201	.757	2.688	F	6870	3.24	13.819
.600	7.949	.236	.199	.765	2.692	F	6900	3.26	13.854
.650	7.923	.230	.197	.774	2.699	F	6940	3.30	13.837
.700	7.902	.223	.195	.785	2.705	F	6990	3.34	13.768
.750	7.881	.217	.192	.802	2.713	F	7020	3.34	13.780
.800	7.853	.209	.188	.827	2.721	F	7063	3.32	13.791
.850	7.810	.198	.184	.860	2.730	A	7120	3.31	13.845
.900	7.758	.187	.180	.895	2.741	A	7180	3.28	13.931
.950	7.710	.177	.176	.294	2.749	A	7240	3.30	14.018

Table 5. Photometric Properties of V1719 Cyg for the Second Period

PHASE	y_0	$(b-y)_0$	$(c_1)_0$	$(m_1)_0$	β	Sp	Te	log g	θ
.000	7.838	.209	.188	.842	2.719	A	7050	3.26	13.934
.050	7.844	.210	.189	.839	2.717	A	7030	3.26	13.951
.100	7.851	.211	.190	.835	2.715	A	7040	3.26	13.890
.150	7.858	.213	.191	.831	2.713	A	7020	3.26	13.905
.200	7.865	.215	.192	.827	2.711	F	7010	3.25	14.922
.250	7.872	.216	.194	.823	2.710	F	7000	3.26	13.904
.300	7.877	.217	.194	.819	2.708	F	7000	3.27	13.895
.350	7.879	.219	.195	.817	2.708	F	6980	3.25	13.951
.400	7.880	.219	.195	.816	2.708	F	6990	3.25	13.906
.450	7.879	.219	.195	.816	2.708	F	6990	3.25	13.913
.500	7.875	.219	.195	.818	2.710	F	6980	3.24	13.975
.550	7.869	.218	.194	.820	2.711	F	6980	3.25	13.994
.600	7.863	.217	.193	.824	2.713	F	6990	3.24	14.009
.650	7.855	.215	.191	.828	2.715	F	7010	3.25	13.986
.700	7.848	.213	.190	.833	2.717	F	7010	3.25	14.003
.750	7.842	.212	.819	.837	2.719	F	7019	3.24	14.020
.800	7.837	.210	.188	.840	2.720	F	7030	3.25	13.994
.850	7.834	.209	.188	.842	2.721	A	7050	3.26	13.960
.900	7.833	.209	.188	.844	2.721	A	7040	3.25	14.000
.950	7.835	.209	.188	.843	2.720	A	7040	3.25	13.987

and therefore *logg* value of V1719 Cyg is necessary to be corrected too. However since it was found that the value of $logg = 3.25$ for the first period is rather underestimated of 0.15 than the value from the theoretical period - gravity relation for dwarf cephids by A83, we did not correct our surface gravity values. This also makes this star peculiar from other ordinary dwarf cepheids. We will discuss about the rotation problem of V1719 Cyg in the last section again.

Finally our effective temperature and surface gravity values can be compared with T_e=7020K and $logg$=3.44 by JJ86 who obtained these with ignoring multiperiodicity, applying Breger model where solar abundence was adopted. Hence although the direct comparison may not be so adequate, the effective temperature is consistent but again the surface gravity is different. The mean temperature of seven high amplitude Scuti stars (A83) is 7420 K and, therefore, we can see that V1719 Cyg is a very cool dwarf cepheid.

III. ABUNDANCE ANALYSIS

By carrying out a systematic study of the behaviour of the m_1 metallicity parameters, it was shown by Rodriguez, et al. (1990) that the behaviour of m_1 and m_1 variation of dwarf cepheids and SX Phe stars is strongly related with the chemical composition. For example, it was found that, fixing T_e and $logg$, the amplitute of the m_1-index curve is larger when the metal abundance is smaller, although an inversion around $[M/H]$=0.5. And in this sense, we can place V1719 Cyg at the boundary where the inversion is just occurring.

Furthermore, they found that the theoretically calculated slope of m_1 index from the Kurucz model atmosophere changes from plus to negative at $[M/H]$=0.0 for fixed T_e=7000k and $logg$=3.5 (see their Figure 4b). For V1719 Cyg, observed strongly negative slope without taking into account different oscillation mode is consistent with this theoretical result. As can be seen in Figure 9, this was again confirmed in our analysis for the first period. In other words, to explain the observed slope, $[M/H]$ should be around 0.5 according to the model atmosphere calculation which is in good agreement with our $[M/H]$=0.4.

However, as we saw in the previous section, the range of $[M/H]$ for a certain pulsation mode and for different spectral region, the variation of metallicity reaches about 0.3 and 0.6 respectively which makes the determination of metallicity very uncertain. Furthermore for V798 Cyg which shows similar unusual light curve variation as V1719 Cyg, it was found by Kim and Joner (1994) that this star is solar abundant. There is no doubt about the fact that the characteristics of V1719 Cyg is strongly related with reliable metallicity. Hence we tried to estimate the metallicity of V1719 Cyg through spectroscopic observation.

As a preliminary work, spectroscopic CCD observations were carried out in July 31, 1991, using the Cassegrain spectrograph of the 1.85m reflector at the Dominion Astrophysical Observatory (DAO). The star was observed through a 2 arc-seconds slit, with a 1200 lines/mm greating blazed at 4100 Åin the first order giving a resolution of about 1.3 Åand producing a spectral dispersion of 15A/mm. A 512x512 Ford Aerospace (pixel size : 20x20μm) CCD was used as a detector.

Among those lines in the observed red region of 5100Å- 5250Å, three MgI lines corresponding to 5183.605Å, 5172.684Å, and 5167.320Åwere strong enough to analyze. Among these, 5167.320Åline which is relatively free from blending was analysed relative to the Sun with a line-blanketed convective model atmosphere. The spectrum is presented on Figure 10. Computations are carried out for 9 assumed abundance of Mg starting from 5.58 (2.0 dex underabundant) with a step of 0.5 dex by adopting the atmospheric parameters of T_e=7000K, $logg$ = 3.3 and classical damping constant. Resulting equivalent widths are given in the Table 6 in lines labelled as EW and they are in unit of mÅfor three values of the microturbulent velocities of 0.0, 1.0, and 2.0 km/s.

We can see that Mg in V1719 Cyg is nearly solar, or underabundant by 0.2 to 0.3 dex by comparision with the observed equivanent width of 270 mÅ. This result is inconsistent with our previous $[Fe/H]$=+0.4 from photometric analysis. However we like to point out that this conclusion is temporal because the analysis was given to the single line of Mg which is not a representative metallicity indicator, and S/N ratio of the data was low due to poor seeing condition. It is therefore absolutely required to observe some other elements such as Ca or Fe to obtain further information with high resolution and high S/N ratio data.

IV. PULSATION PROPERTIES

To investigate the physical characteristics of variables, it is necessary to determine their oscillation mode for each different period. However the identification of oscillation mode is one of the most basic, but one of the most difficult problems in observations of variable stars. The nature of the oscillations in δ Scuti stars is still an unsolved problem. The evidence for radial pulsation is strongest for the dwarf cepheids. These variables either have one stable frequency or pulsate simultaneously in two modes whose frequency ratio is close to that expected for fundamental and first overtone radial pulsation like AI Vel or V703 Sco.

A few different methods have been developed to identify oscillation mode by different investigators. Dziembowski (1977) had suggested, for the first time, that oscillation mode can be identified by utilizing the phase difference between light and color index, and the amplitude ratio. And this method has been applied to different type of variables like δ Scuti (Balona and Stobie, 1979), β Cephei (Stanford and Watson, 1977), and ZZ Ceti (Robinson et

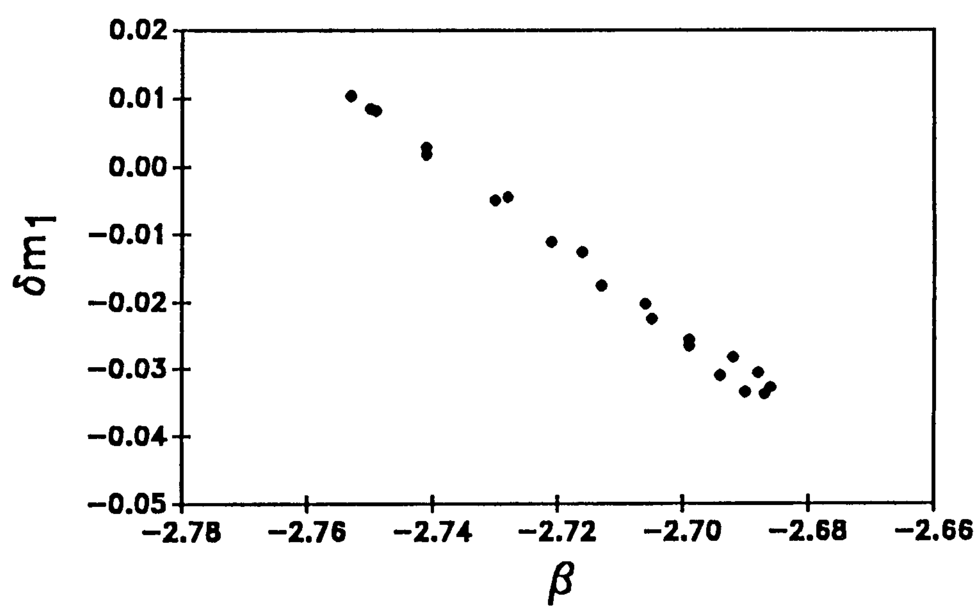

Fig. 9. The variation of m_1–index along Hβ value for the first period.

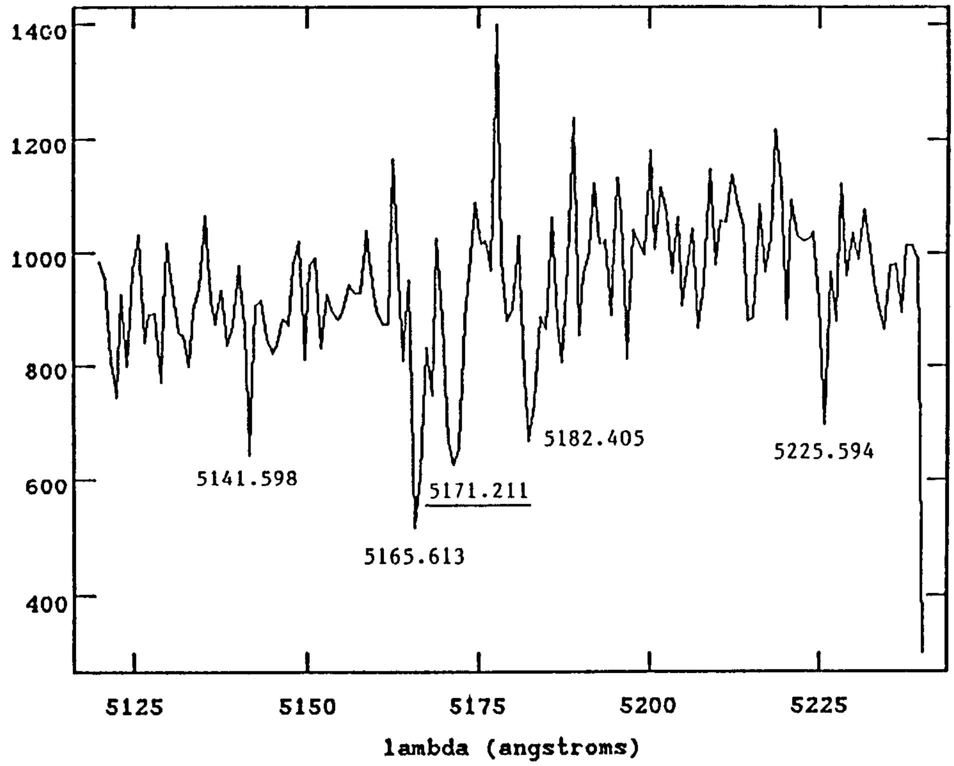

Fig. 10. CCD spectra of V1719 Cyg between 5100-5230Å. The line used in abundance analysis is marked with underline.

al., 1982).

By utilizing the method by Balona and Stobie (1980), PA88 determined the phase difference of $\Delta\phi_1 = \phi_V - \phi_{B-V}$ $= -0.014$ and $\Delta\phi_2 = -0.025$ for the first and second period respectively for V1719 Cyg. From these results and from the values tabulated by Balona and Stobie (1980), they had inferred that the modes are radial. To confirm this result directly, we adopted the method developed by Watson (1988) who outlined a format for the comparison of observational light and colour data with model predictions on an amplitude ratio versus phase difference plane. He showed that this method can be applied to many different type of variable except rapidly oscillating Ap stars. To select the best one corresponding to proper pulsation constant (Q) value for V1719 Cyg among many different figure on $(A_{B-V}/A_V,\ \phi_{B-V} - \phi_V)$ plane, we first calculated Q value for the first and second period. To do this, the equation by Breger and Bregman (1975) was used;

$$logQ = -6.454 + logP(days) + 0.5logg + 0.1M_b + 3.843. \qquad (2)$$

Here the M_b=0.6 mag which was obtained as the bolometric magnitude value in section six was used and we obtained Q = 0.032 ± 0.002 and 0.026 ± 0.002 for the first and second period. Here the errors were calculated by taking the overall error of $logg$ and M_b as the same amount of ± 0.1. Also again $\phi_{B-V} - \phi_V = 0.014 \pm 0.005,\ 0.025 \pm 0.020$ and $A_{B-V}/A_V = 0.364 \pm 0.021,\ 0.383 \pm 0.049$ by PA for the first and second period were used. Hence we selected the Watson model for the standard (T_e, $logg$, Q) = (7400, 3.5, 0.03) test regime which is the closest one to our (7010, 3.25, 0.032). As can be seen in Figure 11, undoubtedly l =0 for the both first and second period repectively and, therefore, two modes are radial which is consistent with that argued by PA88.

On the other hand Fitch (1981) developed the pulsation model at the instability strip for δ Scuti stars and determined pulsation constant depending upon mass and the degree of evolution for l =1,2,3 and he argued that the pulsation constant is not sensitive to the change of mass and the degree of evolution but strongly depends on pulsation mode (n,l), i.e., Q_0 = 0.032 - 0.036, Q_1 = 0.024 - 0.028, Q_2 = 0.0195 - 0.0225, and Q_3 = 0.016 - 0.0185 for n = 0, 1, 2, and 3 respectively. Q = 0.032 for the first period fits to Q_0 and Q = 0.026 for the second period can be corresponding to Q_1. Hence the pulsation modes of V1719 for the first and second period can be identified as the radial fundamental and first overtone mode. And this conclusion supports the inference of PA88 that P_1=0.2673 d for V1719 Cyg is so big as the period of a δ Scuti star that it can not be the first overtone mode. Naturally our direct identification of the pulsation mode makes us to confirm the helium settling within the envelope of V1719 Cyg.

V. RADIUS

A few different methods have been developed to determine the radius of pulsating variables. However these methods are the modification of classical Baade-Wesselink method and it was found that the surface-brightness method is the most reliable one by Carney and Latham (1984) and Jones, et al (1987). However to determine the radius of multiperiodic variable with this method, oscillation mode corresponding to each different period should be known because it was known that resulting Wesselink radius is sensitive to the spherical harmonic order of the oscillation by Dziembowski (1977). Balona and Stobie (1979) have also shown that the Baade-Wesselink technique has to be modified to incorporate oscillation and they showed that for odd values of the projected area variation is zero and the Wesselink technique breaks down. Since the oscillation modes for both periods of V1719 Cyg were determined as radial in previous section, we can apply the surface-brightness method by accepting necessary assumptions (see Balona and Stobie, 1979). Surface-Brightness Method (SBM) developed by Barnes and Evans (1976), gives the relation between the stellar visual surface brightness parameter F_v and $(V - R)_o$ color index. This relation with other color has been applied to a number of astrophysical problems such as establishing independent distance scales for Cepheid variables (Barnes 1980, Gieren 1985), RR Lyrae stars (Manduca et al. 1981, Carney and Latham 1984, Jones et al. 1987). In this method the surface brightness parameter F_v is defined by

$$\begin{aligned} F_v &= 4.2107 - 0.1V_0 - -0.5log\theta \\ &= logT_e + 0.1B.C., \end{aligned} \qquad (3)$$

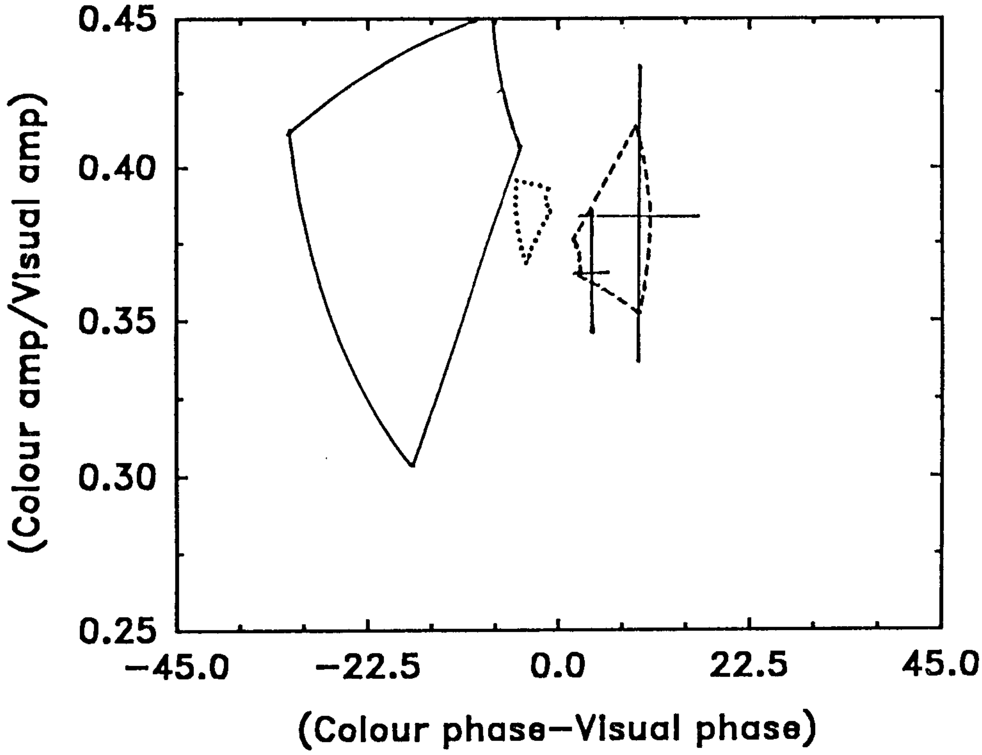

Fig. 11. Data for the star V1719 Cyg compared with the areas of interest on the (A_{B-V}/A_V, ϕ_{B-V}-ϕ_V) plane for δ Scuti test regime. Dashed, dotted, and full lines correspond, respectively, to l= 0, 1, and 2 for the standard (T_e, log g, Q) = (7400, 3.5, 0.03) test regime by Watson (1988). Dot and triangle with error bars correspond to the first and second period.

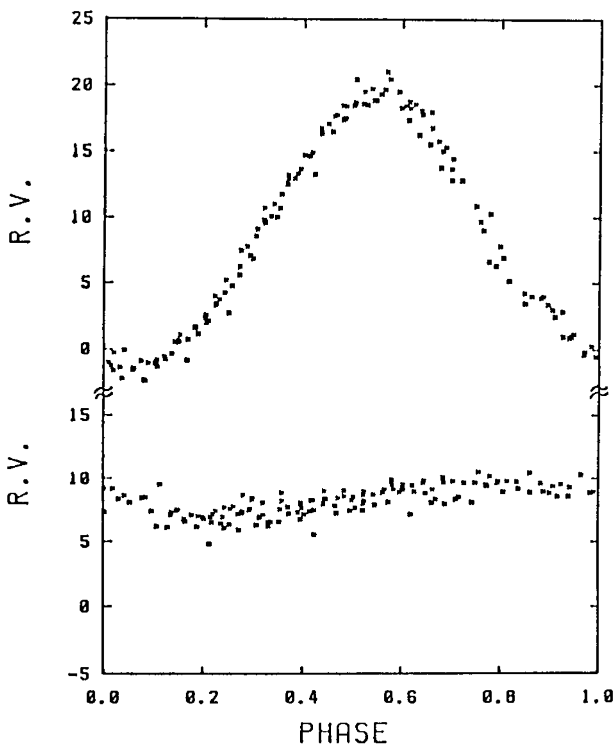

Fig. 12. Radial velocity curves for two periods. Large amplitude corresponds to the first period. Original data from Imbert (1980).

Table 6. Calculated Equivalented Width for Three Different Turbulent Velocities and for Nine Assumed Abundances.*

Micro Turbulence (Km/s)	Abundance								
	5.58	6.08	6.58	7.08	7.58	8.08	8.58	9.08	9.58
	Equivalence Width (mA)								
0.0	91	114	143	192	287	471	791	1359	2482
1.0	98	123	154	203	297	477	803	1395	2477
2.0	117	147	141	229	319	494	826	1397	2477

*Abundance Value of 5.58 is Corresponding to 2.0 dex Underabundant Relative to the Sun.

where V_0 is the unreddened apparent visual magnitude, θ is the angular diameter in milli-arcseconds, T_e is the effective temperature, and B.C. is the bolometric correction. Calibration constant 4.2107 was revised by Kim (1990) by adopting γ Gem, α CMa, α CMi, and α Lyr as the standard stars. This method employs the above equation and the following relation:

$$D(\phi_i) - D(\phi_j) = D(\phi_i, \phi_j) = -\int V_p dt \; = -\int P(V_{rad} - \gamma) dt \tag{4}$$

$$\Delta D + D_m = 10^{-3} r. \tag{5}$$

In this equation, ΔD is the change in diameter measured in astronomical units, D_m is the minimum stellar diameter in astronomical units, V_{rad} is the radial velocity, γ is the mean velocity, P is the projection factor, and r is the distance to the star in parsecs. By matching the angular diameter curve to the radial displacement curve, radius and distance can be obtained simultaneously through least - square fitting.

Through the same procedure as in the light curve analysis, two radial velocity curves corresponding to the first and second frequency were separated (shown at Figure 13) from the radial velocity data by Imbert (1980) and two radial displacement curves were obtained. Radial velocity normals for the first and second period for V1719 Cyg are given in Table 7 and Figure 13 shows the variation of normal points. In this procedure we used 1.31 (Parsons, 1972) as a projection factor and the calculated angular diameter(θ) by using the equation (3) is presented in the last column on Table 4-5.

Another angular diameters were obtained from the empirical relation between F_V and $(b-y)_0$;

$$F_V = 3.961 - 0.529(b-y)_0, \tag{6}$$

derived by Kim (1990) for the comparison with above theoretical result. Here the effect of different metallicity and surface gravity were not taken into consideration. We present the best fit of the radial displacement curve to the angular diameter for the first period in Figure 14. However over all fitting of the theoretical angular variation ($\sqcap$) to the radial displacement variation (solid line) is very poor. Although some phase shift between these two variations was found for other dwarf cepheids like AD CMi by Kim (1990), so poor fitting is beyond expectation. Therefore, with the surface brightness method, we can not derive the reliable distance and radius of this star.

Also the variation of angular radius from the model atmosphere and empirical relationship is abnormally different and even the direction of the variation is opposite for the region of ϕ = 0.2-0.7. In spite of some larger amount of phase shift, for the case of the angular variation (∇) from the empirical relation, the overall variation is similar to that of the radial displacement. Again this makes us to doubt the application of the Kurucz model atmosphere grid to V1719 Cyg as in the determination of surface gravity in section two and, in this sense, V1719 Cyg is again very peculiar.

VI. EVOLUTIONARY STATUS

Now it is possible to determine the mass of V1719 Cyg with theoretical evoluationary tracks in the $logT_e$ versus $logP_0$ diagram. The evolution of the variables is based on the standard evolutionary sequences and the relation

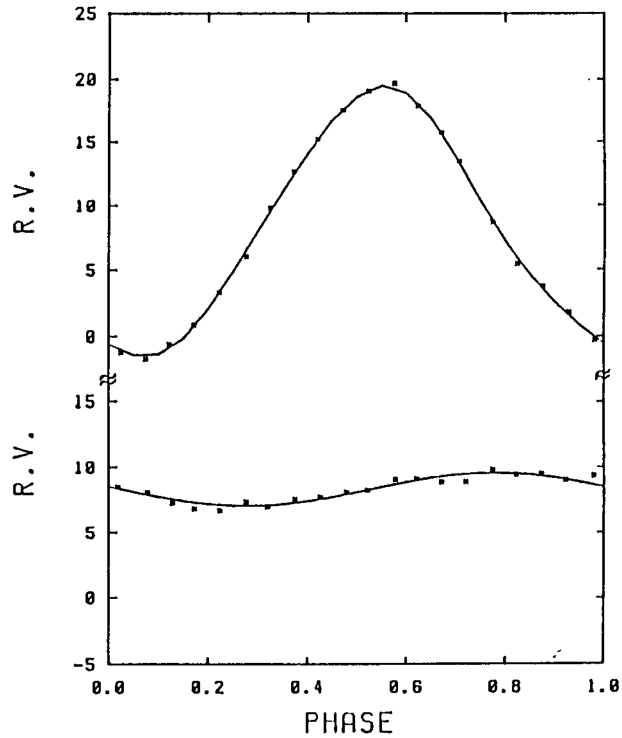

Fig. 13. Radial velocity normals and Fourier fitted curves for two periods.

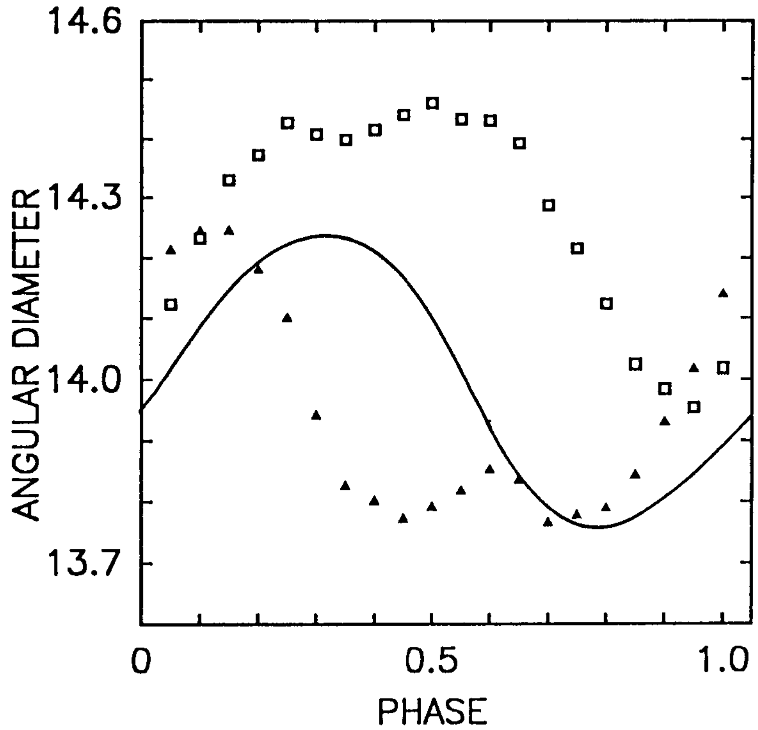

Fig. 14. The angular diameters (θ , units are 10^5 arcsec) and radial displacement curves for the first period. △ and □ are photometric radii from theoretical and empirical methods respectively. Solid curve corresponds to the radial displacement variation.

Table 7. Radial Velocity Normals of V1719 Cyg for Two Periods

PHASE	$V_r(1)(Km/s)$	$V_r(2)$
.00	-0.5	8.5
.05	-1.3	8.1
.10	-1.3	7.7
.15	-0.1	7.4
.20	2.0	7.2
.25	4.8	7.1
.30	7.9	7.1
.35	11.1	7.2
.40	14.0	7.4
.45	16.6	7.7
.50	18.6	8.1
.55	19.5	8.5
.60	18.9	8.8
.65	16.8	9.2
.70	13.7	9.4
.75	10.2	9.5
.80	7.1	9.5
.85	4.5	9.4
.90	2.5	9.2
.95	0.8	8.9

$P_0 \sqrt{M/R^3} = Q_0$ where P_0 is the fundamental period in days, M and R are the mass and radius, and Q_0 is the fundamental pulsation constant respectively. We have first utilized the evolutionary sequences of VandenBerg (1985) for Z = 0.04 and Y = 0.25 and the tracks are exhibited in Figure 15 with the position of V1719 Cyg.

It is evident that the mass of V1719 Cyg is 2.7 solar mass if V1719 Cyg is in a hydrogen shell burning stage of evolution as other members of this group of variables. The 2.7 $M_\odot$ would place V1719 Cyg at the post main sequence evolutionary stage and by referring to the models we find the age is 0.42 Gyr. These values of mass and age can be compared with 2.4 $M_\odot$ and 0.42 Gyr from the standard evolution tracks for Z=0.02 for δ Scuti stars by Andreasen, et al (1983) and there is an excellent agreement between two models. Mass and age of V1719 Cyg are heavier and younger than all other seven double mode dwarf cepheids by A83.

The Crawford calibration relating c_1 to M_v yields $M_v = 0.8 \pm 0.2$ mag. But as MF have pointed out, this absolute magnitude requires a correction of 0.3 mag due to the effect of rotation on the c_1 index which, if applied, yields M_v = 0.5 mag. The bolometric magnitude of M_b=0.7 mag was obtained from VandenBerg Model for Z=0.0169 instead of Z=0.04 for V1719 Cyg because the model corresponding to Z=0.04 was not given. Also from standard evolution track on ($logT_e$, M_b) plane for Z=0.02 for δ Scuti stars by A83, it was found that M_b=0.8 mag and we adopt M_b=0.6 mag as the bolometric magnitude of V1719 Cyg.

Naturally a justification to the rotational correction to V1719 Cyg can be doubted due to the smaller value of absolute magnitude (=0.5 mag) than bolometric magnitude (=0.6 mag). Furthermore M_v=0.7 mag from the absolute magnitude - period relation of $M_v = -3.25\ logP - 1.45$ by MF for dwarf cepheids and again M_v=0.5 mag looks too small. Oppositely as in other dwarf cepheids, if the calibration correction due to slow rotation for both surface gravity and absolute magnitude is not necesary for V1719 Cyg, there is a possibility that this variable may not be a slow rotator. However since it has been known well that all dwarf cepheids are slow rotator, high resolution spectroscopy is required to check this. We will discuss about this thing with the connection to diffusion phenomenon for pulsating variables in the next section.

VII. DISCUSSION

Relatively large m_1 and c_1 values of V1719 Cyg make us to remind some Am/Fm charateristics due to diffusion phenomenon (Kurtz, 1979). In addition, the fact that this variable is an evolved cool star coincides with the characterisics of δ Delphni stars. Cowley (1968) classified a member of stars as δ Delphni stars and stated that the metallic - line spectrum resembles that of an F2IV star but the hydrogen and ionized calcium lines are very

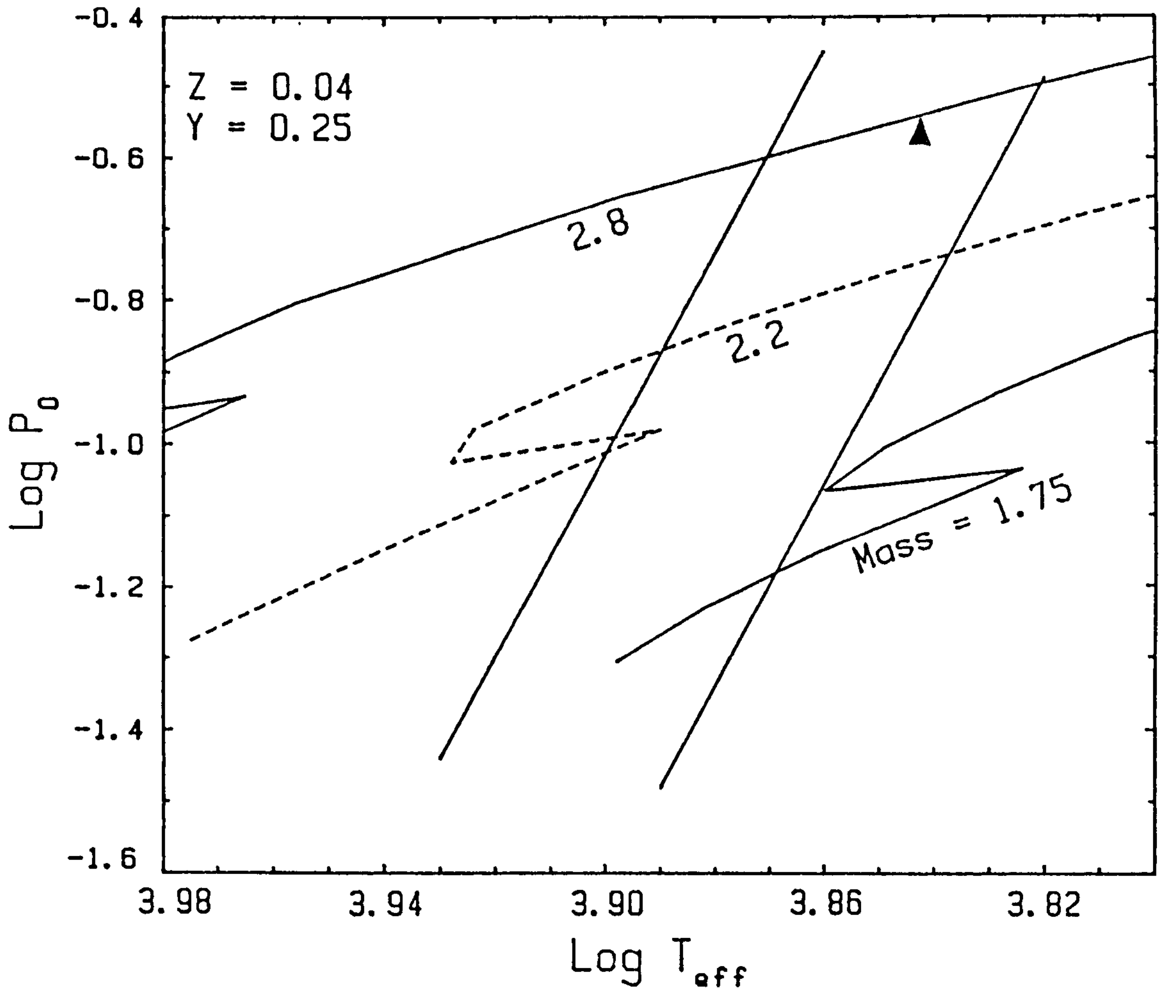

Fig. 15. Evolutionary tracks in the log T_e - log P_o plane. Numbers attracks give model masses and the plus represents V1719 Cyg.

narrow. Morgan and Abt (1972) classified a number of δ Scuti star and the δ Scuti class of stars (which is defined photometrically) was found to be quite inhomogeneous spectroscopically : some of δ Scuti stars show normal spectra, whereas others show a definite weakening of the CaII K and H lines

Furthermore Kurtz (1976, 1978, 1984) had insisted that δ Del stars are the evolved *Am* stars and they can pulsate. Recently Gray and Garrison (1989) suggested that δ Del type should be renamed to ρ Puppis type by defining this class as unusually late, apparently evolved metallic - line stars, because δ Del stars are normal or can not be distingushed from Am stars after the investigation of detailed spectral classification criteria for F-type stars. They also showed that the rotation velocity for A9 and F0 stars is low (v sin i < 100 km/s).

They said the spectroscopic characterisics of ρ Puppis type are that hydrogen lines are F5 type not F2 or earlier as in most of *Am* stars, and luminosity can be classified as F5II-III-F5Ib in $\lambda\lambda$4172-4179 blend. On the other hand, it was also found that β index indicates a siginificantly higher effective temperature than the $b-y$ index in $uvby\beta$ photometric system and in $B-V$ and $b-y$ relation by Crawford (1975), $b-y$ indices is 0.02 mag bluer than $B-V$. This stars actually lie above the main sequence and thus may be examples of unusually cool, evolved *Am* stars. According to luminosity versus c_1 relation, V1719 Cyg can be classified as IIIb and it was also found for V1719 Cyg that the effective temperature is higher about 130 K when the β index was used as a temperature indicator for all cycle. All these things make us to tempt to classify this variable as ρ Puppis type.

With regard this, we like to point out that if the diffusion is the correct explanation for the abundance anomalies in this type variables, then there must be sufficient residual helium left in the He II ionization zone to drive pulsation. Since the oscillation mode of the first and second period was identified as the fundamental and first overtone mode, this fact can be supported by the period ratio of $P_2/P_1 = 0.80$ which is possible when the helium settling phenomenon occurs. However the problem is, although the result is temporal, Mg in V1719 Cyg is nearly solar abundant and also the metallicity of V798 Cyg which shows the similar unusual light curve as in V1719 Cyg is about solar abundant. Then if the metallicity of V1719 Cyg is really solar or under abundant, the period ratio of P_2/P_1=0.80 can not be explained with helium settling within the envelope, and so the different explanation should be provided.

Through the earlier analysis of the unusual light curve of V1719 Cyg by other investigators, it was found that this star is peculiar since the descending branch is steeper than the ascending branch. Also the variations of m_1-index and of the amplitudes and phase differences of Fourier decomposition are different from those of other dwarf cepheids. In addition we showed that there are some other peculiarities, i.e., peculiar variations of surface gravity and angular diameter, and improper rotational correction to the absolute magnitude. However since all these peculiarities are from the photometric analysis, additional spectroscopic investigation is absolutely required. In other words, to clarify and explain all properties related with peculiar characterisics of V1719 Cyg, more detailed investigation via simultaneous photometric and spectroscopic observations, especially extensive high-resolution spectroscopy, should be given to this variable star and also to other similar variables like V798 Cyg and V974 Oph.

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