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DYNAMICAL CHARACTERISTICS OF SUNSPOT CHROMOSPHERES* I. ANALYSIS OF CIRCULAR POLARIZATION MEASURED FROM A SUNSPOT*

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ABSTRACT

We have analyzed a set of high resolution photographic line profiles of a Zeeman sensitive Fe I λ 6302.5 line taken with the Universal Birefringent Filter over a single round sunspot (SPO 5007) at the Sacramento Peak Solar Observatory. The observed spectra recorded on films are traced by PDS and the traced densities are converted to relative intensity by means of IRAF. The Stokes I and V profiles are then constructed by adding together and subtracting from each other the left and right handed circular polarizations, respectively. The reduced I and V profiles are analyzed by means of the coarse analysis(Auer *et al.*(1977), Skumanich and Lites(1987)) with the use of inversion technique. It is found that the umbral field strength is about 3000 gauss and the field distribution follows closely the emperical model proposed by Wittmann(1974).

Key Words : sunspot, circular polarization, Zeeman effect, magnetic field distribution.

I. INTRODUCTION

Babcock invented a photoelectric device which is capable of measuring weak magnetic fields, but this system measured the field only at a single point. After a series of attempts, Leighton finally introduced a technique of making two-dimensional magnetograms. At present many observatories use Stokes polarimeters which provide direct measurements of the profiles of all four Stokes parameters.

The aim of the present work is to derive the magnetic field distribution over a sunspot (SPO 5007) by analyzing the circularly polarized line profiles of Fe I λ 6302.5 by means of the coarse analysis introduced by Auer *et al.*(1977) and later improved by Skumanich and Lites(1987). In section II we describe the processes how I and V profiles are reduced from the observed line profiles. Section III presents how our reduced profiles are analyzed by means of the coarse analysis. In section IV we examine the characteristics of synthetic I and V profiles to obtain the field distribution across the sunspot. Finally, a brief summary and conclusions will be given in section V.

II. DATA REDUCTION

Yun(1980) made high dispersion spectroscopic, observations of a single round sunspot SPO 5007(an angular diameter 36", located nearly at the disk center μ =0.9) with the Universal Birefringent Filter attached to the Echelle spectrograph at Sacramento Peak Observatory. The spectral dispersion is 6.9 mm/Å at Fe I λ 6302.5, and the image scale is 3".46/mm which corresponds to 2500km.

The slit jaw camera in white light are exposed simultaneously with the spectra. For spatial reference a fiducial hair line is inserted in front of the entrance slit of the spectrograph. Figure 1 shows the white light slit-jaw image of the observed sunspot, where the vertical black line is the fiducial mark.

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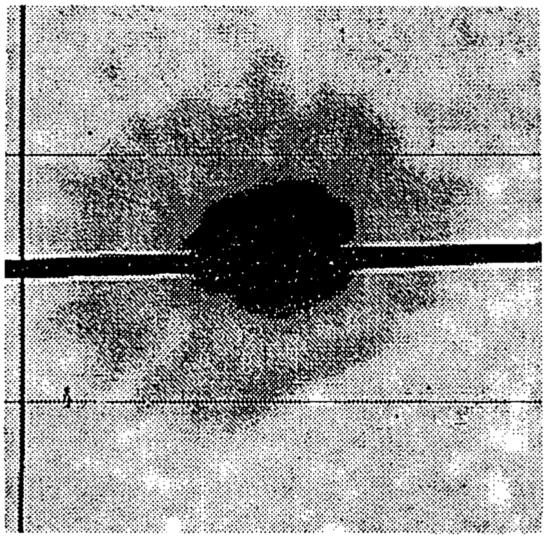

Fig. 1. The slit-jaw image of a sunspot SPO 5007. The black horizontal band is a slit and the vertical line is a fiducial mark.

In order to get a good set of the circularly polarized line profiles the photographic densities recorded on the films have to be measured accurately. For this purpose we have used PDS(Photometric Data System) at the Korea Astronomy Observatory. We scanned the films with a speed of 25 csu (conventional speed unit; Back, 1992) with an aperture size $50\mu m$ x $50\mu m$, which is found to be quite adequate for our purpose. $50\mu m$ corresponds to $0.00742 Å$ in wavelength. The scanning direction is adjusted to perpendicular to the fiducial line and also to the direction of dispersion. The films are scanned to full wavelength ranges in order to include the local continuum.

The stability of the system has been tested by reading the same frame twice. Figure 2 shows their complete concordance. We are convinced that all the small fluctuations shown in the figure are the observational noises and they are not inherent to the PDS. We also noted that the PDS gave the identical result even at a higher speed of 100 csu. A typical frame of the observed image is shown in Figure 3, where the oppositely polarized two σ components are seen to be shifted in the opposite direction. One may note that in addition to the two iron lines there are two telluric O_2 lines, which are used later as the wavelength reference as well as the normalization factor of the intensity measurements.

HD curve is derived to convert photographic densities into relative intensities. Since the transmission of each step wedge and its corresponding density are known, the HD curve can be obtained. When D and I denote the logarithem of the density of each step wedge and the percentage of the transmission, the intensity I can be expressed as $I = A_o + A_1 D + A_2 D^2 + A_3 D^3 + A_4 D^4$. After substituting data points into the equation, the constants A_o, A_1, A_2, A_3, and A_4 are determined by means of least squares. Figure 4 shows a typical HD curve along with the images of the step wedges from which the HD curve has been derived.

In order to obtain I and V profiles we simply added together and subtracted from each other the relative intensities of the right and the left handed circular polarizations. Prior to getting the I and V profiles, some minor adjustments have to be made, since position, wavelength, and strength of relative intensities of spectral features in each individual films differ from each other. The location can be accurately identified by referencing the fiducial mark in the slit-jaw images to that in the spectra. All the reduced relative intensities are normalized to the central intensity of the telluric O_2 λ 6302 line because it is completely independent of the Sun. Figure 5 shows a typical set of the reduced I and V profiles in the penumbra and the umbra, where RHC and LHC refer to the right and left handed circular polarizations , respectively. Figure 6 shows the spectral variations of the I profile across the spot from the umbral center to the outer penumbral boundary. Here we may note that the amount of the shift of the two σ components

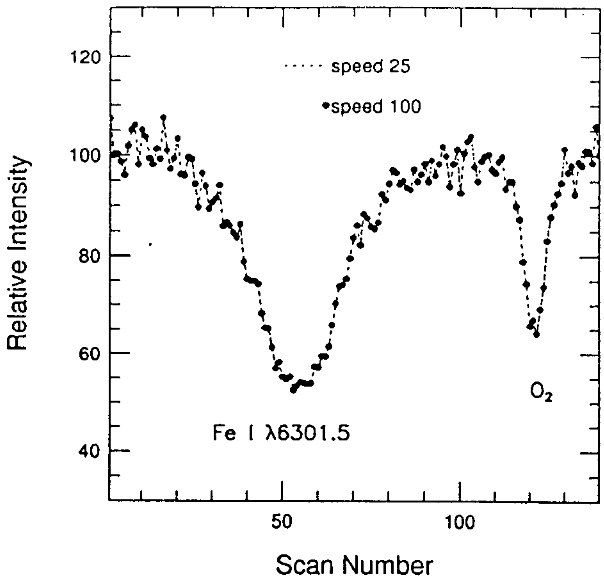

Fig. 2. The same frame is read twice with different speed. It shows a complete concordance, indicating that the small fluctuations are not caused by the PDS reading but by real observational noises.

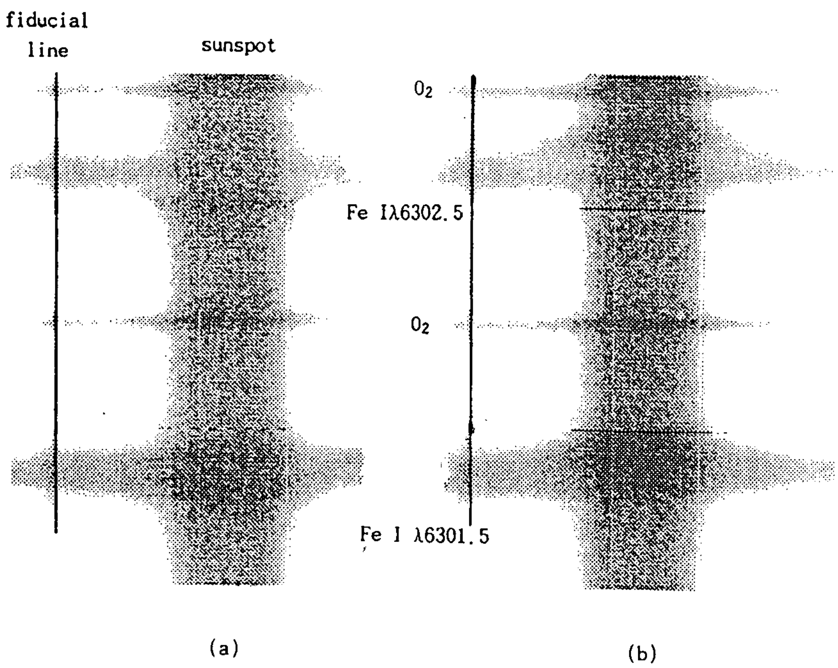

Fig. 3. The images of the right(a) and the left(b) handed circular polarization. We can see the two Fe I lines are shifted to opposite directions.

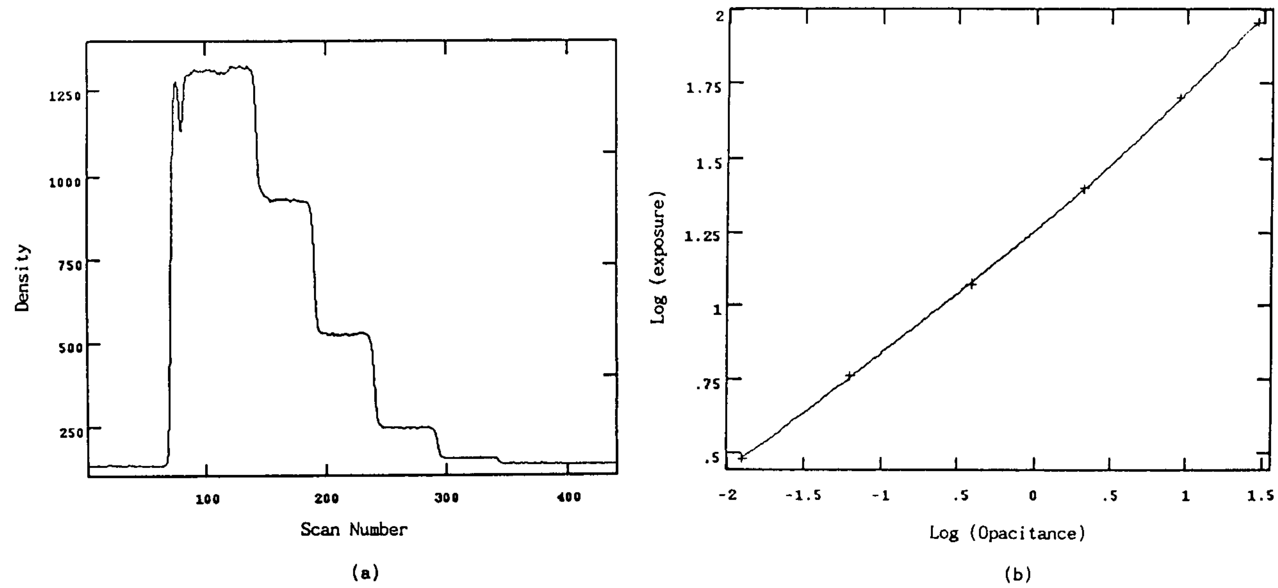

Fig. 4. (a) The density profile of wedges (b) The HD curve.

vary with positions and that the Zeeman splitting in the umbral region is completely apart.

III. THE COARSE ANALYSIS OF STOKES PROFILES

The strength of magnetic fields can be deduced from the Zeeman splitting of a magnetically sensitive spectral lines. In the presence of the magnetic fields, such lines are displaced from its original position λ_o in accordance with

$$\lambda - \lambda_o = \frac{e}{4\pi m_e c}\lambda^2 B g^*,$$

with $g^* = gM - g'M'$, where g, g' and M, M' are the Lande' factors and magnetic quantum numbers of the lower and upper states of the transition under consideration, respectively. Here we may note that the field strength B is directly propotional to the amount of the line splitting.

Thus, a simple way to obtain the field strength is to measure the wavelength separation of the two σ components directly. Beckers and Schröter(1969) used this method to measure the field strength from the Fe I λ 6173.3 line. However, this method is subject to uncertainties arising from the line asymmetries caused by the instrumental polarization and the displacement of the minima of the I and V line profiles through smoothing processes. Another possible approach is to calculate the synthetic I and V profiles with several physical parameters which account for the observed profiles. This method is called the coarse analysis which was developed by Auer *et.al.*(1977; hereafter AHH) and later improved by Skumanich and Lites(1987). It has an advantage of dealing with a large number of Stokes profiles rather quickly.

The AHH's coarse analysis uses the simple Milne-Eddington model atmosphere with the line source function $S(\tau) = B_o + \beta\tau$, where B_o and β are constant. In order to assess the reliability, AHH compared the profiles generated from the simple coarse analysis with the synthetic profiles calculated from the realistic model atmospheres under the assumption of a simple field configuration. From this comparison they confirmed that the two different approaches yield nearly the identical results. Skumanich and Lites(1987; hereafter SL) took similiar calculations and reached the same conclusion. Accordingly, in the present study we have used the coarse analysis similar to the SL's to deduce the field strength and its configuration of SPO 5007 from the observed iron I and V profiles.

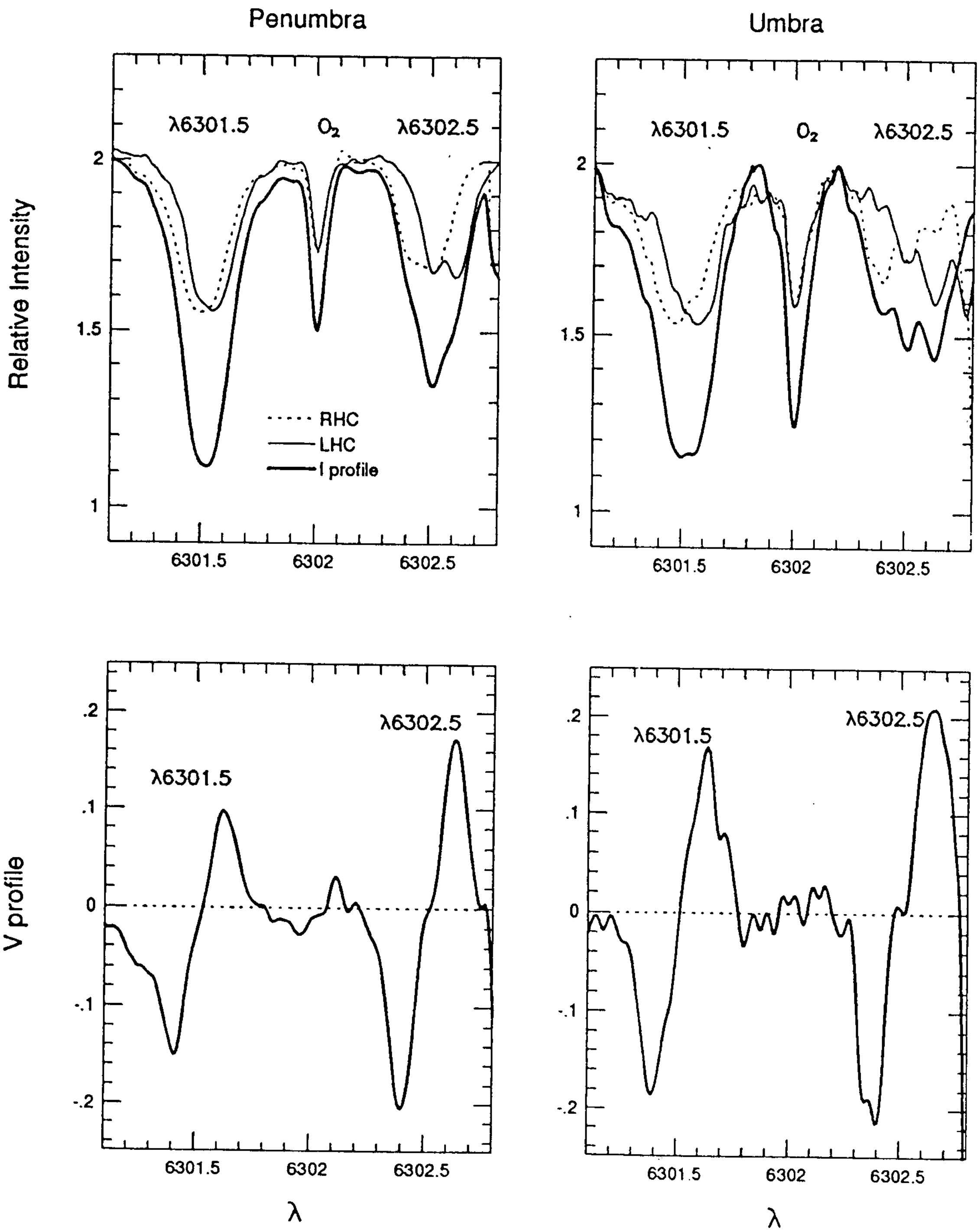

Fig. 5. Reduced RHC, LHC, I, and V profile in the penumbra and the umbra.

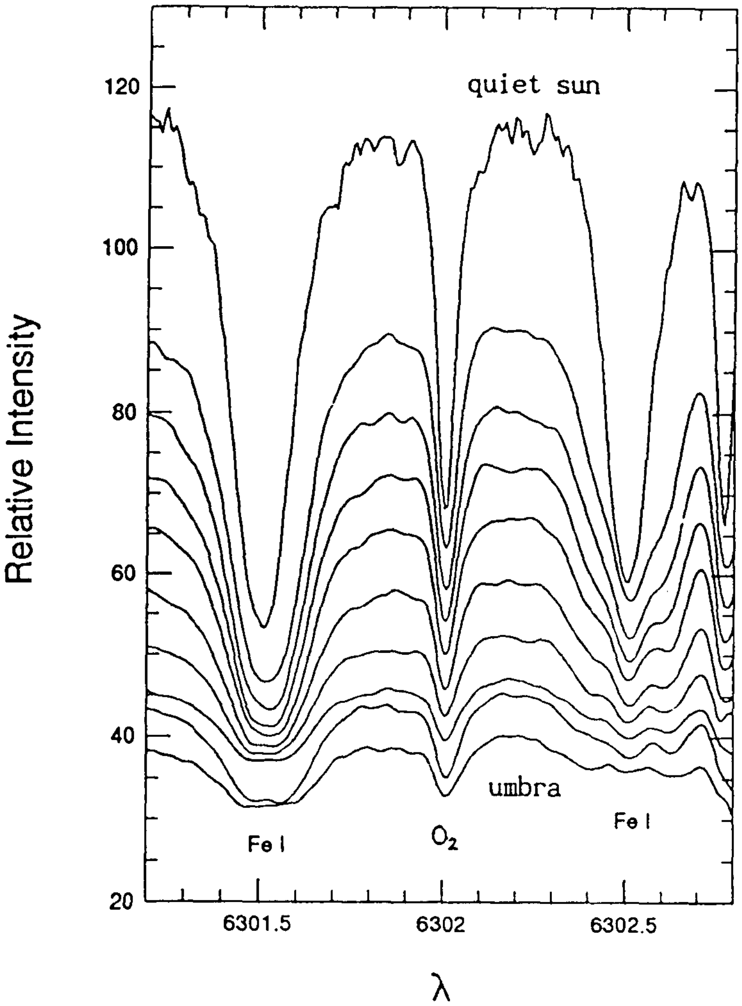

Fig. 6. The spatial variation of intensity. The amount of the shift of the two σ components and the intensities of the σ and the π components vary with positions.

The Marquardt technique is known to be very powerful in a non-linear least squares fitting. The basic concept of the method is to seek the parameters which minimize the differences between the observed and the theoretical quantities. In our case it becomes $\Sigma\ \omega_1(I_{obs} - I) + \omega_2(V_{obs} - V) = Min$. Here ω_1 and ω_2 are weights which put the I and V profiles to an identical intensity scale. Since we observed the circular polarization, only I and V profiles are available. Thus the theoretical I and V profiles normalized by the continuum intensity I_c are simplified to

$$\frac{I}{I_c} = 1 - \frac{\mu\beta}{I_c}(1 - \frac{r\eta_I}{D}), \qquad \frac{V}{I_c} = -\frac{\mu\beta}{I_c}\frac{r\eta_V}{D},$$

$$\eta_I = \frac{1}{2}\eta\ \sin^2\gamma + \frac{1}{4}(\eta^l + \eta^r)(1 + \cos^2\gamma), \qquad \eta_Q = \frac{1}{2}\{\eta - \frac{1}{2}(\eta^l + \eta^r)\}\ \sin^2\gamma\ \cos^2 2\phi,$$

$$\eta_U = \frac{1}{2}\{\eta - \frac{1}{2}(\eta^l + \eta^r)\}\ \sin^2\gamma\ \sin^2 2\phi, \qquad \eta_V = -(\eta^l - \eta^r)\cos\gamma,$$

$$I = B_o + \frac{\mu\beta(1 + r\eta_I)}{D}, \qquad V = -\frac{\mu\beta r\eta_V}{D},$$

and

$$D = (1 + r\eta_I)^2 - r^2({\eta_Q}^2 + {\eta_U}^2 + {\eta_V}^2).$$

The magnetic field strength and orientation are estimated by seeking the physical parameters which match the observed I and V profiles through the inversion scheme. The physical parameters include $\Delta\lambda_D$(Doppler width), a(Damping constant), B(magnetic field strength), $\cos\gamma$ (γ= inclination between B and line of sight), r(line to continuum opacity ratio), and $\mu\beta/I_c$ (normalization factor). In selecting the best set of values of physical parameters, the Marquardt technique is required.

IV. MAGNETIC FIELD CONFIGURATION OF SPO 5007

At first we have examined how accurately our inversion scheme determines the physical parameters. By fixing the line center at $\lambda_o = 6302.5\mathring{A}$, we have calculated a set of theoretical I and V profiles with the initial physical parameters of $\Delta\lambda_D = 0.05$ $\mathring{A}$; a = 0.2; B = 2,500 gauss; $\cos\gamma = 0.3$; r=1; $\mu\beta/I_c = 1$. The computed synthetic I and V profiles are shown in Figure 7, where they are represented by solid lines. The dots in the figure refers to the profiles which have been fitted to the synthetic ones which have been calculated with two sets of trial input values. When a reasonable set of trial input values are taken, the inversion scheme works extremely well(see Figure 7(a)). However, if the trial input values are deviated greatly from the values of the initial physical parameters assigned, this method does not give a good fit to the synthetic profiles(see Figure 7(b)).

Since any least squares inversion technique is prone to select a secondary mininum, the use of only one Stokes parameter does not return the physical parameters accurately. In order to examine how sensitively each individual physical parameters affects I and V profiles, we have calculated theoretical I and V profiles by varing only one parameter, keeping the others constant. The main results are shown in Figure 8, which can be summarized as follows:

(1) As $\Delta\lambda_D$ decreases, two σ components and π component become sharper. The depth of the intensity profile and the amplitude of the circular polarization remain almost unchanged(Figure 8(a)).
(2) As the damping constant decreases, the three components show up more distinctively and the amount of the circular polarization increases(Figure 8(b)).
(3) As the magnetic field strength increases, the separation of the two σ component and the peak of V profile increase quite sensitively(Figure 8(c)).
(4) As the inclination($\cos\gamma$) increases, the σ components get stronger while the π component becomes weaker (Figure 8(d)).
(5) As the line-to-continuum opacity ratio increases, the line depth of the I profile increases, maintaining its shape. The amplitude of the circular polarization increases(Figure 8(e)).
(6) The normalization factor and the line-to-continuum opacity ratio affect I and V profiles in a similar fashion(Figure 8(f)).

Our numerical experiments demonstrate that the inversion scheme works very well for most cases with good convergence. More importantly, the solution is not so sensitive to the initial trial physical parameters.

In the present study we have selected 31 points over the umbra and penumbra to examine the spatial variation of the magnetic field strength. The observed line profiles are not so clean because of the noises, line asymmetries, reduction errors, and so on. Therefore, some refinements have been made for the observed profiles by taking time and spatial averages. For the sake of some detailed discussions we will look into two cases, one for the umbra and the other for the penumbra. Figure 9(a) shows a sample case in which the synthetic I and V profiles best fitted to the observed penumbral I and V profiles of Fe I λ 6302.5 line. In the figure the observed profiles are indicated by the dotted lines. The field strength and the inclination are found to be 2450 gauss and 74^o . For the umbral case, the field strength and the inclination are found to be 2870 gauss and 65^o(Figure 9(b)).

The distribution of the field strength and the orientations estimated from the 31 different locations across the spot are shown in Figure 10. As seen from the figure, the field strength is quite strong in the umbra, which is over 2500 gauss. However, the inclination of the field lines is found to be quite large, particularly in the umbra. Nevertheless, the trend is in the right direction, as noted that it increases with the radial distance toward the outer penumbral

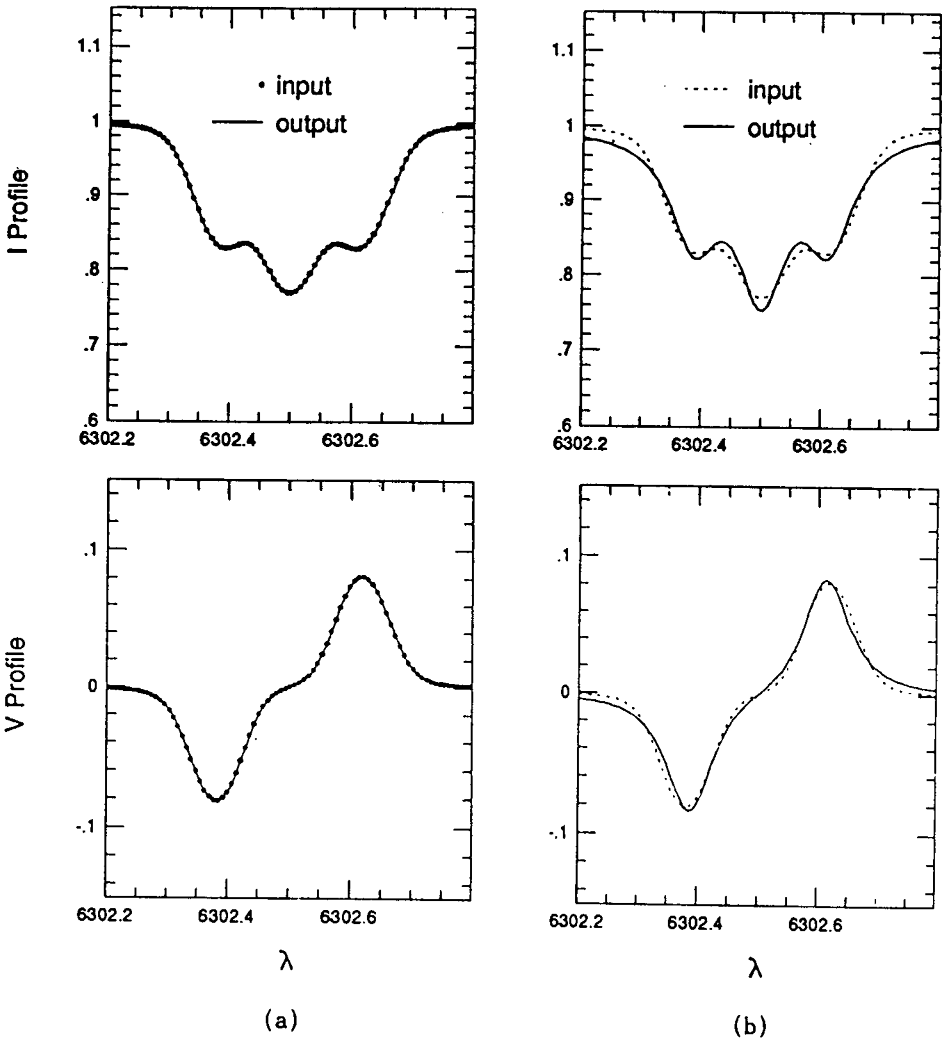

Fig. 7. A test of the convergence of the inversion scheme. (a) With the use of reasonable trial input values; $\Delta\lambda_D = 0.049(0.05)$, $a = 0.207(0.2)$, $B = 2500(2500)$, $cos\gamma = 0.301(0.3)$, $r = 1.01(1.0)$, $\mu\beta/I_c = 1.004(1.0)$. The values in parenthesis are those of the original physical parameters. (b) With the use of unreasonably deviated trial input values; $\Delta\lambda_D = 0.01(0.05)$, $a = 1.5(0.2)$, $B = 2800(2500)$, $cos\gamma = 0.364(0.3)$, $r = 1.44(1.0)$, $\mu\beta/I_c = 3.57(1.0)$.

boundary. The radial variation of the reduced field strength is compared with some emperical laws proposed by several workers(e.g. Broxon(1942), Mattig(1952), Beckers and Schröter(1969), and Wittmann(1974)), which are shown in Figure 11. As we can see from the figure, the field distribution of SPO 5007 follows the Wittmann model more closely.

V. SUMMARY AND CONCLUSIONS

We have analyzed a set of high resolution photographic line profiles of a Zeeman sensitive Fe I λ 6302.5 line taken through the Universal Birefringent Filter over a single round sunspot(SPO 5007) at Sacramento Peak Solar Observatory. In analyzing the data we have used a simple 'coarse analysis' of Skumanich and Lites(1987).

The observed Stokes I and V profiles are constructed by adding together and subtracting from each other the right and left handed circular polarizations. The resulting profiles show some asymmetry, for which the instrumental polarization is considered to be most responsible. The reduced shapes of the total intensity vary with the position across the sunspot. As one goes toward the spot center, the Zeeman splittings between the two σ components increase.

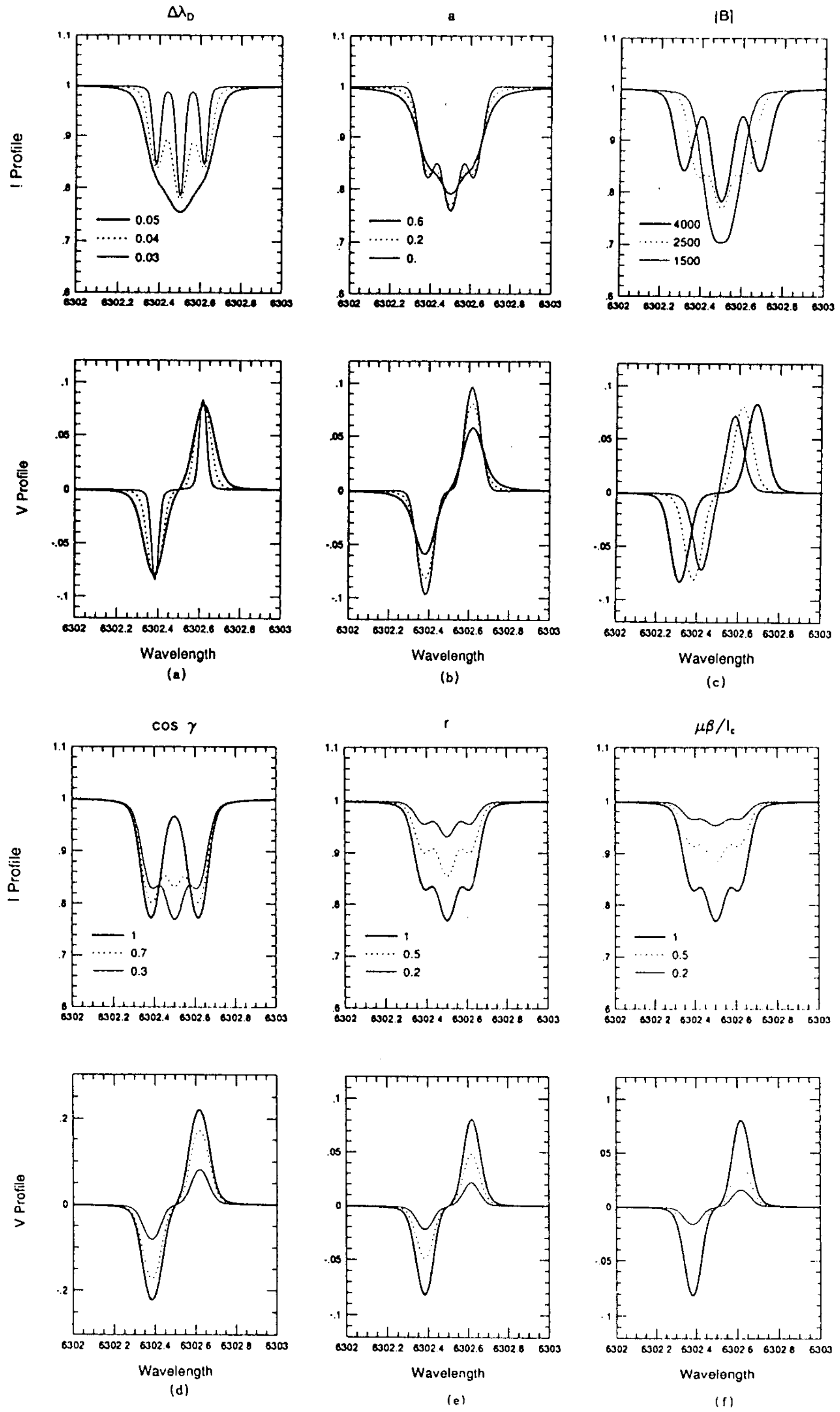

Fig. 8. The effects of each parameter on I and V profiles: (a) Doppler width, (b) damping constant, (c) field strength, (d) inclination, (e) line to continuum opacity ratio, (f) normarization factor.

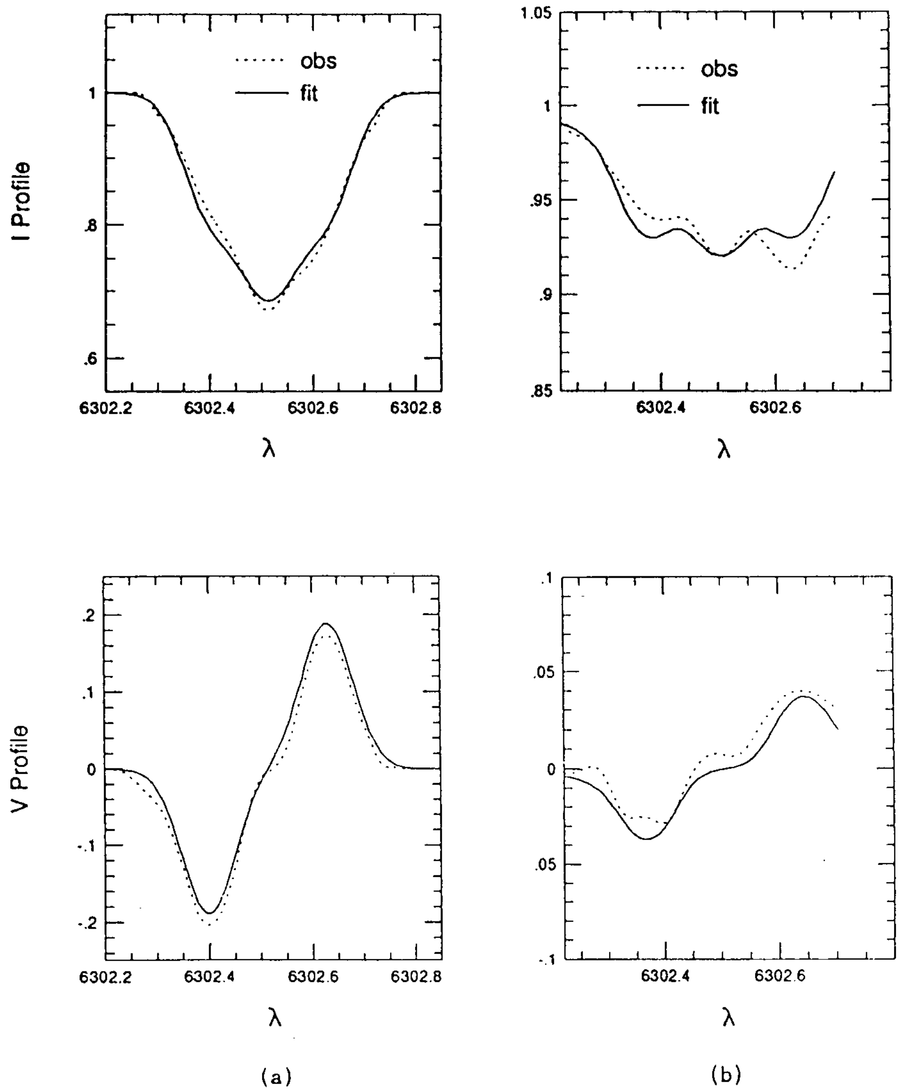

Fig. 9. The comparison of the observed I and V profiles(dashed line) with the fitted profiles(solid line). (a) In penumbra $B = 2450\ gauss,\ \gamma = 74^o$. (b) In umbra $B = 2870\ gauss,\ \gamma = 65^o$.

We have eximined how sensitively of the physical parameters, such as Doppler half width, the line damping constant, the magnetic field strength, the inclination of the field line, line to continuum opacity ratio, and the normalization factor, affect the I and V profiles. From this analysis it is concluded that the field strength can be determined rather accurately by the inversion technique. The field strength in the spot center of SPO 5007 is found to be about 3000 gauss, decreasing rapidly outward. The field distribution of the sunspot SPO 5007 follows closely the Wittmann model(1974).

The line asymmetry appeared in the reduced I and V profiles seems to create the most serious problem. The line asymmetry is most likely to be affected by mass motions and instrumental polarizations. If we consider the fact that the umbral velocities are usually less than 1 km/sec(Solanki and Stenflo(1986), Solanki and Pahlke(1988)), the instrumental polarization may play an important role in the observed line asymmetries. Lites et al.(1990) reported that the instrumental polarization affects line profiles seriously, which in turn affect the field strength and the inclination, in particular the inclination most seriously. This is exactly what we found in the present study. To describe the magnetic field distribution of a sunspot completely, it is desirable to have all of the four Stokes parameters, in which case the reduction errors and the instrumental polarization can be reduced greatly.

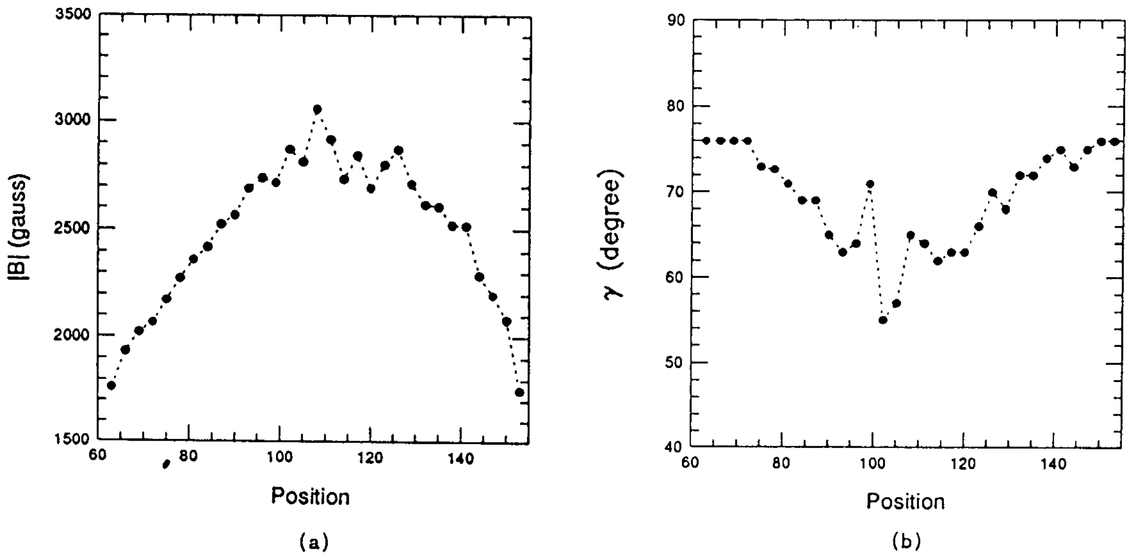

Fig. 10. (a)The estimated field strength and (b)the inclination across a sunspot.

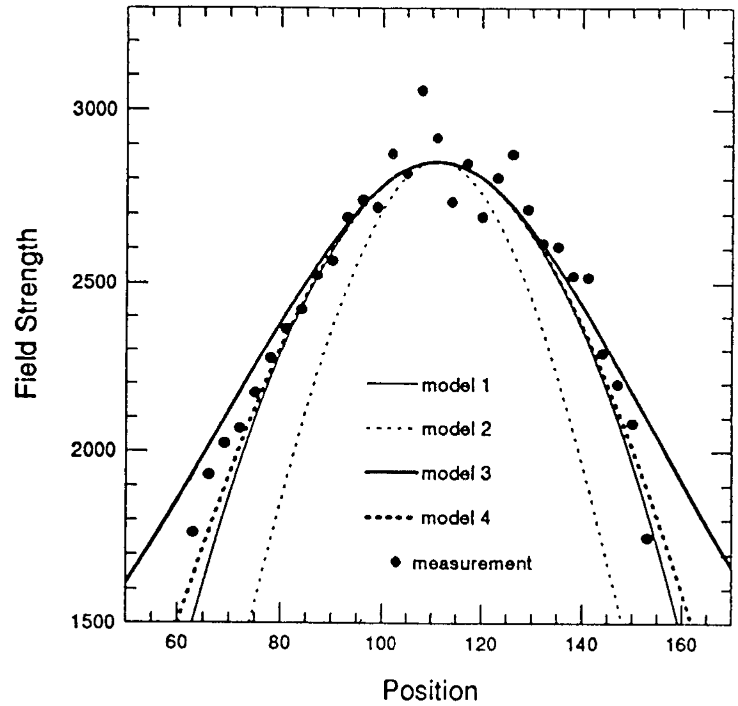

Fig. 11. The comparison of the observed field strength distribution with emperical models: model 1(Broxon); $B = B_m[1 - (r/r_p)^2]$, model 2(Mattig); $B = B_m[1 - (r/r_p)^4]exp\{-2(r/r_p)^2\}$, model 3(Becker and Schöter); $B = B_m[1 + (r/r_p)^2]^{-1}$, model 4(Wittmann); $B = B_m[1 + (r/r_p)^2 + (r/r_p)^4 + (r/r_p)^8 + (r/r_p)^{16}]^{-1}$.

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