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TWO-FLUID CLOSURE PARAMETERS FOR DIFFUSIVE ACCELERATION OF COSMIC RAYS

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ABSTRACT

In order to explore the time dependence of the closure parameters of the two-fluid calculations for supernova remnants and the terminal shocks of stellar winds, we have considered a simple model in which the time evolution of the cosmic-ray distribution function was followed in the test-particle limit using the Bohm diffusion model. The particles are mostly accelerated to relativistic energy either in the free expansion phase of the SNRs or in the early phase of the stellar winds, so the evolution of the closure parameters during these early stages is substantial and should be followed correctly. We have also calculated the maximum momentum which is limited by either the age or the curvature of these spherical shocks. We found that SNRs expanding into the medium where the gas density decreases with the distance from the explosion center might be necessary to explain the observed power-law distribution of the galactic cosmic rays. The energy loss due to the escaping energetic particles has been estimated for the terminal shocks of the stellar winds.

Key Words: shock waves, cosmic rays, acceleration of particles, supernova remnants, stellar winds.

I. INTRODUCTION

The idea that cosmic rays (CRs) can be injected at the astrophysical shocks and the pre-existing particles can be re-energized by the diffusive shock process has been widely recognized and accepted (*e.g.* Axford 1981; Blandford & Eichler 1987; Berezhko & Krymskii 1988). It has been shown that supernova remnants (SNRs) expanding into the hot interstellar medium (ISM) may be able to accelerate the CRs up 10^{14}eV (Lagage and Cesarsky 1983, LC hereafter; Kang & Jones 1991) and replenish galactic CRs by transferring 10% or more of the original supernova explosion energy into energetic particle component (Dorfi 1990, 1991; Drury, Markiewicz & Völk 1989, Markiewicz, Drury & Völk 1990; Jones & Kang 1990, 1992). It has been pointed out, however, the efficiency of the diffuse shock acceleration is rather sensitively dependent upon the details of the various models. In particular, the amount of CR energy gain in time-dependent simulations of cosmic-ray shocks using the two-fluid method varies substantially with the ratio of specific heats for the CRs, γ_c and the mean diffusion coefficient, $\langle\kappa\rangle$ (Jones & Kang 1992; Kang & Drury 1992).

In the two-fluid model of Drury and Völk (1981) the CRs are represented as a diffuse, massless fluid which interacts with the thermal gas through an isotropic pressure. With the appropriate closure parameters, γ_c and $\langle\kappa\rangle$, the two-fluid model provides a powerful and practical tool to explore a wide range of problems of diffusive shock acceleration (Jones and Kang 1990). One should, however, use extreme caution utilizing this method, because in the two-fluid model we lose the information about the particle distribution function which is needed to calculate self-consistently the two-fluid closure parameters, and because the two-fluid calculations actually sensitively depend upon those parameters. Kang and Drury (1992), for example, demonstrated the importance of this aspect of the two-fluid model by showing that the large discrepancy between the results of full numerical hydrodynamic simulations of Jones and Kang (1990) and those of the simplified model for SNRs of Drury, Markiewicz, Völk (1989) would be eliminated when both techniques based on the two-fluid model use similar time-dependent closure parameters.

Jones and Kang (1992) carried out the two-fluid calculation of SNRs with the time-dependent models for the closure parameters which were estimated by assuming a simple time evolution of the distribution function in the test-particle limit.

The mean diffusion coefficient was estimated according to the test-particle distribution function and the Bohm diffusion model, while the specific heat ratio for the CRs was modeled to decrease exponentially from 5/3 to 4/3 on the order of the acceleration time scale. They found that it was physically plausible that SNRs can channel of order 10 % of the initial explosion energy into galactic cosmic rays. This result is consistent with findings of previous studies mentioned the above. In the present paper we estimated the time-dependence of both γ_c and $\langle\kappa\rangle$ from the test-particle distribution function for adiabatic blast waves in SNRs and the terminal shocks in the stellar winds. These models should provide a guide to approximate the time-dependence of closure parameters for two-fluid calculations of these spherical shocks. We focus only on modeling the closure parameters in the present work. The two-fluid calculations using the results of this work will be presented somewhere else. On our way to calculate the distribution function, we also estimate the maximum momentum which is limited by either the age or the curvature effect of the spherical shocks, and the fraction of energy lost due to the escaping energetic particles. In §2 we briefly show how the closure parameters are defined in the two-fluid model. Then the time evolutions of the spherical shocks and the basic assumptions of our models are described in §3, while the results are presented in §4. We discuss the implications of this study in §5.

II. TWO-FLUID APPROXIMATION

The transport equation for the *isotropic* part of the cosmic ray distribution function can be written according to (Skilling 1975)

$$\frac{df}{dt} = \frac{1}{3}(\vec{\nabla}\cdot\vec{u})p\frac{\partial f}{\partial p} + \vec{\nabla}\cdot(\kappa\vec{\nabla}f) + Q, \tag{2.1}$$

where $f(x,p,t)$ is the number density of particles in phase space, d/dt is the total time derivative, κ is the diffusion coefficient, $\vec{u}$ is the background fluid velocity, and Q is the source term representing the particle injection. It is assumed that the isotropic part of the distribution function is dominant due to strong scattering of cosmic rays. In the two-fluid model, Eq. (2.1) is multiplied by the kinetic energy of particles and integrated with respect to momentum. This produces the energy conservation equation for the CR,

$$\frac{dE_c}{dt} = -\gamma_c E_c(\vec{\nabla}\cdot\vec{u}) + \vec{\nabla}\cdot(\langle\kappa\rangle\vec{\nabla}E_c), \tag{2.2}$$

where E_c and γ_c are the CR energy density and the ratio of specific heats, respectively. The mean diffusion coefficient is defined according to

$$\langle\kappa\rangle = \frac{\int_{p_0}^{p_1} f\kappa[\sqrt{p^2+1}-1]p^2dp}{\int_{p_0}^{p_1} f[\sqrt{p^2+1}-1]p^2dp}. \tag{2.3}$$

Hereafter the momentum p is expressed in units of mc. The specific heat ratio is defined as

$$\gamma_c = 1 + \frac{P_c}{E_c},$$

while

$$P_c = \frac{4\pi mc^2}{3}\int_{p_0}^{p_1}\frac{p^4}{\sqrt{p^2+1}}f\ dp, \tag{2.4}$$

$$E_c = 4\pi mc^2\int_{p_0}^{p_1} p^2(\sqrt{p^2+1}-1)f\ dp. \tag{2.5}$$

The gas dynamics equations with terms added to account for CR pressure can be found in many previous papers (*e.g.* Jones & Kang, 1990)

III. MODELS

(a) Similarity Solutions for Spherical Shocks

Our galaxy loses its energy at a rate of $\sim 10^{40}$erg s^{-1} due to the escaping galactic CRs. It is believed that the kinetic energy input from SNRs ($\dot{E} \sim 10^{42}$erg s^{-1}) and the stellar winds from massive stars ($\dot{E} \sim 10^{40} - 10^{41}$erg s^{-1}) could be the major sources of the energy which replenish such loss. In our paper we will concentrate on three types of spherical shocks of astrophysical interests all of which have a well-known solutions: 1) SNR in a uniform ISM of ρ_o, 2) SNR in a medium whose density decreases as $\rho = \rho_o(r/r_n)^{-2}$, and 3) terminal shocks in the stellar winds from massive stars into a uniform ISM. The readers may refer to Chevalier (1976) and Ryu and Vishniac (1991) for the similarity solutions of spherical shocks for a wide range of energy sources at the center and background density distributions.

We approximate the evolution of a SNR with two phases: the free expansion phase and the Sedov phase. We assume that the Sedov phase begins at t_n when the swept up mass equals to the ejected mass. For SNRs in a uniform density with an explosion energy, E_o, ejecta mass, M_{ej}, and ISM density, ρ_o, this assumption (*e.g.* $4\pi r_n^3\rho_o/3 = M_{ej}$) determines the normalization constants which converts the physical flow variables, r, u, and t to dimensionless quantities, $\tilde{r} = r/r_n$, $\tilde{u} = u/u_n$, and $\tilde{t} = t/t_n$. They are

$$r_n = (\frac{3M_{ej}}{4\pi\rho_o})^{1/3}, \tag{3.1a}$$

$$t_n = (\frac{\rho_o r_n^5}{E_o\xi_u^5})^{1/2}, \tag{3.1b}$$

where $\xi_u = 1.15167$. It should be obvious that $u_n = r_n/t_n$. Then the velocity and radius of the shock have the following *approximate*, dimensionless, similarity solutions: for $\tilde{t} < 1$

$$\tilde{u}_s \sim (\frac{3}{2\pi\xi_u^5})^{1/2}, \tag{3.2a}$$

and for $\tilde{t} > 1$

$$\tilde{u}_s \sim \frac{2}{5}\tilde{t}^{-3/5}, \quad \tilde{r}_s \sim \tilde{t}^{2/5}. \tag{3.2b}$$

Here we approximate $E_o \sim M_{ej}u_s^2/2$ to find the shock velocity in the free expansion phase. Only in the limit of $\tilde{t} \gg 1$, for the strong blast wave, the expressions in Eq. (3.2b) become exact.

For SNRs in the inverse-square density distribution we parameterize the background density distribution in terms of a steady stellar wind from the progenitor with a constant mass loss-rate, $\dot{M}_w$, and a constant velocity, u_w as in Jones and Kang (1992), Then the normalization constants can be specified from the condition that $\dot{M}_w r_n/u_w = M_{ej}$ at $t = t_n$ according to

$$r_n = \frac{M_{ej}u_w}{\dot{M}_w}, \tag{3.3a}$$

$$t_n = (\frac{M_{ej}r_n^2}{4\pi E_o\xi_i^5})^{1/2}, \tag{3.3b}$$

where $\xi_i^5 = 3/2\pi$, E_o is the explosion energy, and M_{ej} is the ejected mass. The similarly solutions for this model can be approximated as

$$\tilde{u}_s \sim (\frac{1}{2\pi\xi_i^5})^{1/2}, \tag{3.4a}$$

for $\tilde{t} < 1$, and

$$\tilde{u}_s \sim \frac{2}{3}\tilde{t}^{-1/3}, \quad \tilde{r}_s \sim \tilde{t}^{2/3}, \tag{3.4b}$$

for $\tilde{t} > 1$. Here we also use the approximate relation, $E_o \sim M_{ej}u_s^2/2$ for $\tilde{t} < 1$.

The terminal shock of the stellar winds is different from the SNR shocks in several ways. It is a reverse shock facing toward the star. The upstream side of this shock is the wind, while the downstream side is the shocked wind. There is a contact discontinuity between the terminal shock and the forward shock. For the stellar winds expanding into a uniform ISM, ρ_o, we assume the wind blows out from a shell with a constant speed, v_w and a constant mass loss rate $\dot{M}'_w$. Then the density inside the wind decreases with r as $\rho_w = \dot{M}'_w/(4\pi r^2 v_w)$. One way to specify the normalization constants is to find the radius of the shell where ρ_w equals the ISM density, ρ_o, that is,

$$r_n = (\frac{\dot{M}'_w}{4\pi\rho_o u_w})^{1/2}, \tag{3.5a}$$

$$t_n = \frac{r_n}{v_w}. \tag{3.5b}$$

The wind density ρ_w at the contact discontinuity between the wind and the ISM is greater than ρ_o for $t < t_n$. As discussed in Falle (1975), the evolution in such configuration is rather uncertain due to the Rayleigh-Taylor instability. Thus we assume the stellar wind starts from a sphere of $r = r_n$, since the acceleration before t_n is not important once the age of the wind becomes much larger than t_n. According to Falle (1975), the velocity of the terminal shock is $u_s \sim v_w/3$ initially at r_n. For $t \gg t_n$, the radius of the terminal shock increases as $r_s \propto t^{2/5}$, while the radii of the contact discontinuity and the forward shock increase as $r \propto t^{3/5}$. In the present paper we concentrate on the terminal shock only because the forward shock is much weaker and, so, less efficient in accelerating particle diffusively. According to Weaver *et al.* (1977)

$$r_s \sim 0.743(\frac{\dot{M}'_w}{\rho_o})^{3/10} v_w^{1/10} t^{2/5}, \tag{3.6}$$

for $\tilde{t} \gg 1$. Thus the dimensionless radius and velocity can be expressed by

$$\tilde{r}_s \sim 1.588\tilde{t}^{2/5},$$

$$\tilde{u}_s \sim 0.635\tilde{t}^{-3/5}, \tag{3.7}$$

for $\tilde{t} \gg 1$. Since u_s is the velocity of the terminal shock in the rest frame of the star, the velocity relative to the unshocked wind (upstream region), $u'_s = v_w - u_s$, should be used to calculate the acceleration time scale. Since the detail behavior of u'_s at the early stage is not so important, we take the following approximation :

$$\tilde{u}'_s \sim 2/3 \tag{3.8a}$$

for $\tilde{t} < 3$, and

$$\tilde{u}'_s \sim 1 - 0.635\tilde{t}^{-3/5}, \tag{3.8b}$$

for $\tilde{t} > 3$. Here the relation $\tilde{u}_s = 1/3$ at $\tilde{t} = 0$ is used. The value of u'_s in Eq. (3.8b) is equal to 2/3 at $\tilde{t} \sim 3$.

(b) Distribution Function

In our simplified model we take the momentum distribution function of the particles to be a power law extending from a lower-bound momentum, p_o to an upper-bound momentum, p_1,

$$f = f_o p^{-q}, \tag{3.9}$$

for $p_o < p < p_1$. We assume $q = 4$ as expected in a strong unmodified shock. This turns out to be a pretty good approximation since, in the spherical shocks considered here, most of the acceleration occurs in the early phase of the shock when the compression ratio is still $\sigma = 4$ (LC). The upper-bound momentum, p_1 can be limited by either the finite lifetime of the shock or the curvature effect. It is obvious that the maximum energy gained depends on the ratio of the age of the shock to the acceleration time scale. For the spherical shocks, highly energetic particles cannot be confined by the shock, when the energy is high enough so that their diffusion lengths become comparable to the radius of curvature of the shock front (Blandford & Eichler 1987). In a typical SNR the maximum momentum is likely to be set by the age of the remnants, while it is limited by the curvature of the terminal shock in the stellar winds (LC). The distribution function steepens rather rapidly for $p > p_1$, especially for the Bohm diffusion model considered here (see, for example, Kang & Jones 1991), so that the particles of $p > p_1$ can be safely ignored. Here we assume that the injected particle population is dominant over the pre-existing galactic CRs whose distribution function $f(p)$ is approximately proportional to $p^{-4.3}$.

(c) Maximum Momentum

First we estimate the maximum momentum, p_{1a} limited by the age of the shock, using the test particle theory in which it is assumed that the CR pressure does not influence the dynamics of the shock. The mean time interval for a particle to be accelerated to some momentum $p + dp$ from an initial momentum p can be written by (LC; Drury 1983),

$$dt = \frac{3}{u_1 - u_2}\left(\frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2}\right)\frac{dp}{p}. \tag{3.10}$$

Here the subscripts 1 and 2 refer to the upstream and downstream values of the diffusion coefficient and the velocity as measured in the shock frame. Assuming $\kappa_1 = \kappa_2 = \kappa(p)$ and a compression ratio, $\sigma = u_1/u_2$, Eq. (3.10) becomes

$$\alpha \frac{\kappa(p)}{p} dp = u_s(t)^2 dt, \tag{3.11}$$

where $u_s(t)$ is the shock velocity and $\alpha = 3\sigma(\sigma+1)/(\sigma-1)$. By the same reason as for the power law index q, we can use $\sigma = 4$ ($\alpha = 20$) without sacrificing much accuracy. The maximum momentum, p_{1a} that a particle injected with the momentum p_0 at t_0 can achieve during the lifetime of the shock can be found by integrating Eq. (3.11) if the time dependence of the shock velocity and the momentum dependence of the diffusion coefficients are given assuming the diffusion coefficient depends only on the particle momentum.

If we assume a momentum-dependent diffusion coefficient model in which the CRs scattering mean free path is proportional to the Larmour radius (*e.g.,* Bohm diffusion), then the non-dimensional diffusion coefficient is given by

$$\tilde{\kappa}(p) = \tilde{\kappa}_1 \frac{p^2}{\sqrt{p^2+1}}, \tag{3.12}$$

$$\tilde{\kappa}_1 = \frac{\delta}{B} \frac{3.13 \times 10^{22} \mathrm{cm}^2 \mathrm{s}^{-1}}{\kappa_n}, \tag{3.13}$$

where the magnetic field strength B is expressed in units of μG and $\kappa_n = r_n^2/t_n$ is the normalization constant for the diffusion coefficient. δ is is a constant which is dependent upon details of the microphysics related to the scattering process. For Bohm diffusion coefficient in quasi-parallel shocks the value of δ becomes unity, while it can be less than one for quasi-perpendicular shocks (Jokipii, 1992). The magnetic field strength in ISM could be $B \sim 1 - 10\mu G$. Thus we treat $\frac{\delta}{B}$ as a free parameter to be varied instead of changing each of them separately.

Now we can numerically calculate p_{1a} as a function of $\tilde{t}$ using the similarity solutions given by Eq.'s (3.2), (3.4) and (3.8) and the diffusion coefficient in Eq. (3.12). The approximate behaviors, however, can be easily found in the limit of $p_{1a} \gg 1$,

$$(\tilde{\kappa}_1 p_{1a})_{Su} \sim \frac{1}{25}(1 - \tilde{t}^{-1/5}), \tag{3.14a}$$

$$(\tilde{\kappa}_1 p_{1a})_{Si} \sim \frac{1}{15}(\tilde{t}^{1/3} - 1), \tag{3.14b}$$

$$(\tilde{\kappa}_1 p_{1a})_{W} \sim \frac{1}{20}\tilde{t}, \tag{3.14c}$$

for SNRs in a uniform density, SNRs in the inverse-square density, and the stellar winds, respectively. One should note that $\tilde{\kappa}_1 p_{1a}$ is a function of only the dimensionless time $\tilde{t}$, so that the above relations can be applied to a wide range of physical models of these spherical shocks and p_{1a} (in units of mc) depends on δ, B, and physical dimension of the shock (*e.g.* κ_n) through $\tilde{\kappa}_1$. In particular, for SNRs in a uniform density ISM, $\tilde{\kappa}_1 p_{1a}$ has a maximum value of 1/25, so the firm maximum momentum of this case is

$$(p_{1a})_{Su} = 7.3 \times 10^4 \frac{B E_o^{1/2}}{\delta M_{ej}^{1/6} \rho_o^{1/3}}, \tag{3.15}$$

where B is expressed in units of μG, E_o in units of $10^{51} ergs$, M_{ej} in units of $10 \mathrm{M}_\odot$, and ρ_o in units of $2.4 \times 10^{-27} \mathrm{gcm}^{-3}$. For typical Type II SNRs whose explosion parameters are similar to these fiducial values, the maximum energy of the protons is about 3×10^5GeV if $\delta \sim 1$ and $B \sim 3$.

Secondly, we estimate the maximum momentum set by the curvature of the shock, p_{1r} by finding the momentum which gives a diffusion length similar to the radius of the shock, that is,

$$\frac{\tilde{\kappa}(p_{1r})}{\tilde{u}_s} = \tilde{r}_s. \tag{3.16}$$

Since only highly energetic particles ($p \gg 1$) can escape from the shock, we can use $\tilde{\kappa}(p) = \tilde{\kappa}_1 p$. In the limit of $\tilde{t} \gg 1$, then, we can estimate p_{1r} from

$$(\tilde{\kappa}_1 p_{1r})_{Su} \sim \frac{2}{5}\tilde{t}^{-1/5}, \tag{3.17a}$$

$$(\tilde{\kappa}_1 p_{1r})_{Si} \sim \frac{2}{3}\tilde{t}^{1/3}, \tag{3.17b}$$

$$(\tilde{\kappa}_1 p_{1r})_W \sim 1.588\tilde{t}^{2/5}. \tag{3.17c}$$

It should be remembered that the radius and the velocity given in Eq.'s (3.7) and (3.8b), respectively, are used for the stellar wind model. Here $\tilde{\kappa}_1 p_{1r}$ is also only a function of $\tilde{t}$, so the same statement regarding the dependence of physical models regarding p_{1a} can be made for p_{1r} also. The maximum momentum p_1, then, should be the minimum of p_{1a} and p_{1r}.

(d) Closure Parameters

The mean diffusion coefficient can be estimated according to Eq. (2.3) with the assumed distribution function in Eq. (3.9) by taking a small p_0 and calculating p_1 as described in the previous section. Typically we take $p_0 \sim 10^{-3}$, but the results are insensitive to this value as long as $p_0 \ll 1$. Similarly, in the limit $p_1 \gg 1$ the dimensionless mean diffusion coefficient can be approximated as $\langle\tilde{\kappa}\rangle \approx \tilde{\kappa}_1 p_1/\ln p_1$. With this last form Eq.'s (3.14) and (3.17) give the analytic expressions for $\langle\tilde{\kappa}\rangle$ for each spherical shock. Here we also note that $\langle\tilde{\kappa}\rangle$ is only weakly dependent on $\tilde{\kappa}_1$ through $\ln p_1$.

The pressure and energy of CRs can also be found according to Eq.'s (2.5) and (2.6) by the same procedure as the mean diffusion coefficient. Then the ratio of P_c to E_c should give the specific heat ratio for CRs as in Eq. (2.4).

We have also calculated the ratio, f_{esc}, of the CR energy corrected for escaping energetic particles due to the curvature effect to the CR energy without considering that effect as the following:

$$f_{esc} = \frac{\int_{p_0}^{p_1} p^2(\sqrt{p^2+1}-1)f\,dp}{\int_{p_0}^{p_{1a}} p^2(\sqrt{p^2+1}-1)f\,dp}. \tag{3.18}$$

The denominator of this equation represents the CR energy accelerated during the lifetime of the shock, while the numerator represents the CR energy accelerated either during the lifetime of the shock or until the curvature effect sets in. Since one-dimensional spherical simulations of stellar winds using the two-fluid method cannot treat the curvature effect, the CR energy gain in such simulations corresponds to the CR energy term in the numerator. Thus f_{esc} may provide a way to estimate roughly the CR energy gain when the escaping particles are correctly counted for.

IV. RESULTS

As mentioned in the previous section, our results can be applied to a wide range of physical model parameters for SNRs and stellar winds with the appropriate normalization. In order to discuss the results quantatitively, however, we take a standard model for each kind of spherical shock. For SNRs in a uniform ISM, $E_o = 10^{51}$ergs, $M_{ej} = 10\mathrm{M}_\odot$, and $\rho_o = 7.0 \times 10^{-27}\mathrm{gcm}^{-3}$, so $r_n = 28.5$pc, $t_n = 4.3 \times 10^3$yrs, and $\kappa_n = 5.73 \times 10^{28}\mathrm{cm}^2\mathrm{s}^{-1}$. For SNRs in the inverse-square density distribution which originates from a steady stellar wind, $E_o = 10^{51}$ergs, $M_{ej} = 1\mathrm{M}_\odot$, $\dot{M}_w = 10^{-5}\mathrm{M}_\odot\mathrm{yr}^{-1}$, and $u_w = 10\mathrm{km\ s}^{-1}$, so $r_n = 1.02$ pc, $t_n = 57.7$ yrs, and $\kappa_n = 5.46 \times 10^{27}\mathrm{cm}^2\mathrm{s}^{-1}$. For the terminal shocks of the stellar winds, $\dot{M}_w' = 10^{-5}\mathrm{M}_\odot\mathrm{yr}^{-1}$, $v_w = 2 \times 10^3\mathrm{km\ s}^{-1}$, and $\rho_o = 2.34 \times 10^{-24}\mathrm{g\ cm}^{-3}$, so $r_n = 0.11$pc, $t_n = 52.0$yrs, and $\kappa_n = 6.57 \times 10^{25}\mathrm{cm}^2\ \mathrm{s}^{-1}$. We take $(\delta/B) = 0.2$, 2, and 20 for the diffusion coefficient in Eq. (3.11).

In Fig. 1 we have shown the maximum momenta p_{1a} (solid lines) and p_{1r} (dashed lines) calculated numerically from Eq.'s (3.11) and (3.16) for each standard models with $(\delta/B) = 0.2$. For models with other physical parameters, both p_{1a} and p_{1r} should scale with κ_n. As we have shown in the previous section, the different time behaviors in the maximum momentum among the three dynamical models come about because of the different time dependence in the shock velocity. As seen in Eq. (3.15), for example, p_{1r} for the SNRs in a uniform density distribution decrease with time, because the shock slows down faster compared to the increase of the shock radius. In principle the maximum momentum is determined from the age of the SNRs and the stellar winds which depend on the physical parameters. For the SNRs, however, most of acceleration takes place in the early phase ($\tilde{t}\lesssim 10$), and it is insensitive to when the Sedov phase terminates or the radiatively cooling phase sets in and so on. For our standard models, $p_1 \sim 10^5$ can be achieved in the SNRs in a uniform ISM. This translates into the maximum energy of $p_1 c \sim 10^5$GeV for the cosmic ray protons which constitute most of the galactic CRs. This energy is much lower than the break in the power-law spectrum of the galactic CRs around $pc \sim 5 \times 10^6$GeV.

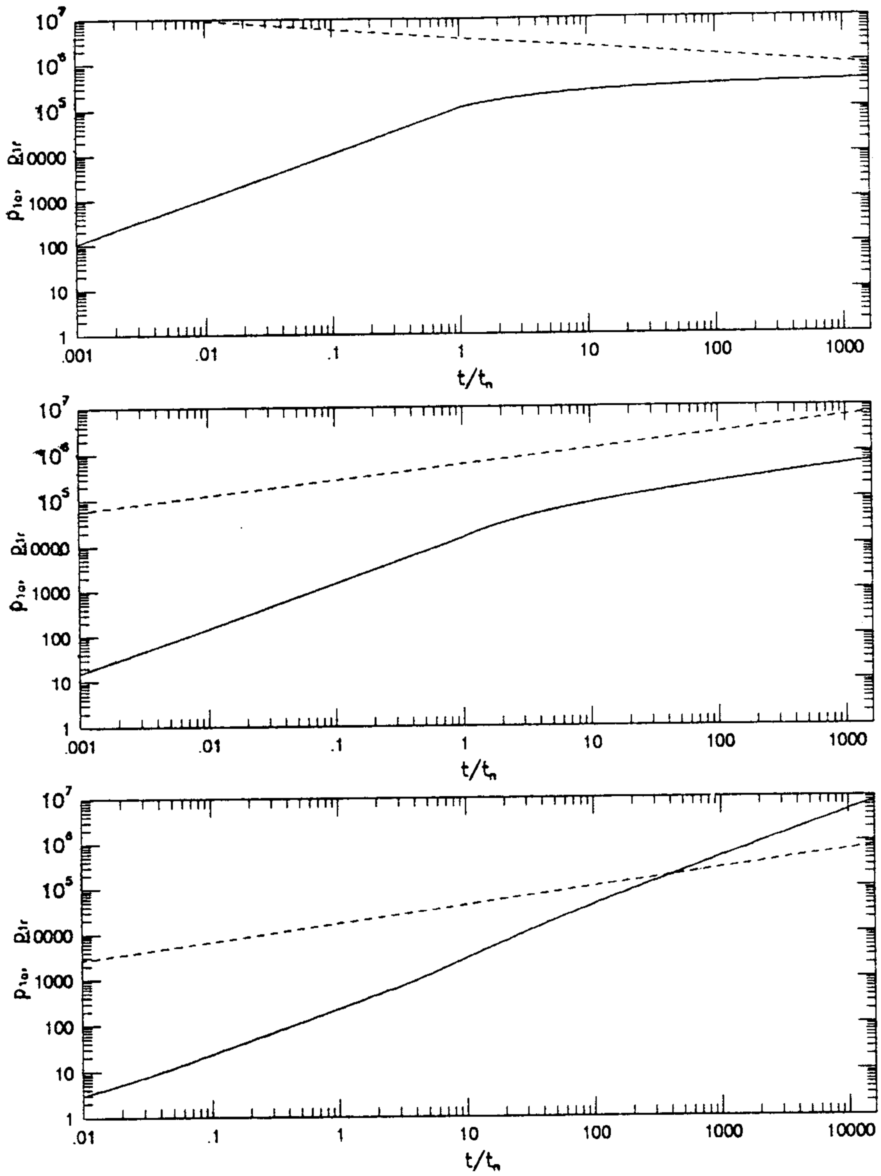

Fig. 1. The maximum momentum set by the age of the shocks, p_{1a} (solid lines), and by the curvature of the shocks, p_{1r} (dashed lines) for SNRs in a uniform density model (top panel), SNRs in the inverse-square density model (middle panel), and the terminal shock in stellar winds (bottom panel). See the text for the normalization constants for each model.

(Fichtel & Linsley 1986). For the SNRs in the inverse-square density distribution, the maximum momentum could be somewhat higher than for the SNRs in a uniform ISM depending on the ages, since p_{1a} increase with time. This indicates that the SNRs in a medium where the density decrease with the distance from the explosion center faster than r^{-2} could accelerate the CRs to higher energy. For the terminal shocks of stellar winds, the maximum age could be determined from the stellar mass and the mass loss rate. For our standard model in which $\dot{M}_w = 10^{-5} M_\odot yr^{-1}$, the age could be order of $\sim 10^5$yrs. Thus the maximum momentum is similar to that of our standard model of SNRs in the inverse-square density

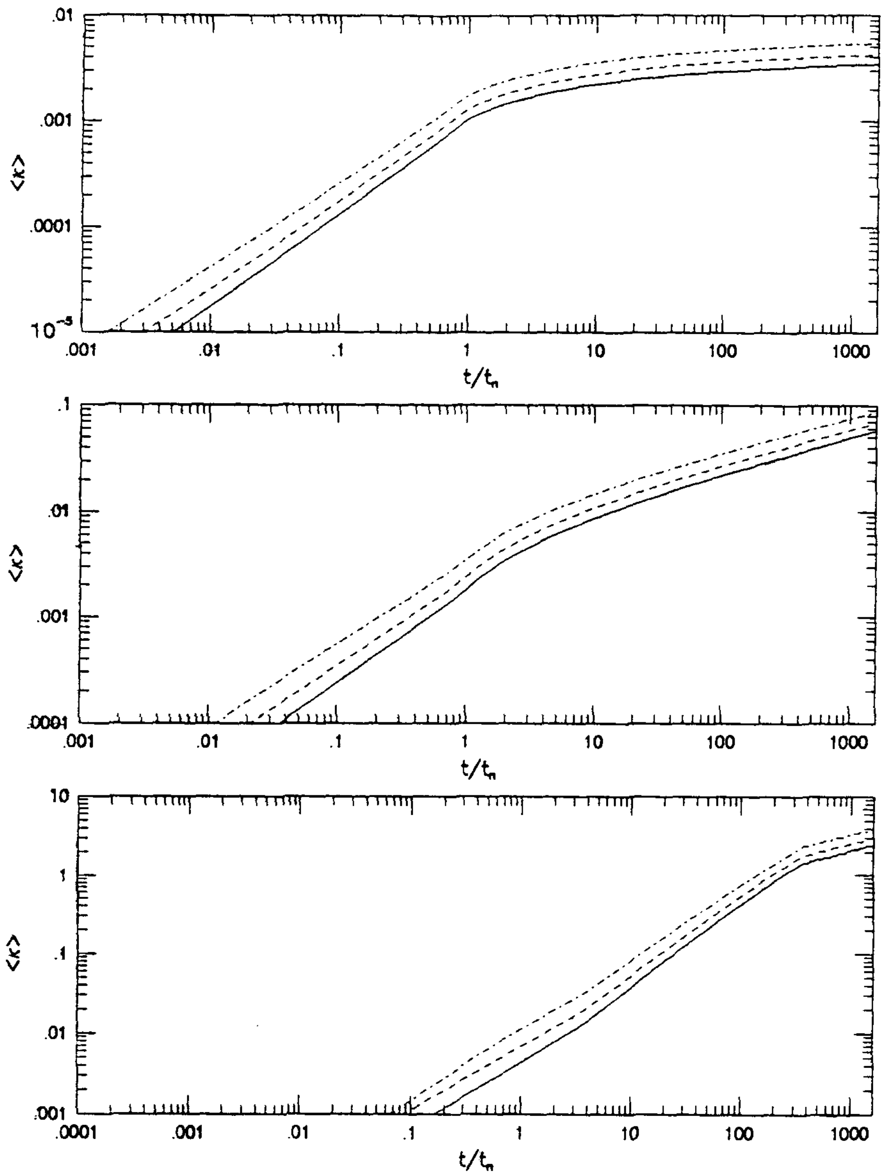

Fig. 2. The mean diffusion coefficient in units of κ_n for each shock model. The solid, dashed, dot-dashed lines represent the models with $(\delta/B) = 0.2, 2,$ and 20, respectively.

distribution or could be a little bit higher for larger stellar mass. Even though the stellar winds from massive stars are less energetic than SNRs in terms of energy budget, they could make a significant contribution to very energetic particles as first suggested by LC.

The dimensionless mean diffusion coefficients, $\langle\tilde{\kappa}\rangle$ are calculated for $(\delta/B) = 0.2$ (solid lines), 2 (dashed lines), and 20 (dot-dashed lines), and plotted in Fig. 2. As mentioned in §3.4, $\langle\tilde{\kappa}\rangle$ is only weakly dependent upon $\tilde{\kappa}_1$ through p_1, so the differences among the lines for different values of (δ/B) are small. Similarly, the specific heat ratios are plotted in Fig. 3. Like the maximum momentum p_{1a}, γ_c decrease close to 4/3 before $\tilde{t}\lesssim 10$, since the particles are accelerated to

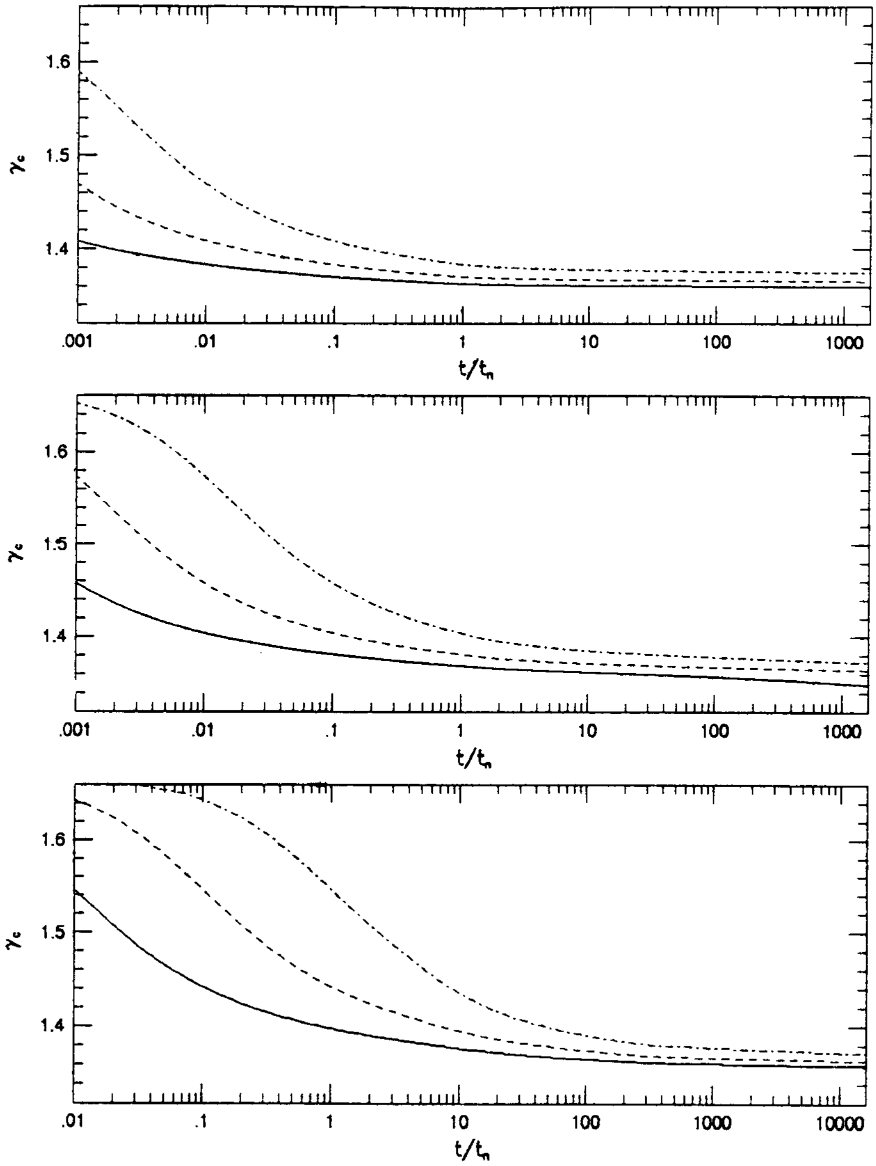

Fig. 3. The specific heat ratio for cosmic rays for each shock model. The lines are the same as in Fig. 2

the relativistic energy already in the early phase. Fig. 2 and 3 should provide a simple time-dependent model for closure parameters for two-fluid calculations of these spherical shocks. It should be noted that the dimensionless mean diffusion coefficient is roughly independent of the details of the scattering processes (*e.g.* δ) and physical models of the shock (*e.g.* B and κ_n).

Finally, in Fig. 4 we have shown the 'retained' fraction of energy which has been corrected for the escaping energetic particles due to the curvature in the stellar wind models (see Eq. [3.18]). For our standard model, less than 20 % energy is lost for $(\delta/B) < 20$. This is because most of the energy is carried by the lower energy particle which remains near the shock, in other words, the number of escaping high energy particles is much less than number of lower energy particles.

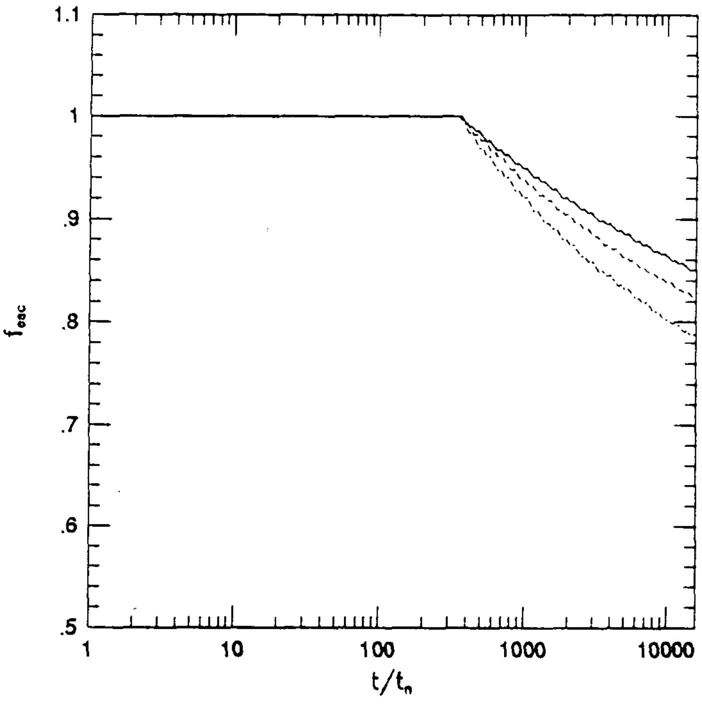

Fig. 4. The estimated fraction of the CR energy corrected for the escaping energetic particles for the stellar wind shock model. The lines are the same as in Fig. 2

V. DISCUSSION

By adopting the simple assumptions in the test-particle theory, we have presented a time dependent model of the closure parameters, the specific heat ratio for CRs and the mean diffusion coefficient, which can be used in the two-fluid calculations of SNRs in a uniform density or the inverse-square density distribution and the terminal shocks of the stellar winds from massive stars.

We found that the time evolution of the mean diffusion coefficient in our model is insensitive to the details of the scattering physics and the size of model parameters. The specific heat ratio for CRs can change somewhat significantly with those details only in the early free expansion phase when the particle changes from being nonrelativistic to being relativistic. Both closure parameters change more rapidly in the free expansion phase than in the Sedov phase. We can see intuitively that what matters most in determining the evolutions of the closure parameters is the evolution of particle distribution function around the transition from being non-relativistic to being relativistic. Thus the closure parameters changes most during this transition which happens in the free-expansion phase of SNRs or in the early age of the stellar winds. We note the wide dynamic range of the mean diffusion coefficient during this transition (at least order of 100) may bring about a technical problem in the two-fluid numerical simulations, since the spatial resolution of such simulations should be a small fraction of the diffusion length (Jones and Kang 1990) unless special techniques such as an expanding grid or an adaptive grid are used.

The Sedov phase plays a more important role in terms of total energy accelerated, since the CRs accelerated in the free expansion phase lose their energy through adiabatic cooling in later phase. Thus some of the previous authors ignored the evolutions of CRs in the free expansion phase and started their calculation from the Sedov phase (Jones and Kang 1990, 1992, Kang and Jones 1991) with the various values of the closure parameters. But the evolution of particle distribution in the free expansion phase should determine the initial closure parameters for the Sedov phase and so it should bear its own significance. Dorfi (1991) started his simulation from the initial explosion of the SNRs in a uniform ISM adopting the time-dependent model of the closure parameters of Drury, Markiewicz & Völk (1989) in which similar assumptions regarding the test-particle distribution and the Bohm diffusion model as ours were made. According to his calculations, 10-30% of the initial explosion energy can be converted into the CRs, and the CR energy increases substantially and the specific heat ratio for CRs becomes 4/3 during the free-expansion phase. On the other hand, Jones and Kang (1992) started the calculations from the Sedov phase, but they kept γ_c at 5/3 for a few dynamical times, so that the CR pressure could build up significantly early in the Sedov phase. They came to the same conclusion as Dorfi (1991) regarding the energy gain in the CRs. In fact, all previous SNR simulations mentioned in §I indicated that it is possible to channel of order 10 % of the initial explosion energy into the galactic CRs despite the differences in the various models. This is because the key to determining the ultimate CR energy gain is the existence of substantial CR pressure early in the Sedov phase, which can be achieved if γ_c is close to 5/3 during the initial acceleration stage, and all previous SNR calculations have at least one model which satisfies such condition.

The maximum momentum for our standard models of the SNRs and the stellar winds are $p_1 c \sim 10^5$GeV. In turns our it is not as high as $pc \sim 5 \times 10^6$GeV above which the galactic CR spectrum steepens (Fichtel & Linsley 1986). The gap between these two energies seems to be rather too big to be explained by simply taking a slightly different physical parameters of these shocks. We point out, however, the SNRs in a medium where the gas density decrease more rapidly than r^{-2} where r is the distance from the supernova could accelerate the particles to higher energy and, therefore, may be able to explain the observed power-law distribution of the galactic cosmic rays if such density distribution is ubiquitous in the interstellar medium. It is also possible to narrow the gap if the drift acceleration in quasi-perpendicular shocks plays an important role in determining the diffusion processes (Jokipii, 1992).

Finally we estimated that the CR energy loss due to the escaping energetic particles should be less than $\sim$ 10% of the total CR energy accelerated in the terminal shocks in the stellar winds, because the number fraction of energetic particles is small. Escape of the energetic particles is not a significant problem in the SNR models considered here.

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