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Optimization Analysis of Flexible Cellular Manufacturing: Route Selection and Determining the Optimal Production Conditions for Ordered Products

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유연한 셀생산을 위한 최적가공경로와 생산조건의 결정

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Abstract

This paper describes a procedure for optimizing the route selection and production conditions in alternate process plans under a cellular manufacturing environment. The type of production is mainly production-to-order which deals with unexpected products as the changes factor. The flexible cellular manufacturing can be viewed as a complete unification of both flexible manufacturing process and flexible production management. The integrated problem for designing flexible cellular manufacturing associated with determining the optimal values of the machining speeds, overtime, and intercell flow is formulated as Nonlinear Mixed Integer Programming(NMIP) in order to minimize total production change cost. This is achieved by introducing the marginal cost analysis into the NMIP, which will compute the optimal machining speed, overtime, intercell flow, and routing. The application of this procedure offers greater flexibility to take advantage of the cellular manufacturing due to the optimum use of resources. A solution procedure for this problem was developed and a numerical example is included.

1. Introduction

Manufacturing industry has adopted cell technology as the strategy for improving poduction operations on the shop floor. However, for successful cell implementation, the flexibility in this manufacturing industry is of considerable importance to cope with dynamic market situa-

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tions. As long as there is progress, managers can count upon changes to constantly disturb cellular manufacturing. The cellular manufacturing is directly affected to varying degree by these changes: changes in demand for the product, changes in the product mix, resulting from unexpected product-order. A flexible cellular manufacturing with the flexibility in the aspect of both hardware and software is an effective way to cope with production-to-order or non-mass jobbing production, and also evens out the level of production and adapts to these changes that come from the market, thus resulting in high utilization of high-priced production facilities.

Although there are numerous studies that address the problem of optimal or nearoptimal machine grouping and parts family classification for cellular manufacturing, little research has been reported that studies a methodology or the concept for designing flexible Cellular manufacturing. Hitomi et al. (*1982*) discussed the machine loading problems for a multistage system: they determined the optimal product-mix as well as the optimal machining speeds under the criteria of the maximum profit rate and minimum cost. Hancock (*1989*) discussed the effects of alternative routings by comparing each of the three strategies: fixed routing, routing by actual process quantity, and routing by resource availability. Egbelu provided a methodology for selecting the best routing for a batch in a multistage production system with material handling consideration in an attempt to minimize total batch manufacturing tiem (*1990a*), and also presented a methodology for selecting the machining parameters at each workstation and the material flow plan in a multistage production system in order to minimize toatal production cost (*1990b*). Nasr et al. (*1990*) proposed the problem of machine assignment and scheduling task in order to minimize the mean flow time in a job shop with alternative machine tool routings.

In this paper, the flexible cellular manufacturing is designed by the flexible production management(information flow) based on the given flexible production facilities, e.g., flexible type NC, machining centre, or flexible manufacturing system(FMS). It is fundamentally important for the efficient and economical execution of production activities to completely unify manufacturing processes(aspect of hardware; material flow) and production mangement(aspect of software; information flow)[Hitomi (*1979*)]. This unified and integrated approach to cellular manufacturing is significant concept to design flexible cellular manufacturing systems. This paper presents an analysis of integrated problem to be considered products-order in a cellular manufacturing.

In this paper, the flexible production management to be viewed as an integrated production model consists of the following four decision-making problems:

(1) ***Production planning problem.*** This determines the optimal overtime and the production quantities of ordered products at each machine adopted in order to satisfy the its desired due dates subject to limited resources.

(2) ***Production process planning.*** This determines the process routes by considering alternative machine routing in the cellular manufacturing systems, and machine speeds for

ordered products at each machine selected.

(3) *Cellular layout.* The integrated production model considers configuration of cellular manufacturing with regard to intercell movement for process required of ordered Products.

(4) *Production scheduling problem.* This determines releasing time of ordered products and the time schedule for its in the process route selected.

The relationship of these four decision-making problems is represented by the slantline box in Figure 1. These flows are the information flow for effective and economical production in the flexible cellular manufacturing. In order to design a flexible cellular manufacturing and optimize an integrated manufacturing systems, Song et al. discussed three problems indicated in this Figure: one is on machine loading problem(mainly, workload balance problem among machines) with alternative machine tools considerations(*1990a, 1990b*), another is a cell formation for minimizing intercell parts flow based on the workload balance among the machine(*1991*), the other is an integrated model unifying the production planning and cellular layout with forecast errors(*1991*). Several attempts have been made to analysed between merely two planning problems(both production plomning and scheduling) as a mixed integer linear program, the so-called hybrid or hierarchical approach[Hax et al. (*1975*), Graves(*1982*)].

In this paper, the four decision-making problems which unify the production management and flexible production facilities are simultaneously analyzed and optimized. Furthermore, this paper proposes a framework and a methodology that is capable of absorbing the high rate of environmental changes as using overtime usage, increasing or decreasing machining speeds, and intercell movement for the following two conditions:

(i) When the cell does not include a machine required for processing the ordered products,

(ii) When the whole machin the cell is not capable of producing total quantities of ordered products.

2. Assumptions

The following preconditions are set for the analysis of an integrated production model by group technology concept.

(1) Ordered product are classified into the same parts family, and products into the same parts family have the same machining function.

(2) Ordered product into the parts family, i has production quantities, q_i[pcs] and must be completed by desired order due dates, d_i[days].

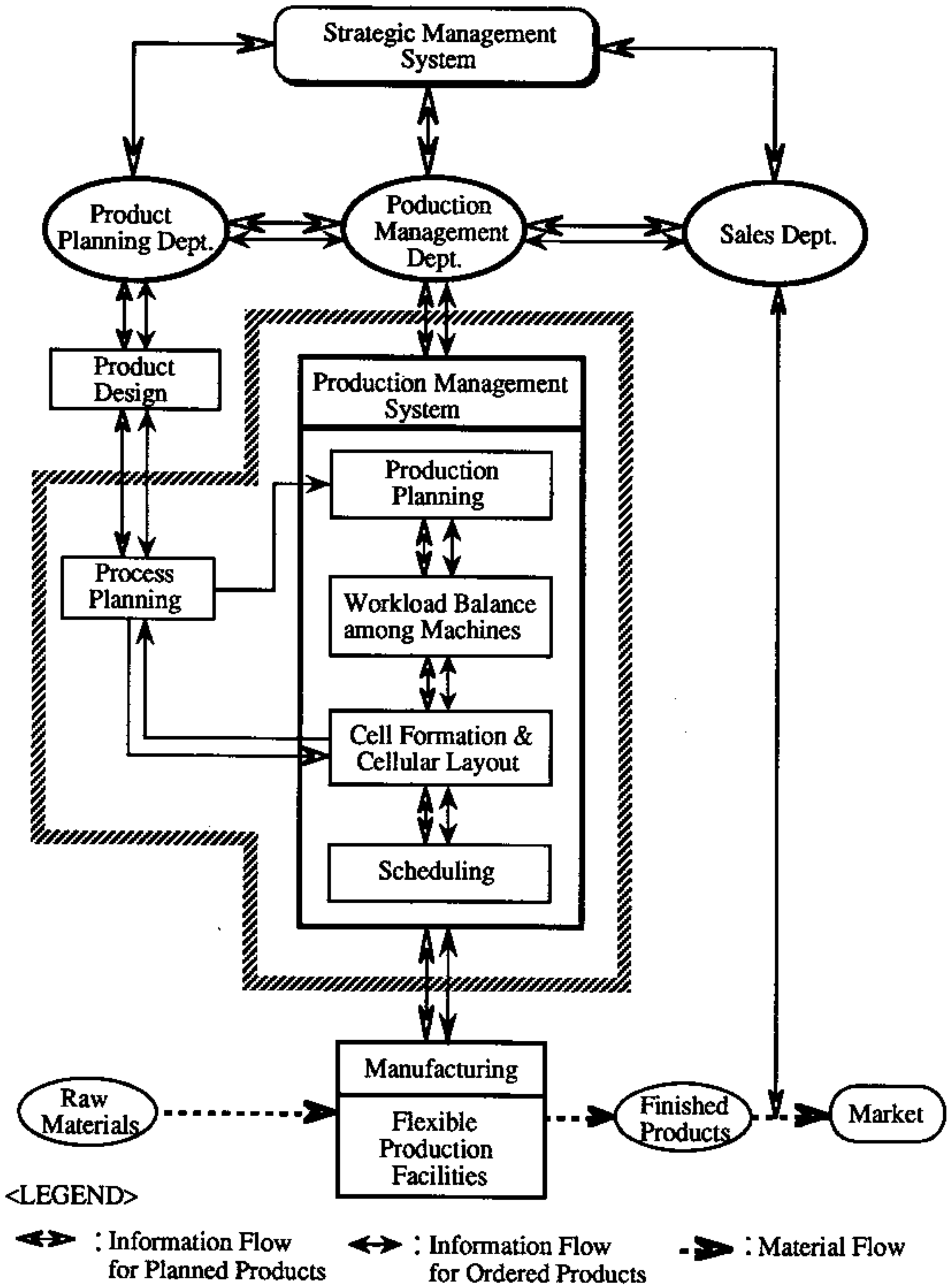
(3) Ordered products have a predetermined desired machining function, $l(=1, 2, \cdots, \sigma)$ and precedence relationships between machining functions should be satisfied.

(4) Each machining function required can be performed by alternative machines, $k_l(=1, 2, \cdots, K_l)$.

(5) To move from one cell to another, group setup time, S_i[min/pc] for processing and setup time, Y_i[min/pc] for intercell products movement are required.

(6) Ordered poducts within parts family i, individually move one by one between cells.

In this paper, the integrated decision-making



[Figure 1] Integrated production management system for designing flexible cellulr manufacturing problems considered are to determine the optimal process routes from alternative machine routing, optimal order releasing time, optimal overtime, and machining speeds at each machine selected, so as to meet specified order due dates subject to meeting production due dates. This integrated production model with a limited time available is to minimize the total incremental production cost which consists of direct labour/machining cost, material handling cost, overhead cost, and overtime cost.

3. Model formulation

3.1 Formulation constraints

Formulations are developed to ensure that a schedule meets the following constraints.

3.1.1 Production capacity

To illustrate the nature of the problem, the length of released production time and available production time at each machine are shown in Figare 2. The Figure depicts arrival period t=0, due dates d_i[days], the eariest possible time t^e_{kl}[min] in which ordered products could be produced under the maximum machining speed v^{max}_{kl}[m/min] on machine k_l, and the latest possible period t^c_{kl} in which ordered products could be produced under the minimum-cost machining speed v^c_{kl} on the machine k_l.

At arrival period $t=0$, planned products are already scheduled on each machine, and are being produced by using minimum-cost machining speeds v^c_{kl}[m/min]. Available production time can be defined as the sum of the time slack $d_i-t^c_{kl}$[min] under the minimum-cost machining speeds and increased time slack $t^c_{kl}-t^e_{kl}$ as increasing machining speed up to maximum speed v^{max}_k. Therefore, the available production time can be formulated as,

$$\{\tau_{ikl}(v_{kl})+(\frac{\eta}{\omega}d_{lE}+Y_i+S_i)\xi_i\}\ x_{ikl}\leq d_i-t^c_{kl},$$

$$\text{for all } k_l,\ l \qquad (1)$$

where, $\tau_{ikl}(v_{kl})$[min/pc]: production time to finish of the ordered product under the machining speed v_{kl}, η: handling coefficient depending upon weight, feature, and surface condition of parts family, ω[m/hour]: velocity of the material handling equipment, d_{lE}[m/pc]: travelling distance between cells, x_{ikl}: production quantity of ordered product i on machine k_l, and variable ξ_i: 0-1 type variable defined as following:

$$\xi_i=\begin{cases}1,\ \sum k_l\in \boldsymbol{m}_r\ x_{ikl}\geq 1,\ (i\in \boldsymbol{G}_r)\\ 0,\ \sum k_l\in \boldsymbol{m}_r\ x_{ikl}=0,\ (i\in \boldsymbol{G}_r)\end{cases} \qquad (2)$$

$\boldsymbol{G}_r$ and $\boldsymbol{m}_r$ are set of parts families to be produced and set of machines within cell r, respectively.

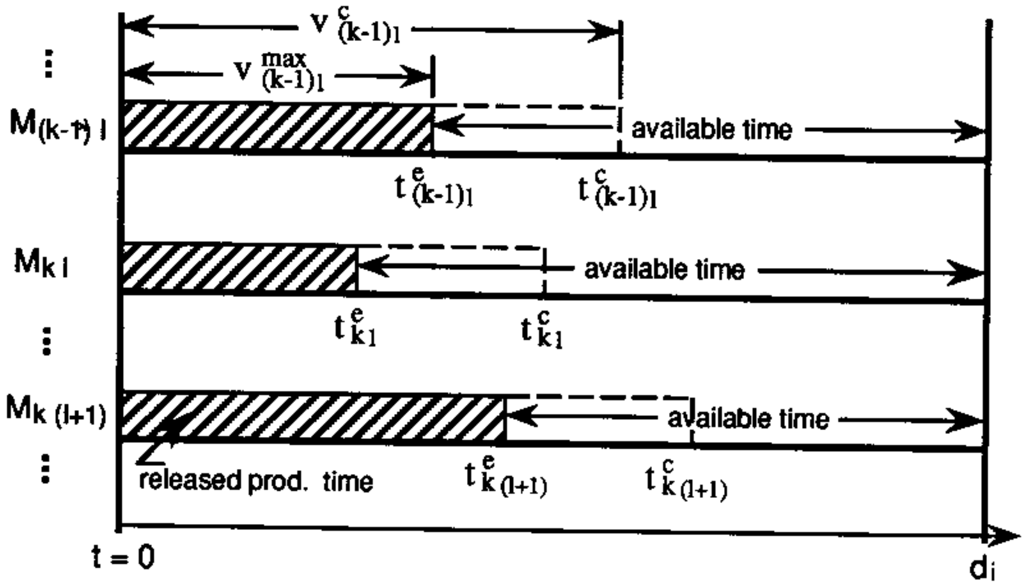
3.1.2 Order quantity

Constraint on quantity to be consisted of both quantity x_{ikl} on regular time and quantity y_{ikl} on overtime is given as follows:

$$\sum_{k_l=1}^{K_l} x_{ikl}+y_{ikl}=q_{il}\quad \text{for all } l \qquad (3)$$

3.1.3 Precedence requirement

A sequencing constraint is required when a following machining function $l+1$ cannot be started until proceding machining function l have been completed. If T_{kl} denotes the start of machining function l of ordered products on



[Figure 2] Available production time with two-types slack time

machine k_l, the appropriate sequencing constraint becomes,

$$T_{kl}+\{\tau_{ikl}(v_{kl})+(\frac{\alpha}{\omega}d_{IE}+Y_i+S_i)\xi_i\}x_{ikl} \le T_{kl+1}, \quad \text{for all } k_l,\ l \tag{4}$$

3.1.4 Completion ordered products

Ordered products included into the parts family i cannot be completed by due dates d_i untill $\sum_{k_l=1}^{K_l} x_{ikl}+y_{ikl}=q_i$ all for last machining function σ of its. This requirement can be written as:

$$\sigma \le \frac{1}{q_i}(\sum_{l=1}^{\sigma}\sum_{k_l=1}^{K_l})\ x_{ikl}+y_{ikl}) \tag{5}$$

3.1.5 Order due dates

A completion of the final machining function σ must satisfy the following constraint:

$$F_\sigma = T_{k\sigma}+\{\tau_{ik\sigma}(v_{k\sigma})+(\frac{\eta}{\omega}d_{IE}+Y_i+S_i)\xi_i\}x_{ik\sigma}\le d_i \tag{6}$$

3.1.6 Machining speed constraint

Where v_{kl}^{max} and v_{kl}^{c} are the upper limited machining speeds and minimum cost machining speeds, respectively. Hence, the possible releasing time for ordered product must exist in the following range of t_{kl}.

$$v_{kl}^{c} \le v_{kl} \le v_{kl}^{max}, \quad \text{for all } k_l,\ l \tag{7}$$

$$t_{kl}^{e} \le t_{kl} \le d_i, \quad \text{for all } k_l,\ l \tag{8}$$

3.1.7 Overtime constraint

The overime should be less than a specific amount, O^{max}:

$$\tau_{ikl}(v_{kl})\ y_{ikl} \le O^{max}, \quad \text{for all } k_l,\ l \tag{9}$$

3.2 Objective function

When ordered products are released, planned products are being produced by using predetermined minimum-cost machining speeds, v^c_{kl} at each machine within their cells. Ordered products are to be scheduled in a manner that minimizes the sum of (1) labor/machining cost to be needed as exceeding the minimum-cost machining speed, v^c_{kl}, (2) material handling cost for intercell movement, (3) group setup cost, and (4) overtime production cost. Therefore, in order to complete the ordered products by their due dates, the objective function for minimizing the total incremental production cost can be formulated as follows:

$$\min \sum_{l=1}^{\sigma} \sum_{k_l=1}^{K_l} f(v_{kl}) + \{(c_P(\frac{\eta}{\omega} d_{lE} + Y_i) + c_l\ S_i)\xi_i\ x_{ikl}\} + g(v_{kl}) y_{ikl} \quad (10)$$

where c_P[$/hour]: material handling cost per unit time; c_l[$/hour]: direct labor cost per unit time; $g(v_{kl})$[$/pc]: overtime production cost per unit. In the above equation, the overtime production cost, $g(v_{kl})$ is defined as follows:

$$g(v_{kl}) = f(v_{kl}) + b_l\ \tau_{ikl}(v_{kl}),\ \text{for all } k_l,\ l \quad (11)$$

where b_l[$/hour] is cost of direct labor on overtime per unit time at machine type l. And the incremental machining cost, $f(v_{kl})$[$/pc] also can be defined using the following two types machining conditions.

$$f(v_{kl}) = \begin{cases} \alpha_{kl}\ v_{kl}^{m_{kl}} + \dfrac{\beta_{kl}}{v_{kl}}, & \text{for } v^c_{kl} < v_{kl} \le v^{max}_{kl} \\ 0, \text{ for } v_{kl} = v^c_{kl} \end{cases} \quad (12)$$

where $m_{kl}(=1/n_{kl}-1)$ is slope constant of the Taylor tool life curve,

α_{kl} = (direct labor cost[$/min]
+ overhead[$/min] + tool cost[$/edge])
λ_{kl}, on machine k_l

β_{kl} = (direct labor cost[$/min]
+ overhead[$/min]
+ machining overhead[$/min]) $\frac{\lambda_{kl}}{C_{kl}^{m_{kl}+1}}$,
on machine k_l

Usually, the slope constant of the Taylor tool-life curve, n_{kl} is in a range of 0.1 to 0.5, hence it is assumed that $m_{kl} \ge 1$.

In the above two cost components, C_{kl} is parameter for the Taylor tool life equation, and λ_{kl} on machine k_l is calculated as,

$$\lambda_{kl} = \begin{cases} \pi DL/1000s, & \text{for turning, boring, drilling, and reaming,} \\ \pi D(L+l)/1000sZ, & \text{for milling} \end{cases} \quad (13)$$

where D is the machining diameter[mm] for turning and boring or the tool diameter[mm] for milling, drilling, and reaming; L is the machining length[mm] for boring, drilling, and reaming; l is the additional feed length[mm] for milling; s is the feed rate per tooth[mm/tooth] for milling or the feed rate per revolution [mm/rev] for othere operations; and Z is the number of teeth of the milling cutter.

4. Analysis of integrated production model

4.1 Formulation of integrated problem

The integrated problem formulated as non-linear mixed integer programming problem deals with determining the process routes, when an ordered product must be released at each determined machine, optimal machining speeds, v_{kl}, optimal production quantities, x_{ikl}, and production quantities, y_{ikl} on the overtime at the each selected machine, so as to minimize equation (10) subejct to resources constraints proposed above. The integrated production model defined above is formulated as follows:

[Problem P]

$$\min\sum_{l=1}^{\sigma}\sum_{k_l=1}^{K_l}\{f(v_{kl})+(c_P(\frac{\eta}{\omega}d_{IE}+Y_i)+c_l S_i)\xi_i\}x_{ikl}+g(v_{kl})y_{ikl}$$

subject to:

$$\{\tau_{ikl}(v_{kl})+(\frac{\eta}{\omega}d_{IE}+Y_i+S_i)\xi_i\}x_{ikl}\le d_i-t^e_{kl},$$
for all k_l, l

$$\sum_{k_l=1}^{K_l}x_{ikl}+y_{ikl}=q_i, \text{ for all } l$$

$$T_{kl}+\{\tau_{ikl}(v_{kl})+(\frac{\eta}{\omega}d_{IE}+Y_i+S_i)\xi_i\}\ x_{ikl}\le T_{kl+1}, \text{ for all } k_l,\ l$$

$$\sigma\le\frac{1}{q_i}\sum_{l=1}^{\sigma}\sum_{k_l=1}^{K_l}x_{ikl}\ y_{ikl},$$

$$F_\sigma=T_{k\sigma}+\{\tau_{ik\sigma}(v_{kl})+(\frac{\eta}{\omega}d_{IE}+Y_i+S_i)\xi_i\}\ x_{ik\sigma}\le d_i,$$

$v^c_{kl}\le v_{kl}\le v^{max}_{kl}$, for all k_l, l

$t^l_{kl}\le t_{kl}\le d_i$, for all k_l, l

$\tau_{ikl}(v_{kl})\ y_{ikl}\le O^{max}$, for all k_l, l

$x_{ikl}\ge 0,\ y_{ikl}\ge 0$, integer, for all k_l, l

4.2 Initial policy

Incremental production cost $f(v_{kl})$ and overtime production cost $g(v_{kl})$ can transformed to $f(t_{kl})$ and $g(t_{kl})$ as function of increasable production time by inversely proportional to machining speed, that is, $\lambda_{kl}/(v_{kl}-v^c_{kl})$. In an attempt to solve the integrated production model defined, the following policy is initially employed by considering the properties of production cost function.

[**Property** 1] The increasable machining cost and overtime production cost functions, $f(t_{kl})$ and $g(t_{kl})$, respectively, are positive, unimodal, twice-differentiable, and strictly convex function of production time t_{kl}, that is,

$f'(t_{kl})>0,\ f''(t_{kl})>0,$

$g'(t_{kl})>0,\ g''(t_{kl})>0.$

[**Property 2**] If overtime is required to meet order due date of ordered products under the minimum-cost machining speed, v^c_{kl}, then the optimal policy is to utilize both increasing machining speed and overtime under the property 1.

[**Property** 3] Let the production time, t^e_{kl} required to manufacture the ordered products be definced by setting the derivative of inverse equation (12) with respect to t_{kl} equal to b_l, that is, $f'(t^e_{kl})=b_l$. From the property 2, if production time required t_{kl} greater than t^e_{kl}, then the optimal policy for meeting desired order due date is given as following conditions:

$0<t_{kl}\le t^e_{kl}$; machining speed, v^e_{kl} is employed for this range,

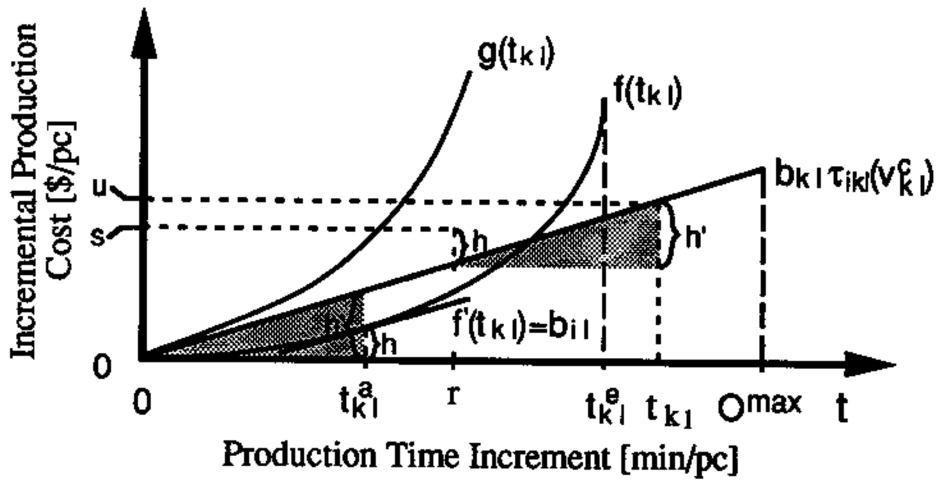
$t^e_{kl}<t_{kl}<d_i$; overtime is employed for this range.

<u>Proof</u> From Figur 2, following conditions can be confirmed:

$b_l\ge f'(t_{kl})$, for the range $t\le t^e_{kl}$

$b_l< f'(t_{kl})$, for the range $t> t^e_{kl}$

Let t_{kl} be production time required for the ordered products under the minimum-cost machining speed, v^c_{kl}. If whole of the production time required, t_{kl} is performed on the overtime, then incremental production cost is u as indicated in Figure 3. However, we can cut down the overtime t_{kl} to production time remained, r $(t_{kl}-t^{e}_{kl})$ by increasing the machining speed up to v^{e}_{kl}. Then incremental production cost becomes Sas indicated in the Figure 3. Hence, production cost required, u always greater than s. QED



[Figure 3] Interrelationship between Incremental production cost $f'(t_{kl})$ and $g'(t_{kl})$

In an attempt to seek the optimal solutions for integrating production model, thess properties are employed. Solution procedure is composed by following four cases.

[Case 1] The ordered products can be completed by their due dates with minimum-cost machining speed, v^{c}_{kl}.

In this case, the optimal solution for production quantities x^{*}_{ikl} is easily obtained by solving the following problem:

[Problem P1]

$$\min \sum_{l=1}^{\sigma} \sum_{k_l=1}^{K_l} \{(c_P(\frac{\eta}{\omega}d_{IE}+Y_i)+c_l\ S_i)\xi_i\} x_{ikl} \tag{14}$$

subject to: equation (1)~(6), $x_{ikl} \geq 0$, integer

The other optimal solutions are given by:

$v^{*}_{ikl}=v^{c}_{ikl},\ y^{*}_{ikl}=0$, for all k_l, l

[Case 2] The ordered products can be completed by their due dates as increasing production time per unit, t^{e}_{kl}[min/pc].

A decomposition technique is employed to solve the following nonlinear mixed integer programming problem(Problem P2) which determines the optimal machining speed, v^{*}_{ikl} and production quantities, x^{*}_{ikl}. To determine optimal production quantities, x^{*}_{ikl}, the problem P2 including fixed machining speeds, v^{e}_{kl} can be formulated as follows:

[Problem P2]

$$\min \sum_{l=1}^{\sigma} \sum_{k_l=1}^{K_l} \{f(v^{e}_{kl})+(c_P(\frac{\eta}{\omega}d_{IE}+Y_i) + c_l\ S_i)\xi_i\} x_{ikl} \tag{15}$$

subject to:

$$\{\tau_{ikl}(v^{e}_{kl})+(\frac{\eta}{\omega}d_{IE}+Y_i+S_i)\xi_i\}\ x_{ikl}$$

$$\leq d_i - t^a_{kl}, \text{ for all } k_l, l \quad (16)$$

$$\sum_{k_l=1}^{K_l} x_{ikl} + y_{ikl} = q_l, \text{ for all } l \quad (3)$$

$$T_{kl} + \{\tau_{ikl}(v^a_{kl}) + (\frac{\eta}{\omega} d_{IE} + Y_i + S_u)\xi_i\} x_{ikl} \leq$$

$$T_{k(l+1)}, \text{ for all } k_l, l \quad (17)$$

$$\sigma \leq \frac{1}{q_i} \sum_{l=1}^{\sigma} \sum_{k_l=1}^{K_l} x_{ikl}\, y_{ikl} \quad (5)$$

$$F_\sigma = T_{k\sigma} + \{\tau_{ik\sigma}(v^a_{kl}) + (\frac{\eta}{\omega}) d_{IE}$$

$$+ Y_i + S_i)\xi_i\} x_{ik\sigma} \leq d_i \quad (18)$$

$x_{ikl} \geq 0$, $y_{ikl} \geq 0$, integer, for all k_l, l

If $F_\sigma = d_i$ from the results, the optimal machining speed at each machine is,

$v^*_{kl} = v^a_{kl}$, for all k_l, l

The optimal production quantities, x^*_{ikl} can be obtained by computational results of problem P2. Otherwise, to determine the optimal machining speed, v^*_{kl}, the following nonlinear programming problem P3 included optimal production quantities x^*_{ikl} obtained from P2 is solved.

[Problem P3]

$$\min \sum_{l=1}^{\sigma} \sum_{k_l=1}^{K_l} x^*_{ikl}(\alpha_{kl} v^{m_{kl}}_{kl} + \frac{\beta_{kl}}{v_{kl}}) \quad (19)$$

subject to:

$$\tau_{ikl}(v_{kl}) x^*_{ikl} \leq d'_i - t^a_{kl}, \text{ for all } k_l, l \quad (20)$$

$$v^a_{kl} < v_{kl} \leq v^a_{kl}, \text{ for all } k_l, l \quad (21)$$

where, $d'_i = d_i - \{(\frac{\alpha}{\omega} d_{IE} + Y_i + S_i)\xi_i\} x^*_{ik1}$. Applying the Kuhn-Tucker's theorem to this problem, nonnegative constants θ_{kl}, which satisfy the following optimality conditions, exist for the optimal machining speed v^*_{kl}.

$$\frac{\partial\ (v^*, \theta)}{\partial v_{kl}} = x^*_{ikl}(m_{kl}\ \alpha_{kl}\ v^{*(m_{kl}-1)}_{kl} \frac{\beta_{kl}}{v^{*2}_{kl}}$$

$$+ \theta^*_{kl}(\frac{\partial \tau_{ikl}(v^*_{kl})}{\partial v_{kl}}) x^*_{kl}) = 0, \quad (22)$$

$$\theta^*_{kl}(\frac{\partial \tau_{ikl}(v^*_{kl})}{\partial v_{kl}}) x^*_{kl}) = \theta^*_{kl}(\tau_{ikl}(v^*_{kl}) x^*_{kl}$$

$$-(d'_i - t^a_{kl})) = 0 \quad (23)$$

where $\mathcal{L}(v^*, Q^*)$ is the Lagrangean function, and Q is an Lagrange multiplier.

From the above optimality conditions, the optimal machining speed at each machine is:

(1) If $\tau_{ikl}(v^*_{kl}) x^*_{kl} < (d'_i - t^a_{kl})$, then $\theta^*_{kl} = 0$ from the above equation (23). Hene,

$$v^*_{kl} = \left(\frac{\beta_{kl}}{m_{kl}\ \alpha_{kl}}\right)^{\frac{1}{m_{kl}+1}}$$

(2) If $\tau_{ikl}(v^*_{kl}) x^*_{kl} = (d'_i - t^a_{kl})$, then

$$v^*_{kl} = \frac{d'_i - t^a_{kl}}{\tau^{-1}_{ikl}\ x^*_{kl}}$$

[case 3] The ordered products can be completed by their due dates with increasing production time per unit from t^a_{kl} to $t^a_{kl} + O^{max}$.

In this case, we should repeatedly solve both the original problem P and P3 defined in Case 2 to determine the production quantities x^*_{ikl}, overtime, and machining speed v^*_{kl} at each machine. According to the above property (3), the optimal policy is to increase machining speed up to t^a_{kl}, and the remaimed production time $t_{kl} - t^a_{kl}$ is performed by overtime production. The optimal solutions can be obtained by applying the same procedures of Case 2. Then, optimal overtime at each machine is:

$$O^*_{kl} = \tau_{ikl}(v^a_{kl})\ y^*_{ikl}$$

[Case 4] The ordered products can be completed by their due dates with increasing production time per unit from $t^a_{kl} + O^{max}$ to $t^a_{kl} + O^{max}$.

Solve the original problem P. If $\tau_{ikl}(v^a_{kl})\ y^*_{ikl} = O^{max}$, then the optimal solution is given:

$v^*_{kl}=v^{max}_{vl}$, $O^*_{kl}=\tau_{ikl}(\frac{a}{k})$ $y^*_{ikl}=O^{max}$, and optimal production quantities: x^*_{ikl}.

Otherwise, repeatedly solve the following problem P4 included optimal production quantities x^*_{ikl} to determine the optimal machining speed, until Z^*(optimal value of P)$=Z_4$(optimal value of P4).

[Problem P4]

$$\min \sum_{i=1}^{\sigma} \sum_{k_l=1}^{K_l} x^*_{ikl}(\alpha_{kl} v^{\pi_{kl}}_{kl}+\frac{\beta_{kl}}{v_{kl}}) \quad (18)$$

subject to:

$$\tau_{ikl}(v_{kl})x^*_{ikl} \leq d'_i - t^s_{kl}, \text{ for all } k_l, l \quad (24)$$

$$v^l_{kl} < v_{kl} \leq v^a_{kl}, \text{ for all } k_l, l \quad (25)$$

4.3 Optimizing algorithm

A computational algorithm determining the optimal process routes, machining speeds, optimal overtime, and production quantities at each adopted machine for an ordered products into the same parts family, cellular manufacturing with alternative machine routing is as follows:

[Step 1] Solve the problem P by fixed machining speed $v^*_{kl}=v^a_{kl}$. If P is infeasible, then go to Step 3. Otherwise, P has an optimal solution($v^*_{kl}=v^a_{kl}$, *and* x^*_{ikl}, and y^*_{ikl}).

[Step 2] If $O_{kl}=0$, solve the P2 and P3 until Z_2(optimal value of P2)$=Z_3$(optimal value of P3) by utilizing the solution procedure of Case 2. If $0<O_{kl}$, solve the P by applying the solution procedure of Case. 3.

[Step 3] Solve the P4 untill $Z^*=Z_4$ by applying the solution procedure of Case 4.

5. Illustration

The optimizing algorithm for the optimal route selection, for determinig optimal overtime, machining speeds to be utilized in a cellular manufacturing systems with alternative machine routing in an attempt to minimize the total incremental production cost is applyed to two products-order having same machining function, two-cells manufacturing systems. The basic production data with negligible machining overhead cost is given as indicated in Table 1. The ordered products have four machining functions, and each machining function can be performed on alternative machines of same machine type as shown in Figure 4. Initial cellular layout and material flow of planned parts families released are also shown in Figure 5.

The other production data for ordered produts are given as follows:

the whole of the jobs are included parts family 1 (that is $i=1$), order quantities:$q_i(=q_{1i}+q_{2i})=$ 50[pcs],

order due date: $d_i=6.7$days($=3200$[min]),

maximum overtime:$O^{max}=8$hours($=480$[min]),

velocity of material handling equipment: $\omega=30$ [m/min],

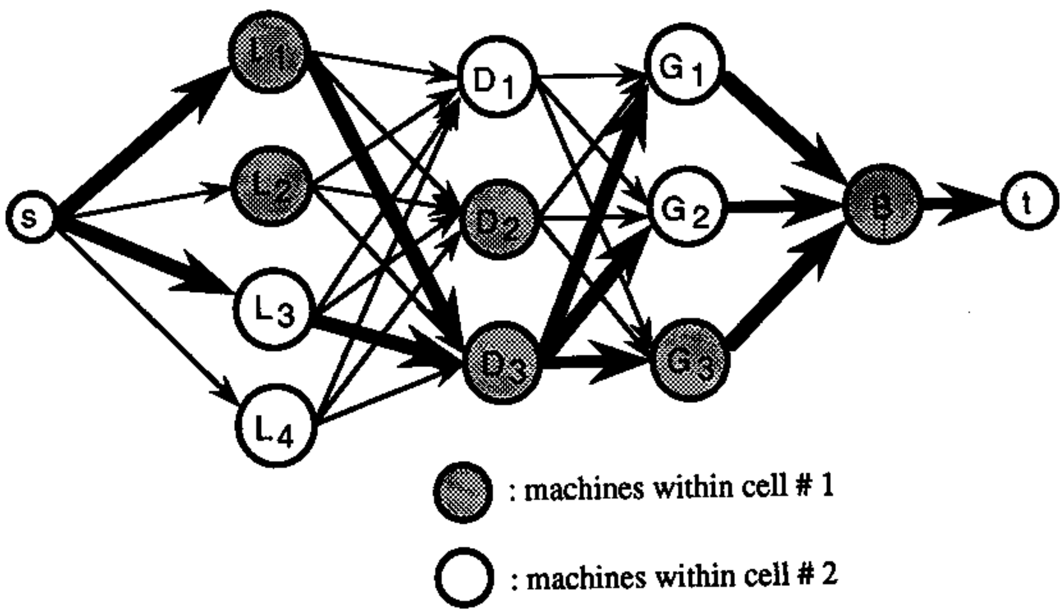
material handling coefficient: $\eta=1$,

group setup time: $S_i=30$[min], average intercell travelling time: 20[min]($=Y_i+d_{iE}/30$),

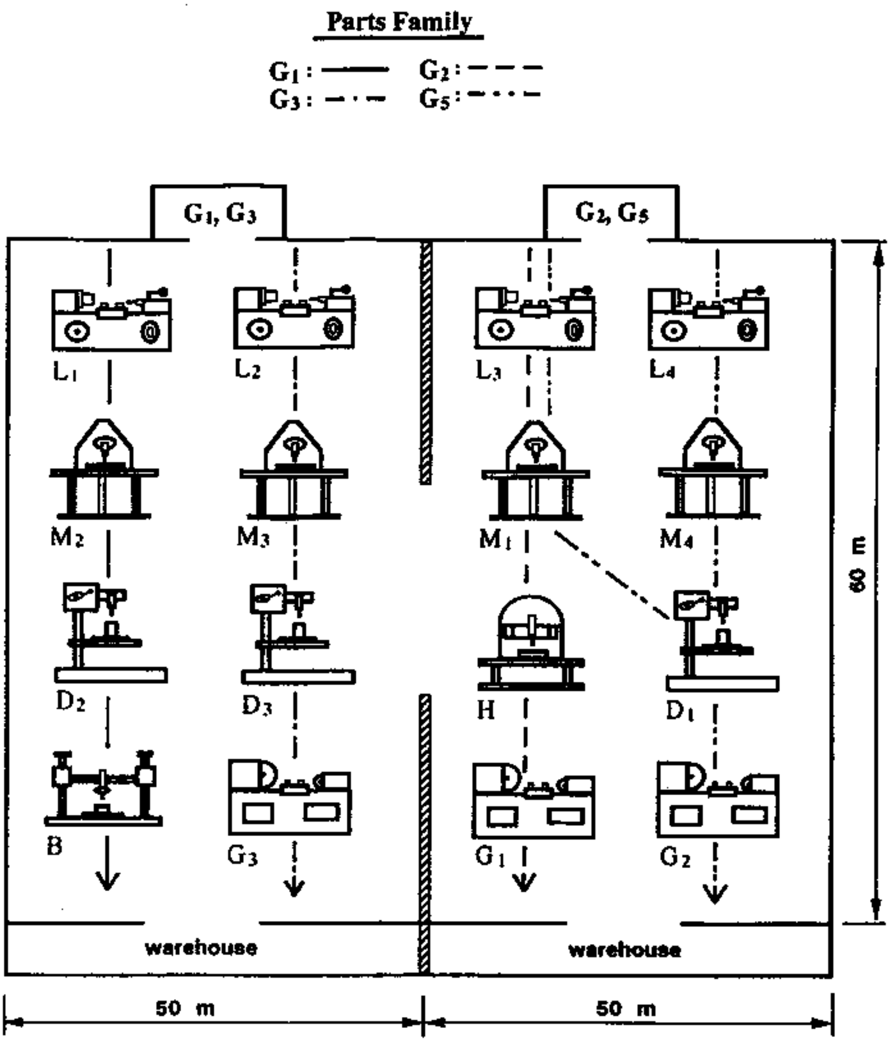
direct labor cost: $c_l=15$[$/hour],

material handling cost: $c_p=19.8$[$/hour], 〈Table 1〉 Basic production data for the four machine types and ordered products

Machine	m_{kl}	Direct labor cost and overheady[$ /min]	Tool cost [$ /edge]	C_{kl} [m/min]	λ_{kl}	v^{c}_{kl} [m/min]	v^{f}_{kl} [m/min]	Released Work Load[min]
L_{11}	1.4	0.5	2.5	400	2.0	100	140	350
L_{21}	1.5	0.4	5	250	2.2	120	150	400
L_{31}	1.7	0.45	3	450	1.9	125	150	280
L_{41}	2.0	0.3	7	350	1.5	150	180	300
D_{12}	2.0	0.5	3	350	1.6	80	120	520
D_{22}	1.8	0.25	3	270	1.5	100	140	460
D_{32}	1.7	0.4	6	400	1.6	130	150	330
G_{13}	1.2	0.7	6	290	3.0	00	150	450
G_{23}	1.6	0.5	4.5	400	2.4	120	150	300
G_{33}	1.4	0.3	5	360	2.4	150	200	370
B	1.5	0.5	8	300	1.6	90	130	420



[Figure 4] Alternative machine routing(→) and optimal process route(➡) for ordered products



[Figure 5] Initial cellular layout and material flow of palnned parts fimilies

overtime production cost: $b_1=300$[$ /hour], $b_2=360$[$ /hour],
$b_3=240$[$ /hour], and $b_4=300$[$ /hour].

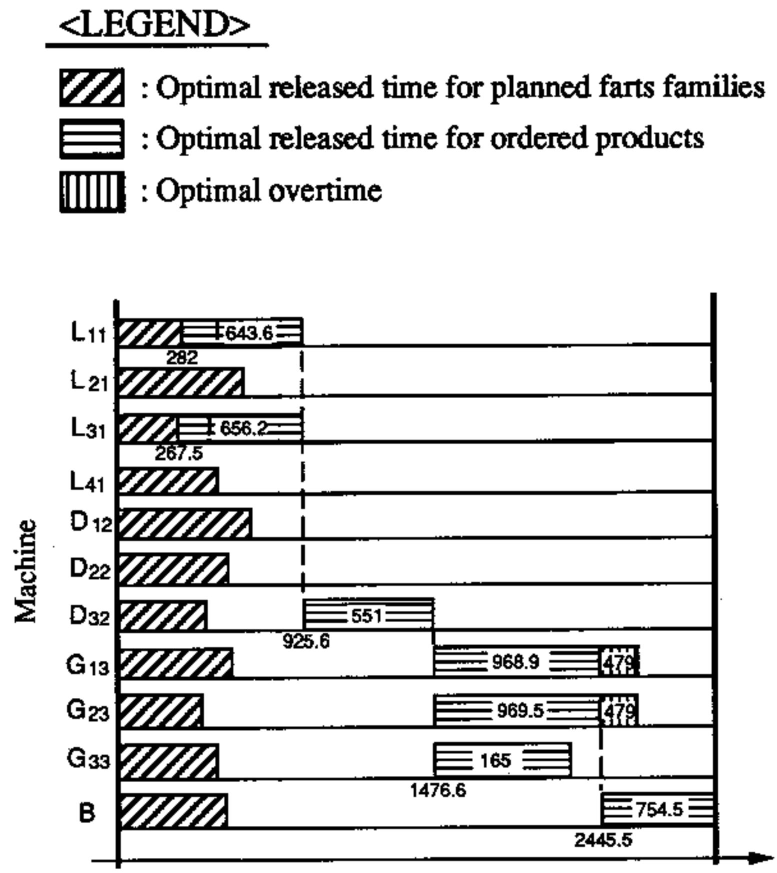
[Step 1] Machining speeds, v^{a}_{kl} at each machine are shown in Table 2. Solve the P including machining speeds v^{a}_{kl} in order to determine production quantities and optimal overtime. The computational results and optimal process route are represented in Table 3 and Figure 4.

⟨Table 2⟩ Machining speed v^{a}_{kl} to be satisfied $f'(v^{a}_{kl}=b_l)$

Machine	L_{11}	L_{21}	L_{31}	L_{41}	D_{12}	D_{22}	D_{32}	G_{13}	G_{23}	G_{33}	B
v^{a}_{kl}[m/min]	124.1	142.4	130.85	155.56	83.75	119.28	141.5	110	123.4	181.37	107.36

〈Table 3〉 Optimal solution for planned parts families and ordered products($Z^* = 46805.4$[$])

Machine		L_{11}	L_{21}	L_{31}	L_{41}	D_{12}	D_{22}	D_{32}	G_{13}	G_{23}	G_{33}	B
Optimal machining speed for planned parts families v^*_{lk}[m/min]		124.1	120	130.85	150	80	100	130	100	120	150	90
Ordered products	Releasing time [min]	282	–	267.5	–	–	–	925.6	1476.6	1476.6	1476.6	2445.5
	Optimal machining time v^*_{ik}[m/min]	124.1	–	130.85	–	–	–	141.5	110	123.4	150	107.36
	Optimal production qunatities x^*_{ik}[pcs]	39.8	–	10.2	–	–	–	50	12.5	13.9	10.4	50
	Optimal overtime O^*_{k}[min]	–	–	–	–	–	–	–	479	479	–	–



t = 0 [Figure 6] Gantt chart for the optimal solutions 3200 [min]

[Step 2] Since $0<O_{H}$ on overtime, solve the P3 defined in Case 2. From the computational results, optimal machining speeds v^{*}_{kl} obtained from Step 2, the optimal value of P(=4805.4 [$]) equal to the optimal value of P3. Thus, botained optimal solution for planned products and for ordered products are shown in Table 3. The releasing time, overtime, and optimal products sequencing at selected machine are represented in Figure 6 showing Gantt chart.

6. Conclusions

The proposed design prodcedure provides a methodology of integrating the four decision-making problems, such as production planning problem, process planning problem, scheduling problem, and cellular layout problem, for lfexible cellular manufacturing systems. The integrated production model for ordered products, cellular manufacturing system with alternative machine routing and cost depending upon both production speed, overtime, and intercell movement cost was developed and analysed as a nonlinear mixed integer programming problem in an attempt to minimize the total increamental production cost. The incremental production cost factors considered in the integrated production model are labor/machining cost, material handling cost for intercell movement, overtime production cost, and overhead cost. To cope with uncertainty market situations, changes in machining speed, overtime, and intercell flow for process required were simultaneously considered as decision-making variables. An optimal algorithm with the marginal cost analysis between machining cost and overtime cost was developed in seeking optimal solution to the model, and it was applied to real-life examples for a cellular manufacturing systems.

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