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A Study on the Storage Systems for the Items Having Different Turnover Rates

Lim, Sanggyu*

회전율 차이를 갖는 제품 저장 시스템에 관한 연구

임 상 규*

Abstract

This work is to study the behavior of certain storage systems in which items with different turnover rates are stored. In particular, we assume that items can be divided into two types: high and low turnover.

We try to find simple policies that induce favorable zoning, which will reduce the storage and retrieval distance of items.

We define a desirable property of a good policy which we call *asymptotically efficient*, and show that the Zone-z policy is asymptotically efficient if the zone size is proportional to the expected number of high turnover items in the system in equilibrium.

1. Introduction

Suppose that we have two types of items to be stored in a stroage system—type h for high turnover and type l for low turnover items. The turnover of an item is defined as the reciprocal of the item's mean storage time[7]. The following assumptions are made throughout the study unless otherwise mentioned.

- The storage rack consists of a sufficiently large number of locations, i.e., every arriving item must be stored somewhere.
- A unique input/output(I/O) point is located at the left corner of the storage rack.
- All storage loations are identical and the n^{th} location is d_n units from the I/O point, where $d_1 \leq d_2 \leq \cdots$. Usually, we will be looking at linear storage with $d_n = n$.
- At any given time, at most one item is

*Department of Industrial Engineering, Gyeongsang National University

stored in a location.

It is assumed that type $h[l]$ items arrive at the storage rack according to a Poission process with rate $\lambda_h[\lambda_l]$ and each arrived type $h[l]$ item is stored for an exponentially distributed length of time with parameter $\mu_h[\mu_l]$, respectively.

Our goal is to spend less time storing and retrieving items in the storage systems by minimizing expected storage/retrieval time (distance). With regards to this objective, we propose to analyze a class of simple storage policies called Zone-z Storage, which we compare to the basic COL(Closest Open Location) storage policy. In all policies, high turnover times are stored in the closest open location. In the COL policy, the low turnover items are also stored in the colsest open location. In the Zone-z policy, low turnover items are stored in the closest open location after location z. (The storage locations are numbered 1, 2, ⋯ in order of their distance from the input/output (I/O) point.)

We attempt to find value of z in terms of the basic system parameters which reduce the overall storage and retrieval distance.

There have been many studies for operating storage systems, especially for the operating of Automated Storage/Retrieval (AS/R) systems, in order to minimize expected storage/retrieval distance.

Some fundamental approaches for determining the expected s/r machine travel time in an AS/R system have been developed and examined by Graves, Hausman and Schwartz [5, 7, 12].

Bozer and White [3] extended the results of Graves, Hausman and Schwartz [5, 7, 12] for Randomized Storage. They derived analytical expressions for the expected travel times of single and dual commands with alternative I/O locations and storage rack configurations.

Newell[11] studied an $M/M/\infty$ service system with ranked servers where the service facility consists of a large(infinite) number of servers in parallel. He is concerned with the stochastic properties of the random process $N(m, t)$ which describe the number of busy servers among first m ranked servers at time t. Newell's model can be specially applied in telephone traffic which has m primary channels and a large number(infinite) of secondary channels.

Coffman et al [4] studied an $M/M/\infty$ queue with ordered servers as a model of dynamic storage alloction in computer memory. He obtained an explicit formula for the stationary distribution of the total occupancy max(N), defined as the highest numbered occupied location.

Aldous [1] studied the process of parking cars which can be interpreted as an $M/M/\infty$ queue with ordered servers(parking spaces). His main concern is to examine the disposition process of parked cars(the distribution of where cars are parked) at steady state.

2. The Objective Function

2.1 Objective Function and Asymptotically Efficient

We wish to minimize the average storage distance and retrieval distance in equilibrium. Later, we will show that under certain assumptions the average storage and retrieval distance are identical. Thus, we can analyze whichever is more convenient, and we will simply use the term *travel distance* which will refer to either storage or retrieval distance. We define the average storage distance under policy π as

$$D\bar{x}_0(\pi)=\lim_{n\to\infty}E\left[\sum_{i=1}^{n}\frac{T_i(\pi)}{n}|\vec{X}_0\right] \quad (1)$$

where $T_i(\pi)$ denotes the storage distance of the i^{th} item stored under policy π and $\vec{X}_0$ is the initial state of the storage system, provided the limit exists. In some cases, the limit in (1) may not exist, so we also define $\underline{D}\bar{x}_0(\pi)$ and $\bar{D}\bar{x}_0(\pi)$ by replacing *lim* with *liminf* and *limsup*, respectively.

$$\underline{D}\bar{x}_0(*)=\inf_{\pi}\underline{D}\bar{x}_0(\pi) \quad (2)$$

where the infimum is taken over all policies π. Our basic idea is to find $r(\lambda)$ and c_1, $0<c_1<\infty$, such that

$$\lim_{\lambda\to\infty}\frac{\underline{D}\bar{x}_0(*)}{c_1 r(\lambda)}=1$$

where $\lambda=\lambda_h+\lambda_l$. Then, we call a policy π *asymptotically efficient* if

$$\lim_{\lambda\to\infty}\frac{\bar{D}\bar{x}_0(\pi)-\underline{D}\bar{x}_0(*)}{c_1 r(\lambda)}=0$$

or equivalently

$$\lim_{\lambda\to\infty}\frac{\bar{D}\bar{x}_0(\pi)-\underline{D}\bar{x}_0(*)}{r(\lambda)}=0 \quad (3)$$

for all initial states $\vec{X}_0$.

Since the optimal policy is unknown, the optimal average travel distance $\underline{D}\bar{x}_0(*)$ is unknown. A critical step in our approach is to determine a sufficiently accurate lower bound D_* for the optimal average travel distance. Given a lower bound D_*, if

$$\frac{D_*}{r(\lambda)}\to c_1$$

with $0<c_1<\infty$ and

$$\frac{\bar{D}\bar{x}_0(\pi)-D_*}{r(\lambda)}\to 0,$$

then π is asymptotically efficient.

2.2 Lower Bound on Travel Time

For $c\in\{h,l\}$, let $A^c(t)$ denote the number of type c items that arrive druing $[0,t]$, Y_i^c denote the storage time of the i^{th} type c arrival, and $T_i^c(\pi)$ denote the storage distance of the i^{th} type c item stored under policy π. Assume that Y_i^c is independent of $A^c(\cdot)$, $A(\cdot)$, $T_i^c(\pi)$, and $\vec{X}_0$ where $A(t)=A^h(t)+A^l(t)$. Let $(N^h(t), N^l(t))$ denote the number of type h and l items in the system at time t, respectively.

Lemma 1 *For every t,*

$$E\left[\frac{\sum_{i=1}^{A^c(t)}T_i^c(\pi)}{A(t)}|\vec{X}_0\right]\geq\frac{1}{E[Y_i^c]}$$

$$E\left[\frac{\sum_{n=1}^{\infty}d_n\int_0^t I_{(c)}[X_s(n)]ds-R_c}{A(t)}|\vec{X}_0\right], \quad (4)$$

where R_c denotes the total remaining storage time of type c items which arrived before time 0 and still are in the system at time 0 weighted by the storage distance.

Proof For every t,

$$\frac{\sum_{i=1}^{A^c(t)}T_i^c(\pi)Y_i^c}{A(t)}\geq\frac{\sum_{n=1}^{\infty}d_n\int_0^t I_{(c)}[X_s(n)]ds-R_c}{A(t)} \quad (5)$$

where $X_t(n)$ denotes state of the n^{th} location at time t as

$$X_t(n)=\begin{cases}0, & \text{if empty}\\ h, & \text{if occupied by a type } h \text{ item}\\ l, & \text{if occupied by a type } l \text{ item}\end{cases}$$

and

$$I_{(c)}[X_s(n)]=\begin{cases}1 & \text{if } X_s(n)=c\\ 0 & \text{otherwise.}\end{cases}$$

This follows since the numerator on the left hand side of (5) gives the total storage time weighted by the storage distance of all arrivals up to time t. The numerator on the right hand side of (5) gives the weighted total storage time incurred during $(0, t)$. The inequality follows since some items may have arrived by time t but not incurred the total weighted storage time since they are still in the system at time t. Now, since Y_i^c is independent of $A^c(t)$, $A(t)$, $T_i^c(\pi)$ and $\vec{X}_0$, we have

$$E\left[\frac{\sum_{i=1}^{A^c(t)} T_i^c(\pi)\, Y_i^c}{A(t)} \mid \vec{X}_0\right]$$

$$=E\left[E\left[\frac{\sum_{i=1}^{A^c(t)} T_i^c(\pi)\, Y_i^c}{A(t)} \mid A^c(t),\ A(t),\ T_i^c(\pi)\right] \mid \vec{X}_0\right]$$

$$=E\left[\sum_{i=1}^{A^c(t)} \frac{T_i^c(\pi)}{A(t)} E[Y_i^c \mid A^c(t),\ A(t),\ T_i^c(\pi)] \mid \vec{X}_0\right]$$

$$=E[Y_i^c]E\left[\frac{\sum_{i=1}^{A^c(t)} T_i^c(\pi)}{A(t)} \mid \vec{X}_0\right]$$

Hence, (4) is obtained.

Theorem 1 *Assume that $A^c(\cdot)$ is independent of $\vec{X}_0$ and*

$$\frac{A(t)}{t} \to \lambda \ a.s.$$

$$\int_0^t \frac{f(N^h(s),\ N^l(s))}{t} \to E\{f(N^h,\ N^l)\}\ a.s.$$

and $\Pr\{R_c<\infty\}=1$. *Then, for every initial state $\vec{X}_0$ and policy π, a lower bound on the expected travel distance $D\bar{x}_0(\pi)$ is:*

$$D\bar{x}_0(\pi) \geq \frac{1}{\lambda}\mathrm{E}\left[\frac{1}{E[Y_i^h]}\sum_{n=1}^{N^h} d_n + \frac{1}{E[Y_i^l]}\sum_{n=N^h+1}^{N^h+N^l} d_n\right] \tag{6}$$

Proof The proof follows from Lemma 1 and the fact that

$$E\left[\frac{\sum_{i=1}^{A(t)} T_i(\pi)}{A(t)} \mid \vec{X}_0\right]=E\left[\frac{\sum_{i=1}^{A^h(t)} T_i^h(\pi)}{A(t)} \mid \vec{X}_0\right]+E\left[\frac{\sum_{i=1}^{A^l(t)} T_i^l(\pi)}{A(t)} \mid \vec{X}_0\right].$$

Corollary 1 *Under linear storage, that is $d_n=n$, a lower bound on the expected travel distance $D\bar{x}_0(\pi)$ is:*

$$D_* \equiv 1+p\cdot\frac{\rho_h}{2}+(1-p)\cdot\left(\rho_h+\frac{\rho_l}{2}\right) \tag{7}$$

where $\rho_h=\lambda_h/\mu_h$, $\rho_l=\lambda_l/\mu_l$ and $p=\lambda_h/(\lambda_h+\lambda_l)$

Proof The right hand side of (6)

$$\frac{1}{\lambda_h+\lambda_l}E\left[\mu_h\sum_{n=1}^{N^h} d_n+\mu_l\sum_{n=N^h+1}^{N^h+N^l} d_n\right]$$

$$=\frac{1}{2(\lambda_h+\lambda_l)}E[\mu_h((N^h)^2+N^h)+\mu_l(2N^hN^l+(N^l)^2+N^l)]=D_*.$$

If we extend the above arguments for a lower bound to the case in which we have $n(>2)$ types of items to store, a lower bound on the expected travel distance will be obtained as

$$D_* \equiv 1+\sum_{c=1}^{n}\left[p_c\left(\sum_{i=1}^{c-1}\rho_i+\frac{\rho_c}{2}\right)\right], \tag{8}$$

where $\rho_c=\lambda_c/\mu_c$ and $p_c=\lambda_c/(\lambda_1+\cdots+\lambda_n)$, and $\sum_{i=1}^{0}\rho_i=0$.

2. 3 Poisson Arrivals and Stationary Policies

We are trying to locate simple policies that are asymptotically efficient. In particular we will be looking only at stationary policies, i.e., storage policies that depend only on the state of the rack and the type of the item to be stored at time t. For these stationary policies with

Poisson arrivals(even compound Poisson arrivals) and exponential storage times, we will be able to show that $D\bar{x}_0(\pi)$ exists and does not depend on $\vec{X}_0$. Furthermore, the expected retrieval distance is also given by $D\bar{x}_0(\pi)$.

Consider the state of the storage rack. Define the process $\vec{X}_t$ as

$$\vec{X}_t=(X_t(1),\ X_t(2),\ \cdots,\ X_t(M_t))$$

where $X_t(n)$ denotes state of the n^{th} location and M_t denotes the farthest occupied location at time t. The state space S of $\vec{X}_t$ can be represented as

$$S=\{0\}\cup\left\{\bigcup_{n=0}^{\infty}(\{0,\ h,\ l\}^n\times\{h,\ l\})\right\}$$

where 0 represents an empty system.
Note that S is countable. Under a stationary policy, the stochastic process $\{\vec{X}_t:t\geq 0\}$ can be represented as a time-homogeneous Markov process with a countable state space, in which transitions occur whenever a type h or type l item is stored or retrieved. Given a policy π, let $S_0(\pi)\subset S$ denote the set of states which communicate with the empty state 0 under policy π. Then the Markov chain on the state space $S_o(\pi)$ is irreducible. Furthermore, since the probability of having an empty system is positive, $\vec{X}_t$ is positive recurrent, and there exists a unique stationary distribution for $\{\vec{X}_t:t\geq 0\}$ on $S_o(\pi)$. For stationary policies π, we have the following result.

Proposition 1 *For stationary policies π, the stationary distributions of storage distance and retrieval distance exist and are identical. In particular, the expected storage and retrieval distance are equal. Furthermore, the expected storage distance does not depend on the initial state $\vec{X}_0$.*

Proof By PASTA(Poisson Arrival Sees Time Average), the distribution imbedded just prior to arrivals is the same as the stationary distribution of $\{\vec{X}_t:t\geq 0\}$. Using this distribution and the storage policy, we can compute the stationary distribution for the storage distance. Now the stationary distribution for the retrieval distance is identical to that of the storage distance since in every busy cycle, exactly the same storages and retrievals are performed.

Since $D\bar{x}_0(\pi)$ doesn't depend on $\vec{X}_0$ for stationary policies π, we simply let $D(\pi)$ denote the average travel distance under the stationary policy π. Note that if π is a stationary policy and $D_*/r(\lambda)\to c_1$ with $0<c_1<\infty$, then we need only show that

$$\lim_{\lambda\to\infty}\frac{D(\pi)-D_*}{r(\lambda)}=0 \qquad (9)$$

to show that π is asymptotically efficient.

3. Closest Open Location(COL) Storage Policy

Assume that items arrive according to a Poisson process with rate λ and each arriving item is stored in the first available(empty) location, and each stored item stays in a location exponentially distributed length of time with parameter μ. Then, from[10], the expected travel distance under COL storage

$$D(COL)=\sum_{m=0}^{\infty}B(m,\ \rho).$$

where $\rho=\lambda/\mu$.

Now consider the following point process on

$[0, 1)$. New points appear according to a Poisson process with rate 1 and are randomly placed on $[0, 1)$. The left most point is removed according to another independent Poisson process with rate 1. Let $M(x, t)$ denote number of points in $[0, x)$ at time t. Aldous [1] calls this process the ***geometric process***. Note that for fixed x, $M(x, t)$ acts like an $M/M/1$ queue with arrival rate x and service rate 1. Hence, the stationary distribution will satisfy

$$\Pr\{M(x)=i\}=(1-x)x^i,\ i=0,\ 1,\ \cdots$$

where $0\leq x<1$.

Let X_k be the position of the k^{th} point from the left in the geometric process. Then

$$\Pr\{X_k\leq x\}=\Pr\{M(x)\geq k\}=x^k.$$

Theorem 2(Aldous) *If L_n is the position of the n^{th} open location,*

$$\left(\frac{L_1}{\rho},\ \frac{L_2}{\rho},\ \cdots\right)\xrightarrow{D}(X_1,\ X_2,\ \cdots) \tag{10}$$

as λ goes to ∞.

As a consequence, we have

$$\frac{L_1}{\rho}\xrightarrow{D}X_1,$$

as $\lambda\to\infty$

where X_1 is uniformly distributed over $[0, 1)$. Thus, the first open location L_1 is approximately uniform on $[0, \rho]$ for large λ.

Since the random variables $\frac{L_1}{\rho}$ are uniformly integrable, we know

$$\frac{E[L_1]}{\rho}\to E[X_1]=\frac{1}{2} \tag{11}$$

as $\lambda\to\infty$, and hence

$$\lim_{\lambda\to\infty}\frac{\sum_{m=0}^{\infty}B(m,\rho)}{\rho}=\frac{1}{2}. \tag{12}$$

4. Zone-z Storage Policy

Assume that type $h[l]$ items arrive according to a Poisson process with rate $\lambda_h[\lambda_l]$, respectively. Each arriving type h item needs to be stored for a random time which is exponentially distributed with mean $1/\mu_h$. Similarly, type l items need to be stored for an exponentially distributed length of time with mean $1/\mu_l$. We assume that the arrival processes and storage times are mutually independent. The policy can be described as follows. Type h items are stored in the closest open location, but type l items are stored in the closest open location beyond location z. Thus, the first z locations are always reserved for high turnover items. This policy is sometimes used in practice to decrease travel distance.

We show that a Zone-z policy with zone size $\lfloor\rho_h\rfloor$ is asymptotically efficient for Poisson arrivals and linear storage.

Theorem 3 *Assume that we have Poisson arrivals, exponential storage times, and linear storage. Then the Zone-z policy with zone size $\hat{z}(\lambda)=\lfloor\rho_h\rfloor$ is asymptotically efficient, and $r(\lambda)=\lambda$. That is, $\frac{D_*}{\lambda}\to c_1$, $0<c_1<\infty$ and*

$$\frac{D(\hat{z}(\lambda))=\lfloor\rho_h\rfloor-D_*}{\lambda}\to 0 \tag{13}$$

as $\lambda\to\infty$, where $\lfloor x\rfloor$ is the greatest integer less than or equal to x.

Proof Recall that all the travel time of all policies for this system is bounded below by D_* given in (1). Now, D_*/λ converges to c_1 where

$$c_1=\frac{p^2}{2\mu_h}+(1-p)\left(\frac{p}{\mu_h}+\frac{1-p}{2\mu_l}\right).$$

Note that $0<c_1<\infty$. To complete the proof, we need only show that $D(\lfloor\rho_h\rfloor)/\lambda$ also converges to c_1. First consider type h items. Let $T_i^h(\lfloor\rho_h\rfloor)$ be the travel distance of the i^{th} type h item under the Zone-$\lfloor\rho_h\rfloor$ policy. Then

$$\begin{aligned}D^h(\lfloor\rho_h\rfloor)&=\Pr\{T_i^h(\lfloor\rho_h\rfloor)\\&\le\lfloor\rho_h\rfloor\}E[T_i^h(\lfloor\rho_h\rfloor)\mid T_i^h\le(\lfloor\rho_h\rfloor)\\&\le\lfloor\rho_h\rfloor]+\Pr\{T_i^h(\lfloor\rho_h\rfloor)>\lfloor\rho_h\rfloor\}\\&E[T_i^h(\lfloor\rho_h\rfloor)\mid T_i^h(\lfloor\rho_h\rfloor)>\lfloor\rho_h\rfloor]\end{aligned}$$

We will show tat the second term converges to zero as λ gets large. The conditional expectation in the second term is certainly smaller than $2\rho_h+\rho_l=O(\lambda)$. The probability of overflowing the high turnover zone $\Pr\{T_i^h(\lfloor\rho_h\rfloor)>\lfloor\rho_h\rfloor\}$ converges to zero. In particular from [10], the overflow probability is $O(1/\sqrt{\lambda})$. After dividing the product by λ, the last term is $O(1/\sqrt{\lambda})$ which becomes negligible as λ increases. Hence, we need only consider the first term. The first term is the same as the expected storage distance in an $M/M/\lfloor\rho_h\rfloor/\lfloor\rho_h\rfloor$ storage system for an arrival that finds a space. Hence, using the results in Section 3,

$$\frac{D^h(\lfloor\rho_h\rfloor)}{\lambda}\to\frac{p}{2\mu_h}$$

as λ goes to ∞.

For the type l items, if we follow similar procedure, we obtain

$$\frac{D^l(\lfloor\rho_h\rfloor)}{\lambda}\to\frac{p}{\mu_h}+\frac{1-p}{2\mu_h}$$

as λ goes to ∞.

Since $D(\pi)=pD^h(\pi)+(1-p)D^l(\pi)$,

$$\lim_{\lambda\to\infty}\frac{D(z(\lambda)=\lfloor\rho_h\rfloor)}{\lambda}=\frac{p^2}{2\mu_h}+(1-p)\left(\frac{p}{\mu_h}+\frac{1-p}{2\mu_l}\right)=c_1$$

Hence, $z(\lambda)=\lfloor\rho_h\rfloor$ is asymptotically efficient. Extending these results to the case of more than two types of items should not be hard.

Theorem 4 *If $\mu_h>\mu_l$, the COL Storage policy is not asymptotically efficient.*

Proof Let $r(\lambda)=\lambda$.

$$\lim_{\lambda\to\infty}\frac{D(COL)-D\check{x}_0(*)}{\lambda}=\lim_{\lambda\to\infty}\frac{D(COL)-D(\lfloor\rho_h\rfloor)}{\lambda}+\lim_{\lambda\to\infty}\frac{D(\lfloor\rho_h\rfloor)-D\check{x}_0(*)}{\lambda}$$

The second term converges to zero by Theorem 3. Now from Aldous [1], with the fact that $M/M/s/s$ is insensitive for generally distributed service time,

$$\lim_{\lambda\to\infty}\frac{D(COL)}{\lambda}=\frac{1}{2}\left(\frac{p}{\mu_h}+\frac{1-p}{\mu_l}\right)$$

and from the proof of Theorem 3

$$\lim_{\lambda\to\infty}\frac{D(\lfloor\rho_h\rfloor)}{\lambda}=\frac{p^2}{2\mu_h}+(1-p)\left(\frac{p}{\mu_h}+\frac{1-p}{2\mu_l}\right)=c_1.$$

Hence

$$\lim_{\lambda\to\infty}\frac{D(COL)-D(\lfloor\rho_h\rfloor)}{\lambda}=\frac{p(1-p)(\mu_h-\mu_l)}{2\mu_h\mu_l}>0.$$

Remark 1 *The improvement per item stored or item retrieved, using the Zone-$\lfloor\rho_h\rfloor$ policy instead of the COL policy with linear storage is on the order of*

$$\frac{p(1-p)(\mu_h-\mu_l)}{2\mu_h\mu_l}r(\lambda)=\frac{p(1-p)(\mu_h-\mu_l)}{2\mu_h\mu_l}\lambda$$

for λ large. Hence, the savings per unit time is on the order of

$$\frac{p(1-p)(\mu_h-\mu_l)}{\mu_h\mu_l}\lambda=\frac{p(1-p)(\mu_h-\mu_l)}{\mu_h\mu_l}\lambda^2$$

Remark 2 *The COL Storage policy is asymptotically efficient when $\mu_h=\mu_l$ for λ large. This makes sense since when $\mu_h=\mu_l$, COL is clearly optimal.*

Numerical examples are shown in Table 1.

5. Suggestions for Further Research

First, starting from liner storage rack, two types of items and Poisson arrivals, we should be able to extend the results to square storage racks with the Tchebychev travel metric, more than two types of items and batch arrivals. (Note that even if square storage rack and Tchebychev travel are assumed, the number of items in the system is the same as the case of linear storage($d_n=n$), only travel distances are different. And $r(\lambda)$ seems to be $\sqrt{\lambda}$.) Second, we have assumed that the arrival rate of items did not depend on the current inventory, and that demand rate was proportional to inventory. It would be interesting to generalize these to situations in which the arrival rate depended on the inventory and the demand might not be

〈Table 1〉 Expected Travel Distances for Zone-z; $\mu_h=1.0$

Parameters	COL	Simulation		Aympt.	Efficient	Bound
	ED(0)	z^*	ED(z^*)	$\hat{z}=\lfloor\rho_h\rfloor$	ED($\hat{z}$)	D.
$\lambda=40$, $p=0.4$, $\mu_l=0.4$	39.97	16	35.82	16	35.82	31.80
$\lambda=80$	77.60	33	67.95	32	68.01	62.60
$\lambda=120$	115.07	49	99.67	48	99.70	93.40
$\lambda=160$	152.63	66	131.37	64	131.40	124.20
$\lambda=200$	189.47	81	162.10	80	162.12	155.00
$\lambda=40$, $p=0.8$, $\mu_l=0.4$	28.12	34	24.84	32	24.86	22.20
$\lambda=80$	53.97	67	46.71	64	46.78	43.40
$\lambda=120$	79.32	100	68.26	96	68.36	64.60
$\lambda=160$	104.71	133	89.83	128	89.94	85.80
$\lambda=200$	130.34	166	111.38	160	111.49	107.00
$\lambda=40$, $p=0.4$, $\mu_l=0.2$	69.66	20	55.01	16	56.01	49.80
$\lambda=80$	136.68	38	105.31	32	107.07	98.60
$\lambda=120$	202.66	55	154.72	48	156.51	147.40
$\lambda=160$	268.89	72	204.08	64	206.12	196.20
$\lambda=200$	333.75	89	252.54	80	255.25	245.00
$\lambda=40$, $p=0.8$, $\mu_l=0.2$	38.05	38	27.43	32	28.15	24.20
$\lambda=80$	73.49	74	51.44	64	52.61	47.40
$\lambda=120$	107.91	108	74.96	96	76.46	70.60
$\lambda=160$	142.95	140	98.79	128	100.55	93.80
$\lambda=200$	177.77	173	122.37	160	124.29	117.00

simply proportional to inventory, e.g., independent of the inventory.

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