Journal of the Korean Institute
of Industrial Engineers
Vol. 18, No.2, Dec., 1992

Optimization of Layout Design in an AS/RS for Maximizing its Throughput Rate+

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ABSTRACT

In this paper, we address a layout design problem for determining a K-class-based dedicated storage layout in an automated storage retrieval system. K-class-based dedicated storage employs K zones in which lots from a class of products are stored randomly. Zones form a partition of storage locations.

Our objective function is to minimize the expected single command travel time, which is expressed as a set function of space requirements for zones, average demand rates from classes, and one-way travel times from the pickup/deposit station to locations.

We construct a heuristic algorithm based on analytical results and a local search method. the methodology deveolped can be used with easily-available data by warehouse planners to improve the throughput capacity of a conventional warehouse as well as an AS/RS.

1. Introduction

Automated storage/retrieval systems(AS/RS) are widely used in warehousing, and often found in manufacturing. An AS/RS is defined to be "a combination of equipment and controls which handles, stores, and retrieves material with precision, accuracy, and speed under a defined degree of automation" [5]. A single aisle of a unit load AS/RS basically consists of two stroage racks, an S/R (storage/retrieval) machine carrying one unit load at one time, and P/D (pickup/deposit) station.

The fundamental design issues in operating an AS/RS include maximizing throughput capacity which can be defined as the maximum number of transactions per unit time. For a singl aisle in a unit load AS/RS, the throughput capacity is the inverse of the average transaction time, i.e., the expected amount of time required for a storage and/or retrieval operation. The average transaction time typically depends upon the particular layout configuration of a storage rack as well as the S/R machine specifications.

In designing the storage layout configuration,

+This paper was supported by NON DIRECTED RESEARCH FUND, Korea Research, Foundation, 1991.

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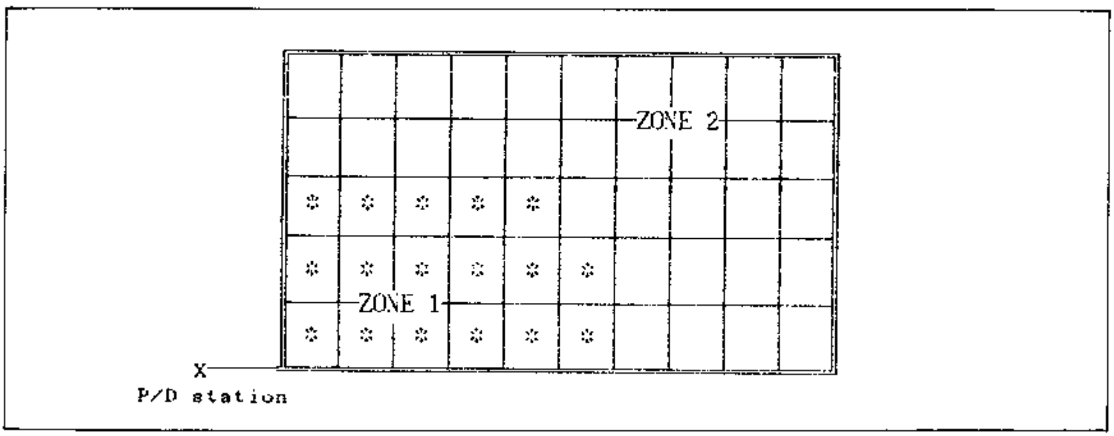
Tompkins and White [8] pointed out that class-based dedicated storage or simply class-based storage with randomized storage within each zone can yield both the throughput benefits of dedicated storage and the space benefits of randomized storage. Also they suggested that in order to achieve both benfits, three to five classes or zones may be defined. Figure 1 shows an example of 2-class-based storage layout configuration where the set of star signs forms zone 1.

There have been few methodologies for determining an appropriate class-based storage layout due to their mathematical intract ability. Hausman et al [3], Graves et al [1], and Schwartz et al [7] first addressed the design problem of determining a 2/3-class-based storage layout in a unit load AS/RS. Based on their continuous square-in-time rack, the EOQ model, and an ABC curve, they reported that significant reductions in the average transaction time were obtainable from a class-based storage policy. For example, the expected dual command (DC) travel time of 3-class-based storage was 56% of that of randomized storage given a 20%/80% ABC curve.

However, they implicitly assumed that total space requirement reserved for all the items is constant irrespective of the number of classes used, the particular partition of products and so on. This "constant-space" assumption is unrealistic. In this paper, the constant-space assumption will be relaxed and different approach will be developed.

In designing a class-based storage layout in a unit load AS/RS, the first question will be (1) "How many zones should we use in order to maximize the throughput capacity?" In fact, there is no reason that three to five zones must be optimal in terms of the average transaction time or other criteria. Suppose that an optimal number of zones is given. The remaining questions will be (2) "How should we partition the products into classes?" (3) "How should we determine each zone size, i.e., the number of storage bays which should be reserved for a class?" In this paper, all three of the questions will be investigated simultaneously.



[Figure 1] An example of a 2-class-based storage layout configuration

2. Problem Definition

Our stoage systems can be described as follows. Arriving replenishment lots in unit load, each containing a single product, are assigned randomly to open storage locations in each zone and the assigned locations are recorded. Retrievals are performed on first-in first-out (FIFO) basis. In order to facilitate the analysis, the following assumptions are made throughout this paper.

(1) The warehouse operation is based on the K-class-based storage policy and within each zone, a storage location is equally likely to be selected for a storage operation, i.e., random assignment rule(RAN rule) is used. Each storage location accommodates only one unit load.

The closest open location (COL) rule, where pallets are stored to the nearest or minimum-distance open storage location, is popular in practice [8]. However, Schwartz et al [7], and Rizo-Patron et al [6] showed by their simulation models that storage with COL is similar to storage with RAN if the storage level remains fairly constant and at a high level of utilization. Hence it can be generally said that there is alomst no difference between RAN and COL in terms of travel time.

(2) S/R machince can carry only one unit load at a time.

(3) The average demand rate for a product i, d_i unit loads/unit time, which is defined as the average number of retrievals per unit time, is given as a constant in advance and the average demand rate from class k, D_k, is obtained as

$$D_k = \sum_{i \in C_k} di$$

where C_k is the set of products assigned to class k. Since practically a class contains more than one product, it is assumed that $|C_k| \geq 1$ Where $|C_k|$ denotes the cardinality of set C_k.

(4) The one-way travel time from the P/D station to storage location j is given as t_j for j= 1, ⋯, M, where M is the number of storage locations. Without loss of generality, it is assumed throughout this paper that $t_1 \leq t_2 \leq \cdots \leq t_M$.

(5) The product overflow effect on the average travel time is trivial and ignored. Whenever space provided for a class is not equal to the maximum space requirement for a class, there is a nonzero probabilty that some or all pallets of an arriving lot cannot be stored in the main storage area. Those unassignable pallets may be stored temporarily in a buffer storage area and later, they can be stored in the main storage or directly retrieved from the buffer storage area. Since it is hard to consider this effect on the average travel time, it is assumed that by supplying appropriate amount of space, the probability of overflow in the main storage area is small enough so that it may be ignored.

(6) Long-term behavior of the storage system is considered. That is, instantaneous replenishment with infinite planning horizon are assumed. Consequently, the probability of visiting a storage location in class k is stationary and given as D_k/D where $D = D_1 + D_2 + \cdots + D_K$.

(7) Time intervals between successive retrievals of each product are random variable and identically independently distributed with an

arbitrary distribution.

Our objective is to minimize the expected single command (SC) travel time. In fact, the dual commands should be performed as often as possibel to reduce the average travel time, since the S/R machine is assumed to carry only one pallet at a time.

The reasons that only single commands are considered are as follows. First, the minimization of the expected DC travel time in class-based storage system is hard in terms of computational complexity. Han et al [2] observed that the minimization of the expected travel time between is equivalent to solving the traveling salesman problem (TSP), which is a well known NP-hard problem. It follows that the minimization of the expected DC travel time is at least as hard as TSP if multiple zones, and /or multiple open locations for storage operations are considered. Second, the expected SC travel time and the expected travel time between can be treated separately in system design and system operation. Hence, the design strategy, by which an optimal class-based storage layout can be obtained through minimizing the expected SC travel time, will be adopted in this paper.

We will derive the expected SC travel time given K zones using the assumptions discussed and describe our K-class-based storage problems. Let A_k be a set of storage locations assigned to zone k for $k=1, \cdots, K$ and $A=\{A_1, \cdots, A_K\}$. We assign the first $|A_1|$ storage locations based on the t_j-nondecreasing ordering to A_1 and in general, assign the k-th $|A_k|$ locations based on the t_j-nondecreasing ordering to A_k for $k=1, \cdots, K$ where $|A_k|$ denotes the zone size for class k. It was shown that the above storage location assignment is best in terms of the expected SC travel time under the constant-space assumption irrespective of K and the particular partition [9].

Assume that each storage location within a zone is equally likely to be selected for a storage or a retrieval operation. Then, given $\{A_k, k=1, \cdots, K\}$ and $\{t_j, j=1, \cdots, M\}$, the expected SC travel time to zone k, T_k can be expressed as:

$$T_k=\frac{2}{|A_k|}\sum_{j\in A_k} t_j \text{ for } k=1, \cdots, K \tag{1}$$

Since the probability of visiting zone k is D_k/D, the expected SC travel time given K zones, $E(SC_K)$, can be expressed as

$$E(SC_K)=\sum_{k=1}^{K}\frac{D_k}{D}T_k \tag{2}$$

Note that $E(SC_K)$ is the weighted average of $\{T_k, k=1, \cdots, K\}$ with weighting factors $\{D_k/D\}$. Replacing T_k with (1), $E(SC_K)$ can be further reduced to

$$E(SC_K)=\frac{2}{D}\sum_{k=1}^{K}\frac{D_k}{|A_k|}\sum_{j\in A_k} t_j \tag{3}$$

Note that $D_k/|A_k|$ can be interpreted as the rate of retrieval operations per storage location assigned to zone k.

It is possible to view t_j as a storage cost (or travel distance) of a pallet from the P/D station to storage location j. Then T_k can be interpreted as the average cost for storing a pallet in zone k and retrieving the pallet to the P/D station. $E(SC_K)$ denotes the average cost for storing a pallet to an arbitrary location and retrieving the pallet to the P/D station. Hence, the minimization of $E(SC_K)$ is exactly equivalent to the minimization of the average cost for storage and retrieval of a pallet if t_j is replaced with a cost for storing in location j.

It can be observed in (3) that $E(SC_K)$ depends on the estimation methods for $|A_k|$. The zone size must accommodate the maxi-mum of the aggregate inventory position of all the products assigned to class k if no space shortage is allowed whenever replenishment lots arrive. Since the maximun aggregate inventory position varies depending on the particular reordering scheduling, the zone size for class k, S_k, can be expressed as:

$$S_k=\min_{C_s}[\max_t \{\sum_{i\in C_k} X_i(t)\}]$$

where C_s denotes the set of reordering schedules and $X_i(t)$ denotes the inventory position of product i at time t.

The difficulty for estimating S_k is that the combination of the replenishment and retrieval processes leads to a time-varying space requirement. In addition, since there is nonzero probability that S_k becomes the sum of the replenishment lot sizes of the products assigned to class k, S_d, estimating S_k smaller than S_d involves risk; specifically the risk that when an inbound lot arrives, there might not be space available for storing some or all of the lots.

A practical approach to determining S_k is to select an acceptible service level, P_k, and select S_k such that $\lim_{t\to\infty} \text{Prob}\{S_k(t)\geq S_k\}\leq 1-P_k$ where $S_k(t)=\sum_{i\in C_k} X_i(t)$, representing the zone size for class k at time t. P_k can be interpreted as the expected proportion of time without space shortage.

Assume that $X_i(t)$'s are independent. Let X_i be a random variable corresponding to the limiting probability mass function of $X_i(t)$. By the central limit theorem for nonidentical random variables[4], it can be shown that

$$S_k=\sum_{i\in C_k} E(X_i)+Z(P_k)\sum_{i\in C_k} Var(X_i)$$

Since $E(X_i)$ and $Var(X_i)$ are not easily available in practice, we further assume that time intervals between successive retrievals of each product are random variable and identically independently distributed with an arbitrary distribution. Then the limiting probability of X_i will be discretely unifrom [4] and it follows that $E(X_i)=1/2Q_i$ and $Var(X_i)=1/12Q_i^2$ where Q_i dentes the replenishment lot size of product i for i=1, ⋯, n. Hence we estimate $|A_k|$ as

$$S_k=\frac{1}{2}\sum_{i\in C_k} Q_i+z(P_k)\{\frac{1}{12}\sum_{i\in C_k} Q_i^2\}^{1/2}$$

$$\text{where } |C_k|>1 \tag{4}$$

Note that the case, $S_k>\sum_{i\in C_k} Q_i$, will be excl-uded since this case rarely occurs in practice. Also, note that if S_k is not an integer, we round it up. Since each class must contain at least one product, our K-class-based storage problem can be described as follows:

PTG[K] : Given n products with $\{(Q_i, d_i), i=1, \cdots, n\}$, $\{t_j, j=1, \cdots, M\}$, and integer $K(\leq n)$, and $\{P_k, k=1, \cdots, K\}$, assign each product to one of the K classes such that we

$$\text{Minimize } Z_G=\frac{2}{D}\sum_{k=1}^{K}\frac{D_k}{S_k}\sum_{j\in A_k} t_j$$

subject to $|C_k|\geq 1$ for $k=1, \cdots, K$

$$S_k=\frac{1}{2}\sum_{i\in C_k} Q_i+z(P_k)\{\frac{1}{12}\sum_{i\in C_k} Q_i^2\}^{1/2}$$

for $k=1, \cdots, K$

$$D_k=\sum_{i\in C_k} d_i \qquad \text{for } k=1, \cdots, K$$

Note that an optimal partition, $\{C_k, k=1, \cdots, K\}$, should be decided in PTG[K]. If it is decided, then $\{S_k, k=1, \cdots, K\}$ and $\{A_k, k=1, \cdots, K\}$ can be determined immediatelty using $\{C_k, k=1, \cdots, K\}$ and (4). It can be observed that the total number of solutions for even two classes will be 2^n-2. This indicates that total enumeration is almost impossible in order to obtain an optimal solution to PTG[K].

3. Basic Properties and Algorithm

Our strong conjecture is that even 2-class-based storage problem is NP-hard. We will show basic properties for PTG[K] so that they can be used for developing a heuristic algorithm for PTG[K]. The algorithm will generate a set of candidate solutions using a starting solution and finally enumerate the set with a local search.

3.1 Basic Properties

t_i is basically composed of the pick-time, the travel time from the P/D station to a storage location, and the deposit time. The travel time includes the acceleration/deceleration time of the S/R machine. The following property implies that the P/D time and the acceleration/deceleration time can be subtracted in PTG[K] if they are constant.

Property 1. For a constant δ, replacing $\{t_j, j=1, \cdots, M\}$ with $\{t_i+\delta, j=1, \cdots, M\}$ does not affect an optimal solution to PTG[K].

Proof : Let $E(SC'_K)$ be the expected SC travel time with $\{t_j+\delta, j=1,..,M\}$. It is enough to show that minimizing $E(SC_K)$ is equivalent to minimizing $E(SC'_K)$. From(3), $E(SC'_K)=E(SC_K)+2\delta$. Q.E.D.

As shown in Figure 2, one-way travel times from the P/D station to storage locations in a typical AS/RS with 40 rows and 10 columns, can be well approximated by the simple linear regression equation, $t_i=0.0491j+5.6686$, with correlation coefficient 0.952. Hence, it can be said that replacing t_j with a linear function of j can give a near-optimal solution to PTG[K].

Without loss of generality, let $t_i=pj+q$ for constants p and q. Then (3) can be reduced to

$$E(SC_K)=p\sum_{k=1}^{K} |A_k| + \frac{P}{D}\sum_{K=1}^{K-1}\sum_{m=K+1}^{K} (|A_k| D_m - D_k |A_m|)+(p+2q) \quad (5)$$

From(5), it can be observed that an optimal solution to PTG[K] is independent of $\{t_j, j=1, \cdots, M\}$ if t_j is a linear function of j. In addition, the first summation of the right side, which represents the total space requirement given K zones, is not generally constant because of the space shrinkage effects from randomized storage. For example, the space shrinkage effects holds if $|A_k|$ is estimated as S_k.

Property 2. For a constant α, replacing $\{Q_i, i=1, \cdots, n\}$ with $\{\alpha Q_i, i=1, \cdots, n\}$ does not affect an optimal solution to PTG[K] if t_i is a linear function of j.

Proof : Let $E(SC_K'')$ be the expected SC travel time given $\{\alpha Q_i, i=1, \cdots, n\}$ and $\{t_i=pj+q, j=1, \cdots, M\}$. It is enough to show that minimizing $E(SC_K)$ is equivalent to minimizing $E(SC_K'')$. Replace Q_i with αQ_i. Using (4) and (5), we have, $E(SC_K'')=\alpha E(SC_K)+(1-\alpha)(p+2q)$. Q.E.D.

Suppose that the space requirement of product i is linearly proportional to Q_i with constant α for all i irrepective of whichever class it has been assigned to. That is, the space requirement of product i is a constant, αQ_i. Let r_i be αQ_i for all i. Then the space requirement for class k will be estimated as

$$|A_k| = \sum_{i \in C_k} r_i = R_k \qquad (6)$$

From Property 2, it can be said that if the value of S_k is close to that of R_k, then replacing S_k with R_k can give a near-optimal solution to PTG[K]. Thus, we introduce a simplified problem, PTL[K], described as below.

PTL[K] : Given n products with $\{(r_i, d_i), i=1, \cdots, n\}$ $\{t_j = pj+q, j=1, \cdots, M\}$, and an integer $K(\leq n)$, assign each product to one of the K classes such that we

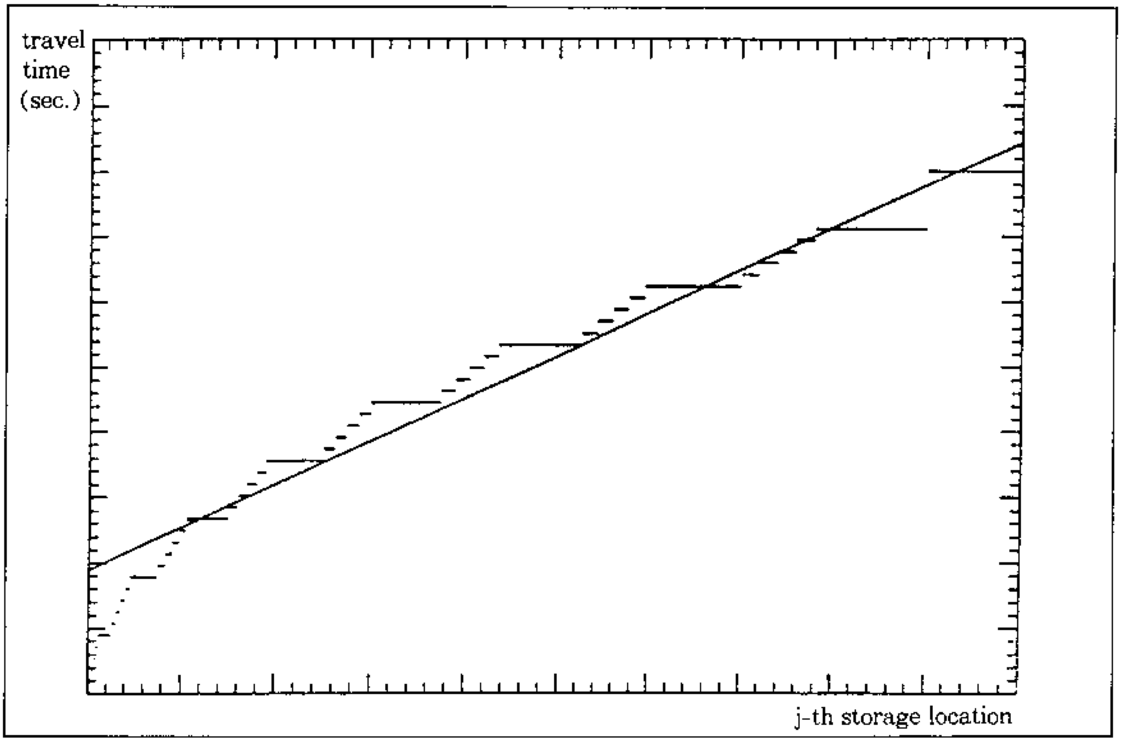
$$\text{Minimize } Z_{L} = \frac{2}{D} \sum_{k=1}^{K} \frac{D_k}{R_k} \sum_{j \in A_k} t_j$$

$$\text{subject to } |C_k| \geq 1 \qquad \text{for } k=1, \cdots, K$$

$$R_k = \sum_{i \in C_k} r_i \qquad \text{for } k=1, \cdots, K$$

$$D_k = \sum_{i \in C_k} d_i \qquad \text{for } k=1, \cdots, K.$$

PTL[K] is one of partition problems. In most cases, partition problems are hard in terms of time complexity. However, it turns out that PTL[K] can be solved efficiently in O(n) for K =2, and in general, in $O(\min\{n^{[k/4]} \lceil \log n \rceil, n^{n-k}\})$ for k=3, ⋯, (n−2) and in $O(n \lceil \log n \rceil)$ for k=(n−1), n [9].



[Figure 2] Plot of one-way travel time in a typical AS/RS with 40 rows and 10 columns. The size of a storage bay is 4x4 feet and the horizontal and vertical speeds of the S/R machine are 450 and 90 fpm respectively.

A necessary condition to PTL[K] is that an optimal solution to PTL[K] is one of the partitions based on a PAI-nonincreasing ordering where PAI_i (product activity index of product i) is defined as d_i/r_i. Note that replacing d_i/r_i with d_i/Q_i does not affect the optimality to PTG[K]. Hence given a PAI-nonincreasing ordering, an optimal solution to PTL[K] can be expressed as $X_L{}^*(K)=(N^L{}_1, N^L{}_2, \cdots, N^L{}_K)$, i.e., we assign $N^L{}_1$ products to class 1 and in general, $(N^L{}_k-N^L{}_{k-1})$ products to class k for $k=2, \cdots, K$ where $N^L{}_k$ denotes the number of products assigned to classes 1 through k. We will use $X_L{}^*(K)$ as a starting solution to PTG [K].

3.2 Algorithm

Alg(T-S) can be described as three phases; initialization, optimization, and termination as shown in Table 1.

In initialization phase, we take products by d_i/Q_i-nonincreasing order. We assign a big value to ESCG and set K as one.

In optimization phase, a starting solution, $X_L{}^*(K)$, to PTG[K] is obtained by solving PTL [K]. From $X_L{}^*(K)=(N^L{}_1 N^L{}_2, \cdots, N^L{}_K)$, we generate a set of candidate partitions, $C_K=\{X_G(K) \mid X_G(K)=(N_1, N_2, \cdots, N_K)\}$, such that for an integer δ and $k=1, \cdots, K$,

(1) $N_k=k$ if $N_k \leq 0$

(2) $N_K=(n-K+k)$ if $N_K \geq n$

(3) $N^L{}_k-\delta \leq N_k \leq N^L{}_k+\delta$

(4) $N_1<N_2<...<N_K=n$

Note that the maximum number of candidate solutions will be $(2\delta+1)^{K-1}$. Next, using local enumeration, we find a heuristic solution, $X_G{}^*(K)$, to PTG[K], which gives the minimum expected SC travel time, MESC, given the set C_k.

In termination phase, Alg(T-S) will be terminated if the currently computed $E(SC_K)$ is not less than $E(SC_1)$ in order to consider the space variation. Otherwise, we go back to step 2 again after increasing K by one and repeat the procedure described above.

4. Computational Results

Input data for 156 products with replenishment lot sizes and average daily demand rates were collected from a company in the United States. The storage bay size is 4 feet x 4 feet. The vertical and horizontal speeds of an S/R machine are given as 90 fpm and 450 fpm respectively.

Using Alg(T-S), some near-optimal solutions for given $K=1, \cdots, 7$ are summarized as shown in Table 2 assuming that $P_k=95\%$ for $k=1, \cdots, K$. Table 3 shows more detailed information on the near-optiman K-class-based storage layouts for $K=1, \cdots, 7$ including the space requirement and demand rate for each class.

It can be observed in Figure 3 that $E(SC_K{}^*)$ curve, which will be referred as a "T curve", is very insensitive to the number of classes close to the optimal number of classes, $K^*=6$. Also note that "S curve" is a strictly increasing function of number of classes used. From our computational experience, it turned out that for an integer constant K', $E(SC_K)$ is a decreasing function of k in the range of $k \leq K'$ and an increasing function of k in the range of $k \geq K'$.

〈Table 1〉 Alg(T-S) for PTG[K]

Initialization Phase

Step 1 : Take products by d_i/Q_i-nonincreasing order. Compute $E(SC_1)$

　　ESCG $\leftarrow$ big value

　　K $\leftarrow$ 2

Optimization Phase

Step 2 : (Obtain a starting solution to PTG[K])

　　Find an exact optimal solution, $X_L^*(K)=(N^L_1, N^L_2, \cdots, N^L_K)$, to PTL[K]

Step 3 : (Local Search Phase)

　　Generate a set of candidate solutions, C_K, from $X_L^*(K)$

　　Given C_K, find a heuristic solution, $X_G^*(K)=(N_1^*, N_2^*, \cdots, N_K^*)$, to PTG[K], which gives the minimum expected SC travel time, MESC.

Step 4 : (Find a near-optimal solution)

　　IF(MESC $<$ ESCG) then

　　　Begin

　　　　$K^* \leftarrow K$

　　　　$X_G^{**}(K) \leftarrow X_G^*(K)$ where $X_G^{**}(K)$ is a global near-optimal solution to PTG[K]

　　　　ESCG $\leftarrow E(SC_K^*)$

　　　End

Termination Phase

Step 5 : IF(ESCG $\geq E(SC_1)$) stop.

　　Otherwise K $\leftarrow$ K+1 and go to step 2

That is, $E(SC_k)$ was "discretely" convex.

Using the S-T curves, a desirable solution can be selected considering both the total space requirement and the expected SC travel time. For example, if the primary interest is to minimize the expected SC travel time, then the optimal number of classes will be six with $E(SC_6^*)=18.4413$ in this case study. On the other hand, if the space requirement is limited to less than 1800 storage locations, then the optimal number of classes will be three in terms of the expected SC travel time. If both travel time and storage space are of interest, the reasonable number of classes might be three. The optimal 3-class-based storage layout saves 37.98% in the expected SC travel time against the randomized storage layout even if S(3) is larger than S(1) by 135 storage bays where S(K) denotes the total space requirement given K. In order to save additional 4.16% in the expected SC travel time, we need additional 57 storage locations. In contrast, the optimal 4-class-based storage layout saves 40.57% against randomized storage at the cost of 192 storage locations. As discussed above, considering specific constraints in warehouses, warehouse managers can make use of the S-T curves in order to decide their appropriate class-based storage layout.

5. Conclusion

Considering both travel time and space requirement, a design procedure for determining a near-optimal class-based storage layout configuration was provided based on the analytical results. Alg(T-S) can be applied to storage systems which satisfies our assumptions, i.e., rack-supported storage systems, unit load conventional storage systems, and so on.

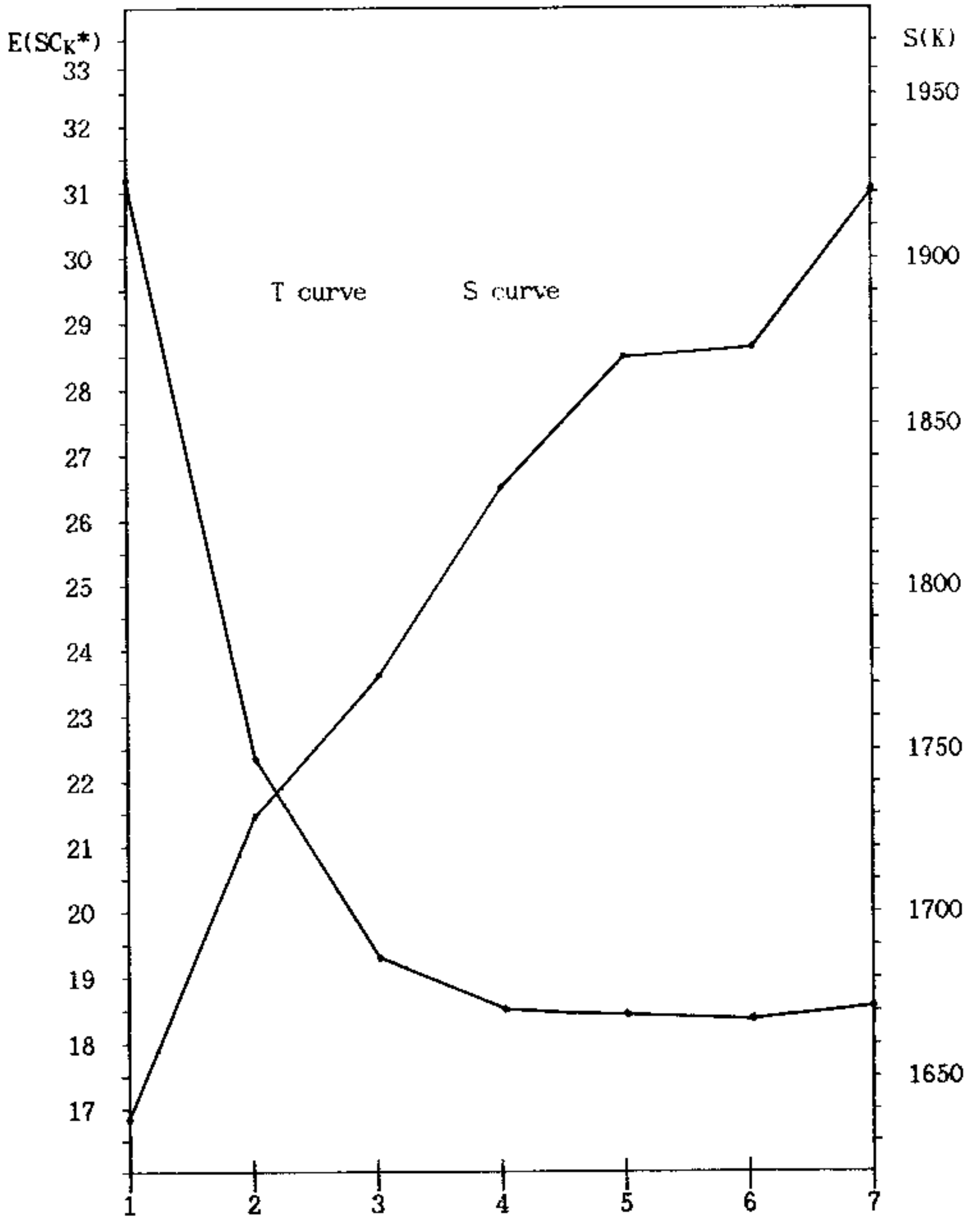
During the course of this research, several topics were recognized as potential directions for further research. (1) It is strongly conjectured that even PTG[2] is NP-hard. (2) Overflow effect may be investigated. (3) The system service level for K-class-based storage is $\prod_{k=1}^{K} P_k$. The case study could be more investigated under the assumption of the same system service level irrespective of the value of K. (4) Further research may be concentrated to the minimization of the expected DC travel time at the cost of time complexity.

〈Table 2〉 Near-optimal solutions to PTG[K] for K=1, ⋯, 7

K	$X_G^*(K)$	S(K)	$E(SC_K^*)$
1		1638	31.1990
2	(54, 156)	1729	22.4427
3	(23, 80, 156)	1773	19.3486
4	(15, 56, 109, 156)	1830	18.5429
5	(10, 31, 60, 113, 156)	1870	18.4786
6	(3, 12, 30, 58, 111, 156)	1872	18.4413
7	(4, 13, 30, 50, 76, 119, 156)	1921	18.5318

〈Table 3〉 Near-optimal K-class-based storage layouts for K=2, ⋯, 7

K	Class	$\|C_k\|$	S_k	D_k	T_k
2	1	54	489	30.3	16.6326
	2	102	1240	11.2	38.1763
3	1	23	100	16.1	7.2533
	2	57	691	20.5	23.4790
	3	76	982	5.0	41.3795
4	1	15	52	12.9	4.8410
	2	41	477	18.1	18.7595
	3	53	418	6.9	31.1451
	4	47	883	3.5	43.1861
5	1	10	44	12.1	4.4606
	2	21	162	8.1	12.2864
	3	29	390	11.7	22.5641
	4	53	427	6.4	32.9368
	5	43	847	3.1	44.0331
6	1	3	11	5.4	2.0364
	2	9	41	7.1	5.5963
	3	18	156	7.5	12.6085
	4	28	371	11.2	22.4115
	5	53	415	6.7	32.3727
	6	45	878	3.4	43.8257
7	1	4	13	6.2	2.2974
	2	9	40	6.4	5.7333
	3	17	155	7.5	12.6417
	4	20	298	8.9	21.3691
	5	26	307	6.2	29.5331
	6	43	288	3.3	35.7963
	7	37	820	2.9	45.1044



[Figure 3] The S-T curve for the case study

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