The Journal of the Korean Society of
Coastal and Ocean Engineers
Vol. 4, No. 4, pp. 225~230, December 1992

Review of Transformation of Wave Spectra Due to Depth and Current
水深 및 흐름에 의한 波浪 스펙트럼의 變化에 대한 考察

Kyung Duck Suh*
徐 慶 德*

Abstract □ An attempt is made to assemble and synthesize recent publications which may contribute to our capability for understanding the transformation of wave spectra in finite-depth water or in the presence of current. This review is limited essentially to the effects of shoaling and current on one-dimensional trnasformation of wave spectra and examining the adequacy of the approximation of irregular waves by a monochromatic wave in modeling of wave transformation in coastal areas.

要 旨 : 水深의 變化 또는 흐름의 영향에 의한 波浪 스펙트럼의 변환을 이해하기 위한 노력으로 최근 발표된 研究論文들의 綜合 및 分析을 시도하였다. 본 論文의 內容은 波浪 스펙트럼의 一次元 변환에 대한 淺水 및 흐름의 영향과 沿岸域에서의 波浪變換을 위한 水理 및 數値 模型에서 不規則波를 規則波로 簡略化시키는 문제의 適合性을 檢討하는데 局限되어 있다.

1. INTRODUCTION

Accurate estimation of design wave condition is essential for both economical and safe design of coastal structures. Long term wave data at the location of interest are needed to properly estimate the design wave condition for a given return period. More often than not, however, the location where the wave data are available is different from the location at which the design wave condition is desired. It then becomes necessary, using the available wave data, to somehow estimate the wave condition at deeper water from which the wave condition at the desired location can be calculated. If no such wave data are available, wave hindcasts are often made. In both cases it is necessary for a coastal engineer to calculate the transformation of waves from one depth to another to obtain the design wave condition at a given location.

The actual wave field in the sea is irregular. Due to its simplicity, however, previous coastal engineers have approximated the irregular wave field by a monochromatic wave and calculated its transformation in the area of interest. This assumption implies that the characteristics of the transformed monochromatic wave at a given location are not substantially different from those calculated by the transformation of the full-wave spectrum. Sometimes the random directional wave is assumed as the superposition of numerous monochromatic waves of different frequency, direction and phase, and the transformation of each of these is calculated separately to obtain the combined wave at any location by superposing the transformed monochromatic waves (see Chae and Song, 1986; and Panchang *et al.*, 1990, among others).

Recently, along with the development of three-dimensional wavemakers capable of generating random directional waves in a basin, the increased capacity of large computers and improved numerical modeling algorithms, and the development of in-site equipments for measurement and analysis of random directional waves, some experimental and numerical works have been reported for the transfor-

*韓國海洋硏究所 海洋工學硏究室(Ocean Engineering Laboratory, Korea Ocean Research & Development Institute, Ansan P.O.Box 29, Seoul 425-600, Korea)

mation of irregular waves in coastal areas or in wave basins with bathymetric change. In this paper, we review some of these works with emphasis on the problems associated with the widely used existing methods for calculating wave transformation in coastal areas for the purpose of raising some questions and proposing desirable directions if possible. This paper deals with two different subjects. In the first part, the effects of shoaling and current on the change of wave spectra are discussed. The second part deals with the problems associated with the approximation of irregular waves by a monochromatic wave in modeling the transformation of waves in coastal areas.

2. EFFECT OF SHOALING ON WAVE SPECTRUM

The unrefracted shoaling of a monochromatic wave train can be calculated using linear wave theory as

$$H_2 = H_1\sqrt{\frac{C_{g1}}{C_{g2}}} \tag{1}$$

in which C_g is the wave group velocity, and the subscripts, 1 and 2, denote the locations of different depths. This linear wave shoaling relationship, giving slight decrease of wave height in the intermediate depth and monotonous increase after that when waves propagate from deep water to shallow water, has been shown to provide reasonable estimates for monochromatic waves generated in laboratory wave tanks. Guza and Thornton (1980) have used this relationship for calculating the transformation of wave spectra at a California beach by describing the local sea-surface elevation as the superposition of numerous independent sinusoids, showing that the significant wave heights calculated by the linear theory generally agree with the measured data for depths greater than about 3 m. The calculated significant wave heights increase monotonically from 10 m depth to about 1 m depth. They also found that between 2 and 3 m depth the linear theory overpredicts the wave height, even though the visually observed breakpoint was generally between 1 and 2 m depth.

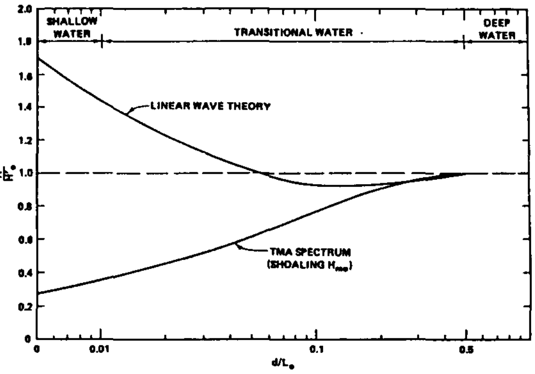

Fig. 1. Shoaling of significant wave heights calculated by linear theory and shallow-water TMA spectrum (after Hughes and Miller (1987)).

On the other hand, starting from the TMA spectrum (see Bouws *et al.* (1985) and Hughes (1984)), an equilibrium spectrum in finite-depth water, Hughes and Miller (1987) have proposed a simple relationship for calculating the significant wave heights of wave spectra shoaled in a finite-depth region:

$$H_{mo1} = H_{mo2}\left(\frac{L_{p1}}{L_{p2}}\right)^{3/4} \tag{2}$$

in which H_{mo} is the energy-based significant wave height determined as four times the square root of the area under the energy spectrum, L_p is the wave length associated with the spectral peak frequency from the linear dispersion relationship, and again the subscripts, 1 and 2, refer to the locations of different depths. The above TMA shoaling relationship, differing from the linear wave theory, gives monotonous decrease of wave height from deep water to shallow water as shown in Fig. 1. The TMA shoaling relationship has been proved to perform better than the linear wave theory for locally generated wind seas and laboratory waves for high energy conditions by Vincent (1984) and Hughes and Miller (1987). For low-steepness swell waves of longer wave periods, however, Hughes and Miller found that the TMA shoaling relationship underpredicts the measured data. Turning back to the field experiment of Guza and Thornton (1980), the disparity between the computed and the measured wave heights in the region between 2 and 3 m depth may be attributed to the failure of linear theory due to the saturation of wave energy in that region.

Conclusively, caution should be made in choosing the method for calculating transformation of wave spectra in shallow water depending on whether they are saturated to reach the equilibrium state or not. Linear theory may give reasonable estimates for the transformation of unsaturated wave spectra, whereas the TMA spectrum should be used for saturated wave spectra. It is noticeable that the design wave conditon is usually associated with the high energy conditions such as typhoon or hurricane in which the wave spectrum is saturated to reach an equilibrium state.

3. EFFECT OF CURRENT ON WAVE SPECTRUM

Ignoring energy dissipation due to wave breaking or botton friction, it is commonly known that when waves encounter a following current the wave height and length decreases and increases, respectively, due to the stretching effect, and the reverse occurs for an opposing current. However, when the bottom roughness increases so that the energy dissipation due to wave-current interacting flow is significant, wave heights often increase when waves are propagating on a following current, and, conversly, they decrease in the presence of an opposing current. These phenomena, looking anomalous at first sight, have been shown to occur by the laboratory experiments of Kemp and Simons (1982 and 1983) and Simons *et al.* (1988), who found that attenuation rates go down if waves propagate with a following current, but that attenuation increases when waves move onto an opposing current. Simons *et al.* additionally proposed a simple method to express the attenuation rates in terms of a wave-current friction factor. Similar results have been reported by the laboratory experiment of Brevik and Aas (1980) and in the field measurement in Chesapeake Bay by Boon *et al.* (1990).

In deep water where the effect of bottom on surface gravity waves is negligible, wave attenuation is governed mainly by wave breaking near water surface, known as white capping. There have been a number of studies for dealing with the change of deep-water wave spectrum due to the presence of current. Ignoring wave breaking and using the energy balance (Huang *et al.*, 1972) or the conservation of wave action (Hedges *et al.*, 1985), it has been shown that the wave spectrum, $S(\omega_a, U)$, under the influence of current, is related to $S_o(\omega_a)$, the spectrum in quiscent water without current, as

$$S(\omega_a, U)=\frac{S_o(\omega_a)}{\left(1+\frac{U\omega_r}{g}\right)^2\left(1+\frac{2U\omega_r}{g}\right)} \tag{3}$$

in which g is the gravitational acceleration, U is the vertically uniform current velocity in the direction of wave propagation, and the apparent angular frequency, ω_a, in the stationary frame of reference is related to the intrinsic wave frequency, ω_r, in the frame of reference moving with the current by

$$\omega_a=\omega_r+kU \tag{4}$$

where $k=\omega_r^2/g$ is wavenumber.

When waves propagate onto an opposing current, their growth may be limited by the breaking process. The upper limit to the wave spectrum has been given by Headges (1981), by the use of Phillips' (1980) equilibrium range constraint, as follows:

$$S_{ER/I}(\omega_a, U)=\frac{\beta g^2}{\omega_r^5}\left(1+\frac{2U\omega_r}{g}\right)^{-1} \tag{5}$$

in which the subscript, ER, refers to the equilibrium range, and β is the equilibrium range constant found to be between 0.008 and 0.015 in the absence of current. Hedges *et al.* (1985) have experimentally shown that the above relationship can successfully predict the spectral energy density in the equilibrium range of wave spectra.

Again by reexamining and extending the Phillips' equilibrium range concept in deep water, Kitaigordskii *et al.* (1975) have proposed an expression for the equilibrium range of wave spectra in water of finite depth and in the presence of current, which later provided the theoretical basis for the development of the TMA shallow-water spectrum of Bouws *et al.* (1985). In the limit of deep water, their expression retaining only the linear term for unidirectional waves can be written as

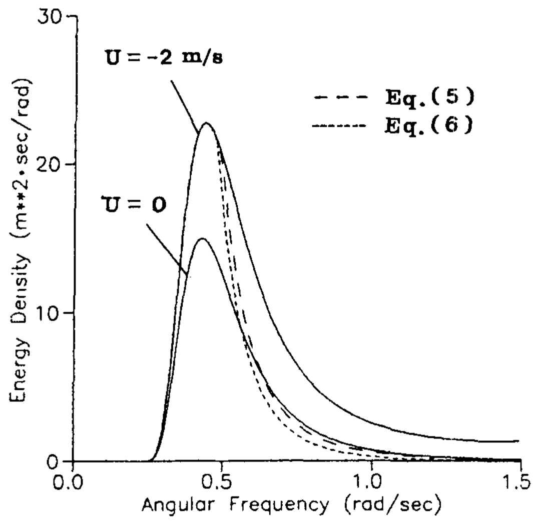

Fig. 2. Change of a Pierson-Moskowitz spectrum due to an opposing current.

$$S_{ERK}(\omega_a, U)=\frac{\beta g^2}{\omega_a^5}\left(1+\frac{3U\omega_a}{g}\right) \quad (6)$$

Comparing this with Eq. (5) given by Hedges (1981), at first sight, one may think that these two expressions will give results opponent each other because of the exponent. -1, in Eq. (5). However, it should be noted that in Eq. (6) ω_a is used in places of ω_r in Eq. (5).

In Fig. 2, the changes of a Pierson-Moskowitz spectrum corresponding to $U_{10}=20$ m/sec and $\beta=0.008$ due to an opposing current of -2 m/sec, calculated by Eqs. (3) to (6), are presented. Whenever S_{ERH} or S_{ERK} was less than the value calculated by Eq. (3), the former was applied for the calculation of the spectral energy density in the equilibrium range. When the effect of wave breaking is ignored, the energy density in the presence of opposing current increases in the entire frequency range compared with the zero-current case. For the following current, of course, the energy density will decrease. In the equilibrium range of high frequency, the energy density significantly decreases due to wave breaking. The energy density in the equilibrium range calculated by Eq. (6) (Kitaigordskii *et al.*, short-dashed line in Fig. 2) is somewhat less than that of Hedges (long-dashed line), though being very close each other.

4. APPROXIMATION OF IRREGULAR WAVES BY A MONOCHROMATIC WAVE

Often an irregular wave field in the offshore area is approximated by a monochromatic wave of representative height, period, and direction. The transformation of the monochromatic wave due to such important phenomena as refraction and diffraction is then calculated to determine the corresponding wave parameters in the shallow water. This approximation implies that the characteristics of the transformed monochromatic wave at a given location are not substantially different from those calculated by the transformation of the full-wave spectrum. A number of numerical models has been developed to calculate the transformation of a monochromatic wave. These include the development of the mild-slope equation by Berkhoff (1972), its parabolic approximation (Radder, 1979; Kirby and Dalrymple, 1983; Liu and Tsay, 1984), its change to the form of a hyperbolic system (Copeland, 1985; Madsen and Larsen, 1987), angular spectrum models (Dalrymple and Kirby, 1988; Dalrymple *et al.*, 1989; Suh *et al.*, 1990), and models solving the mild-slope equation by different methods (Ebersole, 1985; Panchange *et al.*, 1987).

Even though these numerical models have been shown to be extremely useful in modeling the transformation of monochromatic waves in a wide variety of situations, it has been questionable how accurately the wave parameters calculated by the monochromatic-wave approximation can represent those of real random waves. Goda (1985) has shown that there can be substantial differences in both wave height and period between the monochromatic-wave approximation and the spectral calculation when they are applied to the simulation of wave refraction over a submerged shoal or wave diffraction behind breakwaters.

In order to learn more about this difference, Vincent and Briggs (1989) have performed hydraulic model experiments for the transformation of both monochromatic and directionally-spread irregular waves passing over an elliptic shoal resting on a flat bottom which later have been used by Pan-

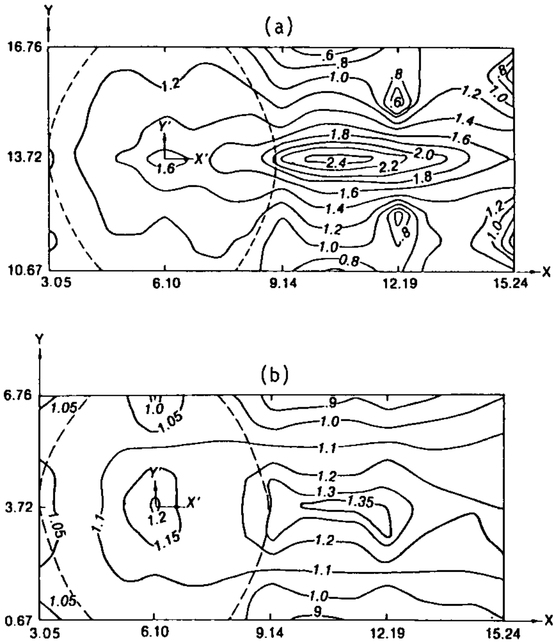

Fig. 3. Comparison of normalized wave heights of monochromatic and irregular waves: (a) monochromatic, (b) irregular (after Vincent and Briggs (1989)).

chang *et al.* (1990) and Suh and Dalrymple (1992) to compare with the predictions of a parabolic equation model and an angular spectrum model, respectively. They used the TMA shallow-water spectrum as the target frequency spectrum and a wrapped normal function (Borgman, 1984) as the directional spreading function. They tested a number of cases of various combination of total energy and peakedness of frequency spectrum and directional spreading function. The wave height of the monochromatic wave was taken as $H_{mo}/\sqrt{2}$ or H_{mo}, and the corresponding wave period was taken as the period at the peak of the frequency spectrum.

The contour plots in Fig. 3 shows a comparison of normalized (with respect to the incident wave height) wave heights of the monochromatic and directionally-spreading irregular waves in the vicinity of the elliptic shoal (designated by dashed lines). Waves propagate from left to right in the figures. The monochromatic-wave approximation gives much stronger wave focusing behind the shoal than the directionally-spreading irregular waves. The similar results have been obtained by the numerical models of Panchang *et al.* (1990) and Suh and Dalrymple (1992). Vincent and Briggs (1989) stated that in terms of the normalized wave height the monochromatic waves deviate by as much as 50 to over 100% from irregular waves for typical spectral shapes and directional spreads.

5. CONCLUSION

An attempt has been made to assemble and synthesize recent publications which may contribute to our capability for understanding the transformation of wave spectra in finite-depth water or in the presence of current. This review has been limited essentially to the effects of shoaling and current on one-dimensional transforamtion of wave spectra and examining the adequacy of the approximation of irregular waves by a monochromatic wave in modeling of wave transformation in coastal areas. Conventional approaches such as linear theory and monochromatic-wave approximation may give erroneous results in some circumstances. Further studies will require combined efforts of coastal engineers and oceanographers in numerical and hydraulic modeling together with comprehensive field experiments.

ACKNOWLEDGEMENT

This work is a part of the project, "Wave Transformation Due to Wave Breaking", funded by the Korea Ocean Research & Development Institute under Project No. PE00336.

REFERENCES

Berkhoff, J.C.W., 1972. Computation of combined refraction-diffraction. *Proc. 13th Coast. Engrg. Conf.*, ASCE, 471-490.

Boon, J.D., Kimball, S.M., Suh, K.D. and Hepworth, D.A., 1990. Chesapeake Bay wave climate: Thimble Shoals wave station. Report and summary of wave observations September 27, 1988 through October 17, 1989, *Data Rep. No. 32*, Virginia Inst. of Marine Sci., Gloucester Point, Va.

Borgman, L.E., 1984. Directional spectrum estimation for the S_{xy} gauges. *Technical Report*, US Army Corps of Engrs., Wtrways. Experiment Station, Coast. Engrg. Res. Ctr., Vicksburg, Miss., 1-104.

Bouws, E., Günther, H., Rosenthal, W. and Vincent, C.L., 1985. Similarity of the wind wave spectrum in finite depth water 1. Spectral form, *J. Geophys. Res.*, **90**(C1): 975-986.

Brevik, V. and Aas, B., 1980. Flume experiment on waves and currents. I. Rippled bed, *Coast. Engrg.*, 3: 149-177.

Chae, J.W. and Song, W.O., 1986. Current-depth refraction of directional wave spectra, *Proc. 5th Congress Asian and Pacific Regional Division*, IAHR, **3**: 15-34.

Copeland, G.J.M., 1985. A practical alternative to the mild-slope equation, *Coast. Engrg.*, **9**: 125-149.

Dalrymple, R.A. and Kirby, J.T., 1988. Models for very wide-angle water waves and wave diffraction, *J. Fluid Mech.*, **192**: 33-50.

Dalrymple, R.A., Suh, K.D., Kirby, J.T. and Chae, J.W., 1989. Models for very wide-angle water waves and wave diffaction. Part 2. Irregular bathymetry, *J. Fluid Mech.*, **201**: 299-322.

Ebersole, B.A., 1985. Refraction-diffraction model for linear water waves, *J. Wtrwy. Port, Coast. and Oc. Engrg.*, **111** (6): 939-953.

Goda, Y., 1985. *Random seas and design of maritime structure*, Univ. of Tokyo Press.

Guza, R.T. and Thornton, E.B., 1980. Local and shoaled comparisons of sea surface elevations, pressures, and velocities, *J. Geophys. Res.*, **85**(C3): 1524-1530.

Hedges, T.S., 1981. Some effects of currents on wave spectra, *Proc. 1st Indian Conf. in Oc. Engrg.*, Indian Inst. of Technology, Madras, **1**: 30-35.

Hedges, T.S., Anastasiou, K. and Gabriel, D., 1985. Interaction of random waves and currents, *J. Wtrwy. Port, Coast. and Oc. Engrg.*, **111**(2): 275-288.

Huang, N.E., Chen, D.T., Tung, C.-C. and Smith, J.R., 1972. Interactions between steady non-uniform currents and gravity waves with applications for current measurements, *J. Phys. Oceanogr.*, **2**, 420-431.

Hughes, S.A., 1984. The TMA shallow-water spectrum description and applications, *Tech. Rep. CERC-84-7*, US Army Corps of Engrs., Wtrways. Experiment Station, Coast. Engrg. Res. Ctr., Vicksburg, Miss.

Hughes, S.A. and Miller, H.C., 1987. Transformation of significant wave heights, *J. Wtrwy. Port, Coast. and Oc. Engrg.*, **113**(6): 588-605.

Kemp, P.H. and Simons, R.R., 1982. The interaction between waves and a turbulent current: waves propagating with the current, *J. Fluid Mech.*, **116**: 227-250.

Kemp, P.H. and Simons, R.R., 1983. The interaction of waves and a turbulent current: waves propagating against the current, *J Fluid Mech.*, **130**: 73-89.

Kirby, J.T. and Dalrymple, R.A., 1983. A parabolic equation for the combined refraction-diffraction of Stokes waves by mildly varying topography, *J. Fluid Mech.*, **136**: 453-466.

Kitaigordskii, S.A., Krasitskiii, V.P. and Zaslavskii, M.M., 1975. On Phillips' theory of equilibrium range in the spectra of wind-generated gravity waves, *J. Phys. Oceanogr.*, **5**: 410-420.

Liu, P.L.-F. and Tsay, T.K., 1984. Refraction-diffraction model for weakly nonlinear water waves, *J. Fluid Mech.*, **141**: 265-274.

Madsen, P.M. and Larsen, J., 1987. An efficient finite-difference approach to the mild-slope equation, *Coast. Engrg.*, **11**: 329-351.

Panchang, V., Cushman-Roisin, B. and Pearce, B.R., 1987. A finite-difference model for combined refraction-diffraction of water waves, *Proc. ASCE Specialty Conf. on Coast. Hydrodynamics* (ed. R.A. Dalrymple), 60-70.

Panchang, V.G., Wei, G., Pearce, B.R. and Briggs, M.J., 1990. Numerical simulation of irregular wave propagation over shoal, *J. Wtrwy. Port, Coast. and Oc. Engrg.*, **116**(3): 324-340.

Phillips, O.M., 1980. *The Dynamics of the Upper Ocean*. 2nd ed., Cambridge University Press.

Radder, A.C., 1979. On the parabolic equation method for water-wave propagation, *J. Fluid Mech.*, **95**: 159-176.

Simons, R.R., Grass, A.J. and Kyriacou, A., 1988. The influence of currents on wave attenuation, *Proc. 21st Coast. Engrg. Conf.*, ASCE, 363-376.

Suh, K.D. and Dalrymple, R.A., 1992. Application of an angular spectrum model to simulation of irregular wave propagation, *J. Wtrwy. Port, Coast. and Oc. Engrg.* (tentatively approved for publication).

Suh, K.D., Dalrymple, R.A. and Kirby, J.T., 1990. An angular spectrum model for propagation of Stokes waves. *J. Fluid Mech.*, **221**: 205-232.

Vincent, C.L., 1984. Shallow water waves: a spectral approach, *Proc. 19th Coast. Engrg. Conf.*, ASCE, 370-382.

Vincent, C.L. and Briggs, M.J., 1989. Refraction-diffraction of irregular waves over a mound, *J. Wtrwy. Port, Coast. and Oc. Engrg.*, **115**(2): 269-284.