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Eigensensitivity Synthesis and Its Applications

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Abstract

The new method, termed the substructural eigensensitivity synthesis method, utilizes the computational merits of the component mode synthesis technique and of sensitivity analysis for the design sensitivities of the dynamic characteristics of substructurally combined structures. It is shown that the eigensensitivities of the entire structure can be obtained by synthesizing the substructural eigensolution and the sensitivities of the eigensolution for the design variables of the modifiable substructure.

The sensitivities of the eigenvalues and eigenvectors obtained by the new method are compared to exact eigensolutions in terms of accuracy and computational efficiency. The small errors in eigensensitivity due to the truncation of higher modes remain within a manageable and permissible range for further analysis. The advantage of the newly proposed method as compared to the direct application of sensitivity analysis of the whole structure is demonstrated through examples.

요 약

한 구조물을 제작 설계할 경우, 단번에 모든 설계 조건들을 다 만족시키기는 쉽지 않고 몇번의 설계 변경을 통하게 된다. 이 경우 만족스러운 기능을 위한 설계 변경 방향 설정이 필요하고 그 설계 변경량도 결정해 주어야 한다. 마찬가지로 구조물의 진동 등 동적 거동 변경을 위해서는 어느 부분의 미소 변화에 대한 동적 성능 변화량(동특성 민감도)을 쉽게 추출해 낼 수 있어야 한다.

이를 위해 본고에서는 부분 구조물들로 조합된 전체 구조물에 대하여 설계변수에 대한 동적성능의 민감도를 좀더 쉽게 계산할 수 있는 새로운 방법을 제시하였다. 이 방법은 부분 구조 진동형 합성법의 계산 이점을 충분히 이용하였으며 부분 구조 동특성 민감도 합성법이라고 일컬어진다. 본고의 주 내용으로, 전체 구조물의 진동 특성치 민감도가 부분 구조물의 진동 특성치 및 변경가능 부분 구조물의

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내부 진동 민감도 등으로부터 합성되어 구해질 수 있음을 나타내 보였다. 예시 구조물로서 트러스 구조물이 선정되었고 통상적인 방법과 여기서 새로이 제시된 방법에 의한 해석결과 들을 비교하였으며, 결론적으로 새로운 방법이 정확도와 계산효율 면에서 충분한 이점이 있음을 보여 주었다.

1. Introduction

In recent years, the field of structural or mechanical design modification or optimization has grown and gained broad popularity. The substructure synthesis method has been developed to alleviate the high computational cost and the difficulty in re-designing the model with partial modifications. As a main stream of substructure synthesis method, the component mode synthesis (CMS) methods were developed by adopting the constrained modes as the substructural vibration modes[1].

On the other hand, among the various techniques for efficient re-design of large complex structures, a more rigorous approach to predict the system's dynamic characteristics with respect to design modifications emerged in the form of sensitivity analysis[2,3]. After a famous paper by Fox and Kapoor[4], Nelson[5] developed better approach which expresses the eigensensitivity in terms of corresponding eigenvalue and the associated eigenvector that is to be differentiated.

Even though there is some feasibility in implanting CMS into sensitivity analysis for optimum dynamic structural design, there are only a few paper in this field such as the work of Hasselman and Hart[6], and the Huang and Huang's paper[7]. However, those works result in a less efficient and accurate method than Nelson's method. For this reason, the development of a scheme to efficiently calculate the accurate eigensensitivity of a structural system which can be used in optimal design by using the CMS method would be the main scope in the subsequent sections of this paper. The second objective is to establish a design scheme for structural modifications taking advantage of the substructurally synthesized sensitivities.

2. CMS, Sensitivity and the Combination

2.1 Formulation of the CMS Method

For simple derivation, a structure with only two substructures, consisting of the unchanged original system Sub 0 and a modifiable appendant system Sub1, will be considered. Assuming that damping is negligible, after partitioning them into boundary(junction) degrees of freedom and interior degrees of freedom with superscripts b and i respectively, the governing equation can be written as :

$$\begin{bmatrix} [m^{bb}] & 0 \\ 0 & [m^{ii}] \end{bmatrix}_J \begin{Bmatrix} \ddot{u}^b \\ \ddot{u}^i \end{Bmatrix}_J + \begin{bmatrix} [K^{bb}] & [k^{bi}] \\ [k^{ib}] & [k^{ii}] \end{bmatrix}_J \begin{Bmatrix} u^b \\ u^i \end{Bmatrix}_J = \{0\} \tag{1}$$

where, subscript J(0 or 1) stands for substructures SubJ. The equation for free vibrations of the entire structure gives ;

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{2}$$

where

$$[M] = \begin{bmatrix} [m^{bb}]_1 + [m^{bb}]_0 & 0 & 0 \\ 0 & [m^{ii}]_1 & 0 \\ 0 & 0 & [m^{ii}]_0 \end{bmatrix} \tag{3}$$

$$[K] = \begin{bmatrix} [k^{bb}]_1 + [k^{bb}]_0 & [k^{bi}]_1 & [k^{bi}]_0 \\ [k^{ib}]_1 & [k^{ii}]_1 & 0 \\ [k^{ib}]_0 & 0 & [k^{ii}]_0 \end{bmatrix} \tag{4}$$

$$\{x\} = \{u^b, u^i_1, u^i_0\}^T \tag{5}$$

Introducing the fixed interface normal modes $[\phi^{iN}]$, the final eigensystem equation for the total structural system with boundary degrees of freedom $\{u^b\}$ and generalized coordinates for SubJ, $\{p\}_J$, can be obtained for the case of a lumped mass matrix as follows[8,9] :

$$[[\bar{K}] - \bar{\lambda}[\bar{M}]]\ \{\bar{X}\} = \{0\} \tag{6}$$

where

$$[\bar{M}] = \begin{bmatrix} [\bar{m}^{bb}]_1 + [\bar{m}^{bb}]_0 & [\bar{m}^{bi}]_1 & [\bar{m}^{bi}]_0 \\ [\bar{m}^{ib}]_1 & [\bar{m}^{ii}]_1 & 0 \\ [\bar{m}^{ib}]_0 & 0 & [\bar{m}^{ii}]_0 \end{bmatrix} \tag{7}$$

$$[\bar{K}] = \begin{bmatrix} [\bar{k}^{bb}]_1 + [\bar{k}^{bb}]_0 & 0 & 0 \\ 0 & [\bar{k}^{ii}]_1 & 0 \\ 0 & 0 & [\bar{k}^{ii}]_0 \end{bmatrix} \tag{8}$$

$$\{X\} = \{u^b, p_1, p_2\}^T \tag{9}$$

and the relation between $\{u\}_J$ and the generalized coordinates $\{p\}_J$ is expressed with the constraint mode $[\phi^{iC}]_J$:

$$[\phi^{iC}]_J = -[k^{ii}]_J^{-1}[k^{ib}]_J \tag{10}$$

$$\{u\}_J = \begin{Bmatrix} u^b \\ u^i \end{Bmatrix}_J = \begin{bmatrix} [I] & 0 \\ [\phi^{iC}]_J & [\phi^{iN}]_J \end{bmatrix} \begin{Bmatrix} u^b \\ p \end{Bmatrix}_J, \; J = 0,1 \tag{11}$$

$$[\bar{m}^{bb}]_J = [m^{bb}]_J + [\phi^{iC}]_J^T[m^{ii}]_J[\phi^{iC}]_J \tag{12}$$

$$[\bar{m}^{bi}]_J = [\bar{m}^{ib}]_J^T = [\phi^{iC}]_J^T[m^{ii}]_J[\phi^{iN}]_J \tag{13}$$

$$[\bar{m}^{ii}]_J = [\phi^{iN}]_J^T[m^{ii}]_J[\phi^{iN}]_J \tag{14}$$

$$[\bar{k}^{bb}]_J = [k^{bb}]_J + [k^{bi}]_J[\phi^{iC}]_J \tag{15}$$

$$[\bar{k}^{ii}]_J = [\phi^{iN}]_J^T[k^{ii}]_J[\phi^{iN}]_J \tag{16}$$

When $[\phi^{iN}]_J$ is a mass-normalized fixed interface mode of SubJ, then :

$$[\bar{m}^{ii}]_J = [I] \quad : \text{unit matrix} \tag{17}$$

$$[\bar{k}^{ii}]_J = [\lambda^i]_J \tag{18}$$

where $[\lambda^i]_J$ is the diagonalized eigenvalue matrix for the interior part with a fixed boundary. The total number of DOF of the final eigensystem equation, Eq.6, is determined by the number of chosen normal modes for the interior part and the number of boundary DOF.

2.2 Synthesis of Eigensensitivity

With respect to the design variable, v, the sensitivity equation of the above eigensystem for the case of distinct eigenvalues can be expressed by following Nelson's method[5] as :

$$\frac{\partial \bar{\lambda}_i}{\partial v} = \{\bar{X}\}_i^T \left(\frac{\partial[\bar{K}]}{\partial v} - \bar{\lambda}_i \frac{\partial[\bar{M}]}{\partial v} \right) \{\bar{X}\}_i \tag{19}$$

$$\frac{\partial\{\bar{X}\}_i}{\partial v} = \{V\}_i + c_i\{\bar{X}\}_i \tag{20}$$

or

$$\frac{\partial \bar{X}_i}{\partial v} = \begin{Bmatrix} V_A \\ 0 \\ V_B \end{Bmatrix}_i + C_i \begin{Bmatrix} \bar{X}_A \\ \bar{X}_k \\ \bar{X}_B \end{Bmatrix}_i \tag{21}$$

where the location k is selected such that the kth element of $\{\bar{X}\}_i$ has the largest absolute value. The partitioned complementary vectors with the pivotal location k, subvectors V_A and V_B, will be obtained from the following equations :

$$\begin{bmatrix} [\bar{K} - \bar{\lambda}_i\bar{M}]_{AA} & 0 & [\bar{K}-\bar{\lambda}_i\bar{M}]_{AB} \\ 0 & 1 & 0 \\ [\bar{K}-\bar{\lambda}_i\bar{M}]_{BA} & 0 & [\bar{K}-\bar{\lambda}_i\bar{M}]_{BB} \end{bmatrix} \begin{Bmatrix} V_A \\ V_k \\ V_B \end{Bmatrix}_i = \begin{Bmatrix} F_A \\ 0 \\ F_B \end{Bmatrix}_i \tag{22}$$

where

$$\begin{aligned} \{F\}_i &\equiv [[\bar{K}] - \bar{\lambda}_i[\bar{M}]] \frac{\partial\{\bar{X}\}_i}{\partial v} \\ &= -\left(\frac{\partial[\bar{K}]}{\partial v} - \frac{\partial\bar{\lambda}_i}{\partial v}[\bar{M}] - \bar{\lambda}_i \frac{\partial[\bar{M}]}{\partial v} \right) \{\bar{X}\}_i \end{aligned} \tag{23}$$

$$c_i = -\{\bar{X}\}_i^T[\bar{M}]\{V\}_i - \frac{1}{2}\{\bar{X}\}_i^T \frac{\partial[\bar{M}]}{\partial v}\{\bar{X}\}_i \tag{24}$$

The number of DOF for the eigensystem equation, Eq.6, and for the sensitivity equations, Eq. 19-20, can be drastically reduced by selecting some normal modes as basis vectors. A reasonable condensation of the system matrix size by CMS will result in significant computational savings during sensitivity synthesis. For the iterative design modification process, extra savings in computational steps can be expected by virtue of the simple recalculation of the eigensolutions only for the modifiable substructure.

To obtain the eigensensitivity, the rate of change of the stiffness and mass matrix of the combined structure should be calculated first[10].

$$\frac{\partial[\bar{K}]}{\partial v} = \begin{pmatrix} \frac{\partial[\bar{k}^{bb}]_1}{\partial v} & 0 & 0 \\ 0 & \frac{\partial[\lambda^i]_1}{\partial v} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{25}$$

$$\frac{\partial[\bar{K}]}{\partial v} = \begin{pmatrix} \frac{\partial[\bar{m}^{bb}]_1}{\partial v} & \frac{\partial[\bar{m}^{bi}]_1}{\partial v} & 0 \\ \frac{\partial[\bar{m}^{bi}]_1^T}{\partial v} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{26}$$

where

$$\frac{\partial[\bar{k}^{bb}]_1}{\partial v} = \frac{\partial[k^{bi}]_1}{\partial v} + \left[\frac{\partial[k^{bi}]_1}{\partial v}[\phi^{iC}]_1\right] + \left[\frac{\partial[k^{bi}]_1}{\partial v}[\phi^{iC}]_1\right]^T + [\phi^{iC}]^T_1\frac{\partial[k^{ii}]_1}{\partial v}[\phi^{iC}]_1 \tag{27}$$

$$\frac{\partial[\bar{\lambda}^i]_1}{\partial v} = \frac{\partial[\lambda^i]_1}{\partial v} = [\mathrm{diag}(\frac{\partial\lambda^i_1}{\partial v}, \frac{\partial\lambda^i_2}{\partial v}, \ldots, \frac{\partial\lambda^i_n}{\partial v})]_1 \tag{28}$$

$$\frac{\partial[\bar{m}^{bb}]_1}{\partial v} = \frac{\partial[m^{bb}]_1}{\partial v} + \left[\frac{\partial[\phi^{iC}]^T_1}{\partial v}[m^{ii}]_1[\phi^{iC}]_1\right] + \left[\frac{\partial[\phi^{iC}]^T_1}{\partial v}[m^{ii}]_1[\phi^{iC}]_1\right]^T + [\phi^{iC}]^T_1\frac{\partial[m^{ii}]_1}{\partial v}[\phi^{iC}]_1 \tag{29}$$

$$\frac{\partial[\bar{m}^{bi}]_1}{\partial v} = \frac{\partial[\phi^{iC}]^T_1}{\partial v}[m^{ii}]_1[\phi^{iN}]_1 + [\phi^{iC}]^T_1\frac{\partial[m^{ii}]_1}{\partial v}[\phi^{iN}]_1 + [\phi^{iC}]^T_1[m^{ii}]_1\frac{\partial[\phi^{iN}]_1}{\partial v} \tag{30}$$

The rate of change of the constraint mode is :

$$\frac{\partial[\phi^{iC}]_1}{\partial v} = -[k^{ii}]^{-1}_1\left[\frac{\partial[k^{ii}]_1}{\partial v}[\phi^{iC}]_1 + \frac{\partial[k^{ib}]_1}{\partial v}\right] \tag{31}$$

where the mass-normalized mode $[\phi^{iN}]_1$ should be in place.

The number of multiplications for the eigensensitivities as expressed by Eq. 19-24 can be also drastically reduced because of the sparsity of $\frac{\partial[\bar{K}]}{\partial v}$ and $\frac{\partial[\bar{M}]}{\partial v}$.

The benefits from the use of the substructural eigensensitivity synthesis method would be :

- Reduction in calculation time and in the size of computer memory for sensitivity analysis.
- Easy design modification of existing dynamic structures by simply modifying the attached substructure with the use of the eigensensitivities for the attached substructure.

3. Analysis with Numerical Example

The validity, accuracy and computational efficiency of the proposed method have been evaluated for different variants of a simple truss structure.

3.1 Investigation of Numerical Efficiency

For comparison purpose the computational load in terms of the number of multiplications for the full model sensitivity analysis and the substructural eigensensitivity synthesis were considered and tabulated in Table 1.

A 100 DOF structural system consisting of two equally sized substructures and a junction part with 10 DOF is considered as an example. With the basic assumption of $3n^3$ computing steps for pure eigenanalysis, the number of computing steps is shown in the numerical example of Table 1. It can be noticed that when 10 fundamental modes out of the 45 normal modes for each substructure are selected, the number of computing steps for the proposed method would be reduced to 1/4 or even lower to about 1/20. For sensitivity analysis, when a full basis of 45 modes for each substructure is used, the number of computation steps is about 1.05×10^6, while when 10 modes are used in conjunction with the previously obtained modal data for both substructures, the number of computations reduces to about 1.7×10^5 steps ; the calculations by using the proposed method are faster by a factor of about 6 for each design variable. Furthermore, it shows that the proposed method is more efficient than the finite difference method applied to two CMS results with a perturbed substructure (0.73 vs. 2x0.56=1.12). When considering the enormous computing time needed for the dynamic analysis of large structures, the saving in computing cost with the proposed method reveals its efficiency.

3.2 Scheme of the Computer Program

A Fortran program has been written to evaluate the feasibility of the substructural eigensensitivity synthesis method on a structural system consisting of two substructures. A brief description of the major stages of the computer program is shown in Fig.1.

Table 1 Number of computing operation for one design element with a numerical example

Analysis Type	Present	Proposal
	Conventional	CMS
EIGENANALYSIS	❷ ❶ $F(L) + E(L)$ $\approx 4L^3 + 3L^2$	Sub0 : $2F(M) + E(M)$ Sub1 : $2F(N) + E(N)$ SYN : $2(M^2 + N^2) + K(M + N) + E(L)$
	Nelson's	Sub. Sens. Synthesis
SENSITIVITY ANALYSIS	❸ $G(L) \approx al/L^3 + 5L^2$	Sub0 : 0 Sub1 : $G(N)$ SYN : $2Kl(N + n) + G(l)$

❶ $E(x) \approx 3x^3$ number of multiplication steps for pure E-analysis.
❷ $F(x) \approx x^3 + 3x^2$ auxiliary steps including steps for matrix inversion
❸ $G(x) \approx F(x) + 2x^2$ steps for E-sensitivity analysis

	Sub0	Sub1	Junction	Total
Number of DOF	M	N	K	$L = M + N + K$
Number of Bases	m	n	K	$l = m + n + K$

Numerical Example for 1 design variable $L = 100(M = N = 45,\ K = 10)$ $l = 30(m = n = 10,\ K = 10)$

# of steps ($\times 10^{-6}$)	Conventional	Substructural Sensitivity Syn.			
		Case1	Case2	Case3	Case4
E-ANALYSIS	4.03	1.03	0.56	0.09	3.01
SENSITIVITY	1.05	0.17	0.17	0.17	1.33
Total	5.08	1.20	0.73	0.26	4.34

Case1 : Without using the previously obtained modal data
Case2 : Using substructural modal data for one substructure
Case3 : Using substructural modal data for both substructures
Case4 : Same as Case3 but using full basis($l = 100 = L$)

3.3 Application to the Vibration of Truss Structure

A mass-spring model which consists of two truss substructures is introduced as shown in Fig. 2 along with the corresponding element stiffness and mass matrices. Based on the numerical model, first, a full model eigensystem analysis was carried out with a total number of DOF equal to 8, followed by a sensitivity analysis for the five elementwise cross sectional areas.

Next, based on the proposed theory, the effect of the number of chosen modes on the accuracy and efficiency was examined for two cases :

(1) Full basis synthesis : By using all fixed interface normal mode vectors as basis vectors, i.e., 3 interior modes for SUB0 and 3 interior modes for SUB1 with 2 junction DOF.

(2) Reduced basis synthesis : By selecting the lowest 2 modes for each substructure as basis vectors.

The mode shapes and sensitivities of the shapes with corresponding eigenvalues for both the case of full model and of a reduced basis vector (2-2 modes) are shown

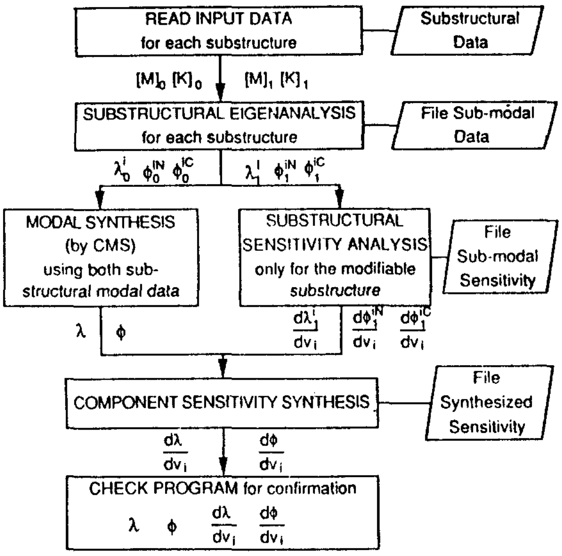
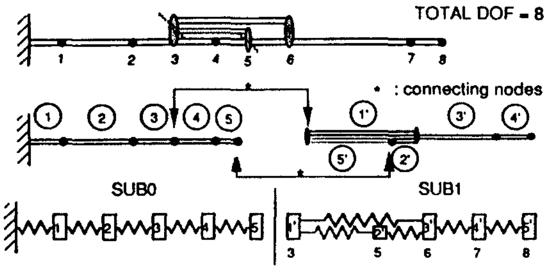


Fig. 1 Schematic flow chart of the substructural eigen-sensitivity synthesis method



$$[Ke] = Ke \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad [Me] = Me \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(ρ = 1.0)

Element Number		Area Ae	Length Le	Element spring Ke	Element mass Me	dKe/dAe	dMe/dAe
SUB0	1	2.0	1.0	20.	1.0	.	.
	2	2.0	2.0	10.	2.0	.	.
	3	1.0	1.0	10.	0.5	.	.
	*4	1.0	1.0	20.	0.5	.	.
	5	1.0	0.5	20.	0.25	.	.
SUB1	1'	1.0	2.0	5.	1.0	5.	1.0
	2'	1.0	0.5	20.	0.25	20.	0.25
	3'	2.0	2.0	10.	2.0	5.	1.0
	4'	1.0	1.0	10.	0.5	10.	0.5
	▲5'	.	.	.	.	15.	0.37

* : Young's Modulus of element 4 is 20.0, while the others are 10.0.
▲ : Element 5' is selected only to show the effect of change in the original structural elements 4 and 5 which have connecting nodes.

Fig. 2 Truss Model and equivalent system

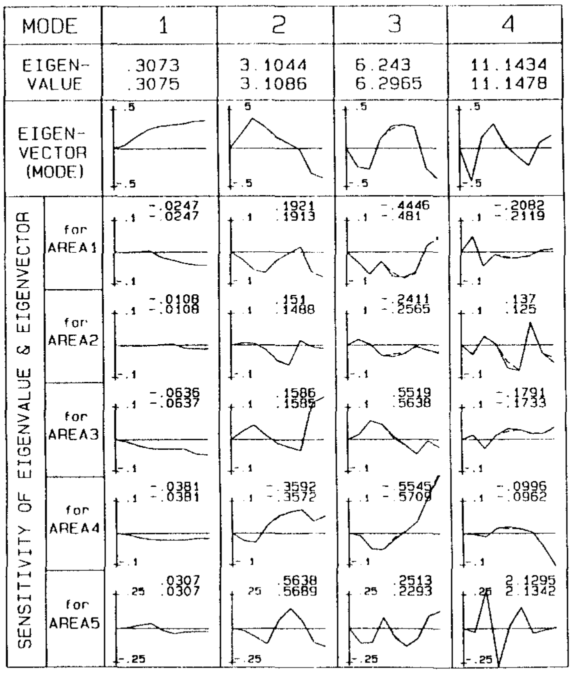


Fig. 3 Eigenvalue, eigenvector and the eigensensitivities for full model analysis and 2-2 reduced basis synthesis

in Fig. 3(the result for full basis synthesis is exactly the same as for the full model analysis). The results show only slight differences which were expected due to the adoption of the CMS technique to the sensitivity synthesis.

3.3.1 Error Analysis

To ascertain the accuracy of the method, the deviation of the substructurally synthesized eigensolutions and sensitivities from those of the full analysis was obtained and expressed in terms of the relative error and percent of absolute difference(PAD). The relative error is generally defined as :

$$\text{Relative Error} = \frac{(\lambda^e - \lambda^a)}{\lambda^e} \tag{32}$$

where, λ^e is the exact eigenvalue and λ^a is the analytically (or by any other means) obtained eigenvalue.

Similarly, by denoting the exact normalized mode shape by $\{\phi^e\}$ and the analytically obtained corresponding normalized mode by $\{\phi^a\}$, the mode shape difference vector $\{\phi^d\}$ can be expressed by jth nodal difference ϕ^d_j :

$$\phi^d_j = \phi^e_j - \phi^a_j, \qquad j = 1,2,\cdots\cdots,n \tag{33}$$

where n is the number of DOF for the mode shape, and subsequently the PAD can be defined as :

$$\mathrm{PAD} = \frac{(\sum_{j=1}^{n} |\phi^d_j|)/n}{(\sum_{j=1}^{n} |\phi^e_j|)/n} \times 100 = \frac{\sum_{j=1}^{n} |\phi^d_j|}{\sum_{j=1}^{n} |\phi^e_j|} \times 100 \tag{34}$$

By using the above concept of relative error and PAD, the results of substructurally synthesized eigensolutions and their sensitivities were examined and tabulated in Table 2 only for the reduced basis (2-2 modes) case.

The results show inherent modal turncation effects [11], i.e., the smaller the number of selected basis modes the larger the errors in the eigensolution. However, the sensitivities obtained by the proposed method demonstrate a very good accuracy and the errors resemble the results of eigensolution analysis with the CMS technique. From the point of view of efficiency, the analysis revealed that if the number of chosen modes is small, less calculation time is required as shown Table 3. The actual calculation time is obtained by running the program on an IBM AT compatible computer.

3.3.2 Larger Truss Model

As another example, a larger truss structure with 149 DOF has been analyzed. The results obtained on a VAX 11/785 computer are presented in Table 4. The eigensolutions and their sensitivities are obtained by full model analysis and both full and reduced basis synthesis but

Table 2 Error analysis with reduced basis(2-2 modes)

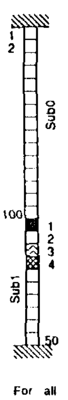
ERRORS from the Result of Full Analysis		Relative Error for Eigenvalue (%)				PAD for Eigenvector (%)			
		Mode Number				Mode Number			
		1	2	3	4	1	2	3	4
E-analysis (CMS)		0.07	0.14	0.86	0.04	0.14	0.96	3.40	1.69
Sensitivities for Variables (E-sensitivity Synthesis)	1	0.00	-.42	8.19	1.78	0.43	0.77	8.05	9.08
	2	0.00	-1.46	6.39	-8.76	1.35	3.55	16.12	14.39
	3	0.16	-.06	2.16	-3.24	0.51	0.32	5.57	10.15
	4	0.00	-.56	2.96	-3.41	0.67	1.24	4.39	4.97
	5	0.00	0.90	-8.75	0.22	0.05	1.78	2.12	3.01

Table 3 Comparison of calculation times

ANALYSIS TYPE	DOF		Upper: w/o previous submodal data / Lower: with previous submodal data	
	Sub0	Sub1	CPU Time † (sec)	% time for Full Analysis
Substructural Eigensensitivity Synthesis (Junction DOF:2)	3	3	5.86 5.55	115 109
	3	2	4.13 3.79	81 74
	2	3	4.18 3.84	82 75
	2	2	2.90 2.57	57 50
Full Sensitivity Analysis	Total 8		5.10	100

†: CPU time was obtained on an IBM AT compatible computer

Table 4 Eigenvalue sensitivities and the elapsed time for the first 4 modes for each analysis type

Test Structure	Design Variable	Number of Chosen Modes		Eigenvalue Sensitivity				Calculation Time (sec)	
	Area #	Sub0	Sub1	Mode1	Mode2	Mode3	Mode4	E-Anal.	Sens.
Truss Model 	1 ■ (A=2.0)	Full Analysis		-.0028	-.0125	.0525	-.0398	162.10	61.63
		99 (all)	49	-.0029	-.0122	.0525	-.0393	173.24	76.25
		99	15	-.0028	-.0125	.0525	-.0397	82.61	33.60
		99	5	-.0028	-.0125	.0525	-.0388	64.79	23.74
		25	49	-.0028	-.0125	.0525	-.0398	30.89	23.63
		25	15	-.0028	-.0125	.0525	-.0397	12.78	6.60
		25	5	-.0028	-.0125	.0525	-.0388	10.20	2.75
		5	49	-.0028	-.0126	.0525	-.0406	19.57	18.73
		5	15	-.0028	-.0125	.0525	-.0405	8.81	5.04
		5	5	-.0028	-.0125	.0525	-.0397	8.66	1.81
	2 □ (A=2.0)	Full Analysis		-.0026	-.0142	.0516	-.0252		61.09
		99 (all)	49	-.0027	-.0138	.0516	-.0247	E-anal. is not required	76.05
		99	15	-.0026	-.0142	.0516	-.0252		33.48
		99	5	-.0026	-.0140	.0516	-.0234		23.79
		25	49	-.0026	-.0141	.0516	-.0252		23.59
		25	15	-.0026	-.0141	.0516	-.0252		6.59
		25	5	-.0026	-.0140	.0516	-.0234		2.72
		5	49	-.0026	-.0142	.0516	-.0261		18.78
		5	15	-.0026	-.0142	.0516	-.0262		5.03
		5	5	-.0026	-.0142	.0516	-.0252		1.78
	3 (A=2.0)	Full Analysis		-.0024	-.0157	.0500	-.0098		61.22
		99 (all)	49	-.0024	-.0153	.0500	-.0094	E-anal. is not required	76.05
		99	15	-.0024	-.0157	.0500	-.0098		33.52
		99	5	-.0024	-.0156	.0500	-.0088		23.81
		25	49	-.0024	-.0157	.0500	-.0098		23.56
		25	15	-.0024	-.0157	.0500	-.0098		6.60
		25	5	-.0024	-.0156	.0500	-.0088		2.85
		5	49	-.0024	-.0157	.0500	-.0109		18.75
		5	15	-.0024	-.0157	.0500	-.0110		5.02
		5	5	-.0024	-.0156	.0500	-.0099		1.79
For all Elements ρ = 1.0 A = 2.0 Le = 1.0 E = 2000	4 (A=2.0)	Full Analysis		-.0022	-.0171	.0476	.0058		61.14
		99 (all)	49	-.0022	-.0168	.0476	.0064	E-anal. is not required	76.37
		99	15	-.0022	-.0171	.0476	.0059		33.68
		99	5	-.0022	-.0171	.0476	.0057		23.79
		25	49	-.0022	-.0171	.0476	.0058		23.52
		25	15	-.0022	-.0170	.0476	.0059		6.67
		25	5	-.0022	-.0171	.0476	.0057		2.71
		5	49	-.0022	-.0171	.0476	.0046		18.85
		5	15	-.0022	-.0171	.0476	.0047		5.03
		5	5	-.0022	-.0171	.0476	.0045		1.78

*1 : Exact eigenvalues are 0.8809, 3.5122, 7.8967, 14.0322, e.t.c.
*2 : Calculation time is for both E-value and E-vector sensitivities to the total DOF.
*3 : For the full sensitivity analysis, total DOF is 149.
*4 : Assembled total DOF varies with the number of chosen modes for each substructures. (eg. for [25:15] case , the number of total DOF is 25+15+1=41)
*5 : The calculation time for the first design variable includes the time for eigenanalysis.
*6 : For eigenanalysis, previously obtained modal data for Sub0 is used for the synthesis.

shown only for the eigenvalue sensitivity. For full analysis, one eigensolution set is obtained in about 162.1 sec and the time for one sensitivity calculation is about 61.5 sec. However, for example, when 25 and 15 modes are selected for each substructure the elapsed time for the sensitivity reduces at most 6.70 sec and the sensitivities are still calculated with good accuracy.

4. Estimation of Modified Eigensolution

By using the eigensensitivities as the derivatives of the current eigensolution, the dynamics of the perturbed structure can be easily estimated In this section, two types of analysis have been performed. First, the estimated eigensolutions are compared to the exact eigensolutions obtained by the full model eigensystem analysis for the perturbed structure. Second, examinations of the estimated eigensolutions by the sensitivities either from full or reduced bases are carried out.

The modified eigenvalues and corresponding eigenvectors, λ_i and $\{\phi_i\}$, are predicted with the original eigenvalues and eigenvectors, λ_i^0 and $\{\phi_i^0\}$, based on the following first order Taylor equations :

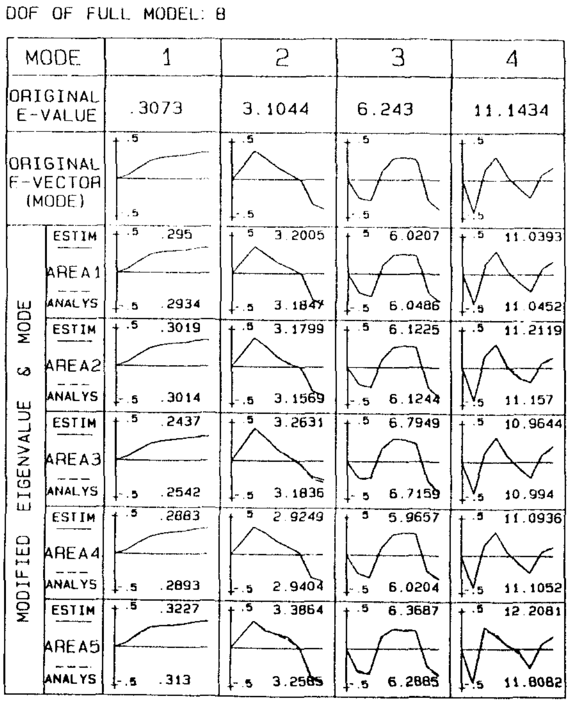


Fig. 4 Estimated eigenvalue and eigenvector for full model with 50% elementwise modification comparing with the result of total eigenanalysis

$$\lambda_i = \lambda_i^0 + \frac{\partial \lambda_i}{\partial v} \Delta v \tag{35}$$

$$\{\phi_i\} = \{\phi_i^0\} + \frac{\partial \{\phi_i\}}{\partial v} \Delta v \tag{36}$$

where, Δv is small design variable change.

4.1 Estimation with First-Order Approximation

For the structure as shown in Fig. 2, exact eigensolutions with full model and estimated solutions based on total sensitivity analysis with a 50% increment in the elementwise cross sectional areas are comparatively shown in Fig. 4. The results reveal that there are some errors due to the first-order linear expansion. Even though a 50% elementwise change rate may seem like a considerably large modification, the example of a 50% elementwise change rate is introduced to show the exaggerated output for the sensitivities and resulting modified eigensollutions. The changes in eigenvalues and eigenvectors for certain modes are checked and tabulated in Table 5 for the 50% elementwise design increase. From the evaluations, it is generally noticed that the most effective design variables are :

i) area number 3 for the first and third mode,
ii) area number 5 for the second and fourth mode.

Even though design variables numbered 3 and 5 are seen to be dominant, it must be born in mind that the other design variables also contribute to the change of structural characteristics and they must also be considered to obtain the optimal design for the structure. For

Table 5 Percentage change from the original values

DESIGN CHANGE (50% Increment)	CHANGE OF E-VALUE (Relative Error, %)				CHANGE OF E-VECTOR (PAD, %)			
	Mode Number				Mode Number			
	1	2	3	4	1	2	3	4
Area # 1	-4.00	3.10	-3.56	-0.93	3.95	8.95	9.47	5.41
Area # 2	-1.76	2.43	-1.93	0.61	1.07	4.66	3.72	12.25
Area # 3	-20.70	5.11	8.84	-1.61	11.39	22.22	10.22	13.02
Area # 4	-6.18	-5.79	-4.44	-0.45	3.24	10.04	10.85	7.15
Area # 5	5.01	9.08	2.01	9.55	4.54	19.24	18.20	27.11

example, design variable number 4 can increase the second eigenvalue of the structure by reducing the area while the others lead to the same effect by increasing the area which causes an increase in the total structural weight.

4.2 Errors from full CMS

The estimated eigensolutions from the full basis CMS and substructural sensitivity synthesis technique and the results with a 50% elementwise change are shown clear resemblance of the results of the total analysis as depicted in Fig. 4. This proves the feasibility of the proposed substructural sensitivity synthesis technique.

4.3 Truncation Error due to the Reduced Basis

By selecting 2-2 reduced basis for the CMS and eigensensitivity synthesis, estimations were carried out again. The results are shown in Fig. 5 to compare with the results of the full basis CMS. As noted before, there are

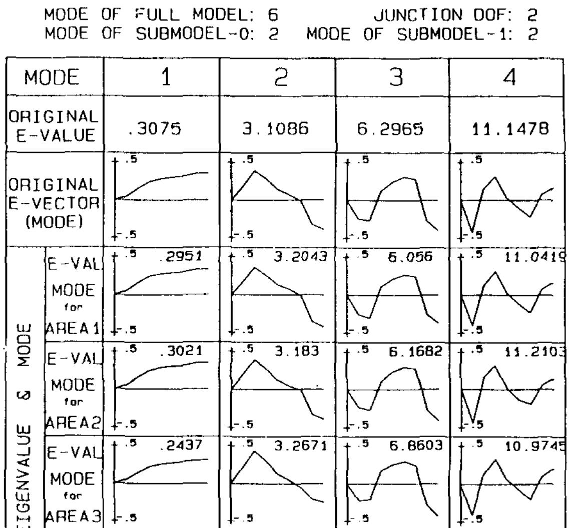


Fig. 5 Estimated eigensolutions by using synthesized sensitivity with 2-2 reduced basis model for 50% elementwise change

small errors due to the modal truncation effect but not significant for the scope of this analysis. For the smaller 10% elementwise modifications (not shown here), the estimated eigensolutions obtained by using the sensitivities for the full basis CMS and reduced basis CMS show a smaller modal truncation error compared to the case with the 50% elementwise modification.

4.4 Brief Comments on the Errors

The differences of estimated results by the substructural sensitivity synthesis technique and their deviation from the exact eigensolution are summarized. The errors for the estimated eigensolutions show, while the sensitivities are influenced only by the modal truncation error, the estimated eigensolutions with varying change rates are influenced by both the truncation error and the error due to the Taylor approximation. The estimated eigensolutions show slight inaccuracies in the case of a small number of bases but the error are permissible for further iterative design changes. Even though the modal truncation effect from the use of reduced bases results in less accurate estimation, the reduced bases require less computation time and, especially for iterative design modifications, the synthesized eigensensitivities can be utilized as computationally fast and fruitful design tools.

5. Concluding Remarks

This paper presented a new eigensensitivity synthesis method which utilizes the basic ideas of CMS to synthesize the eigensensitivities by using substructural eigensensitivities. It has been shown that the newly proposed method works well and offers the following advantages as compared to the conventional sensitivity analysis :

- Simple re-analysis of an entire structure by using the substructural modal sensitivities of the appendant modifiable substructure only.
- Compatibility with both analytically or experimentally obtained modal data.
- Shorter execution time and memory space.
- Sufficient accuracy even for relatively small number of basis vectors.

For the estimation of structural dynamics using the eigensensitivity, the estimated eigensolutions fit very well when the modification is small. The truncation of higher modes leads to a small error but within a manageable and permissible range for further analysis. So, the proposed substructural eigensensitivity synthesis promises of being a fruitful tool for improving the dynamics of large structures.

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正 誤 表

대한조선학회 논문집 제29권 1호(1992년 3월 20일)에 게재된 논문 "Timoshenko 보함수 성질을 갖는 다항식을 이용한 Mindlin 판유추 구조계의 진동해석"(저자 : 정정훈, 정태영, 김극천)에서 편집과정작업의 착오로 인하여 [16]이하 참고문헌이 누락되었기 이에 바로 잡습니다.

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