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A Computational Method of Wave Resistance of Ships in Water of Finite Depth

by

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유한수심에서의 조파저항계산에 관하여

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Abstract

A computational method of the Michell integral for water of finite depth is developed and the method makes use of the expansion of the hull form by the Legendre polynomial in both the longitudinal and the vertical directions. The wave resistance coefficient is given as a quadruple summation of the product of the shape factor and the hydrodynamic factor. The shape factor depends only upon the geometry of the hull form, and the hydrodynamic factor upon the depth-based Froude number and the ratios of the water depth and the draft to the ship length. Example calculations are done for the Wigley parabolic hull and the Series 60 C_B 0.6, and the comparison of our results with the existing experimental data is shown.

요 약

유한수심에 대한 Michell적분의 계산을 위해 선각함수를 종방향 및 수직방향에 대해 Legendre다항식으로 전개하여 조파저항계수를 형상계수와 유체동력학적계수의 곱에 대한 4중급수로 구할 수 있는 식을 얻었다. 여기서 형상계수는 선각의 기하학적 형상만의 함수이고, 유체동력학적계수는 수심에 근거한 Fn와 수심과 흘수의 배의 길이에 대한 비들만의 함수이다. Wigley의 포물선형 선각과 Series 60의 C_B 0.6에 대한 계산을 수행하고 그 결과를 기존의 실험결과(무한수심)및 다른 이론결과(유한수심)와 비교하였다.

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1. Introduction

When a ship moves with a constant speed(U) on the surface of water, her resistance increases as the depth(H) of water decreases. It is believed that most of the increase is due to that of the wave resistance, if the water is not too shallow so that the viscous effect does not play an important role. Although the thin ship theory(Michell, 1898) does not give an accurate prediction of the wave resistance of a ship with a practical hull form due to various reasons(Wehausen, 1973), as a recent study(Millward and Bevan, 1986)showed, it is accurate enough for predicting a relative change in the wave resistance as H varies.

Since Sretensky(1937) applied the thin ship theory to obtain an appropriate formula for the ship wave resistance in water of finite depth, there have been many studies using his or similar formulae. However, most of the previous studies assumed the hull form $Y=F(X,Z)$, Y being a product of a function of X(longitudinal coordinate) and a function of Z(vertical coordinate) only, for which the main reason was probably the numerical burden. This is evidenced by the fact that the result of Schlichting and Strohbusch(1934) is still in use at many shipyards even today. Now, due to the fast development of computers, we are no longer under such restriction, however, an efficient and accurate algorithm suitable to the computation of Michell integral for the finite depth is not as easy as it seems. In our study we employed the method developed by Sendagorta and Grases(1988) for computing the ship wave resistance in deep water. One of the characteristics of this method is that the hull form is expanded in terms of Legendre polynomials in each direction so that the effect of the hull shape can be separated from the hydrodynamic effect.

In the sequel, we first discuss the wave resistance formula we used and the algorithm employed for it, and follow the numerical results and the comparison with other available experimental data.

2. Wave Resistance Formula and the Numerical Algorithm

There are various forms of Michell integral for the ship wave resistance(R_W) in water of finite depth. One form we found convenient for the method employed is as follows(see Kostyukov, 1968)

$$C_W=\frac{R_W}{\frac{1}{2}\rho U^2 S}=C_0\int_{\mu_0}^{\infty}(I^2+J^2)G(\mu)\ d\mu,$$

where

$$C_0=\frac{2}{\pi}\frac{B^2T^2}{L^2S}\frac{1}{Fn^4}$$

$$\mu_0=\begin{cases}0, & (Fh<1),\\ \tan\{\cos^{-1}(Fh^{-1})\}, & (Fh>1),\end{cases}$$

$$I+iJ=\int_{-1}^{1}\int_{-1}^{0}\frac{\partial f}{\partial x}\frac{\cosh k_0(h+tz)}{\cosh k_0h}e^{ik_0x(1+\mu^2)^{-\frac{1}{2}}}\,dxdz$$

$$G(\mu\,;k_0h)=\frac{\sqrt{1+\mu^2}\tanh k_0h}{1-2k_0h(\sinh 2k_0h)^{-1}}$$

$$k_0h=Fh^{-2}(1+\mu^2)\tanh\ k_0h,$$

in which the following dimensionless variables are found useful.

$$(x,\ y,\ z)=(\frac{X}{l},\frac{Y}{b},\frac{Z}{T}),\quad (l,\ b)=\frac{1}{2}(L,B),$$

$$(f,k_0,h,t)=(\frac{F}{b},K_0 l,\frac{H}{l},\frac{T}{l}),\ Fn=\frac{U}{\sqrt{gL}},\quad Fh=\frac{U}{\sqrt{gH}}.$$

Here, $i=\sqrt{-1}$, ρ is the density of water, g is the gravitational acceleration, L, B, T, S, are LWL, breadth, draft, wetted surface area of a ship, respectively, and H is the depth of the water, K_0 is the dimensional wave number, and we note that k_0, a function of μ and Fh, is sought as a solution of the transcendental equation given above.

de Sendagorta and Grases(1988) noted that the term $I+iJ$ can be transformed into a double series by using the well-known expansions (Gradshteyn and Ryzhik, 1980),

$$\exp(i\alpha x)=\sum_{n=0}^{\infty}(2n+1)i^n j_n(\alpha)P_n(x),$$

$$\exp(\beta z)=\sum_{n=0}^{\infty}(2n+1)i_n(\beta)P_n(z),$$

where j_n, i_n, P_n are the spherical Bessel function of the lst kind, the modified spherical Bessel function of the lst kind, and the Legendre polynomial, respectively, all being the nth order.

Noting that

$$\cosh k_0(h+tz)=\frac{1}{2}e^{k_0h}e^{-\beta}\{e^{\beta(1+2z)}+e^{-k_0h_1}e^{-\beta(1+2z)}\},$$

where

$$2\beta=k_0t,\ h_1=2h-t>0,$$

we can get $I+iJ$ in the form of double series

$$I+iJ=D(k_0h)\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}i^m(2m+1)(2n+1)j_m(\alpha)$$

$$e^{-\beta}\{1+(-1)^n e^{-k_0h_1}\}i_n(\beta)S(m,n),$$

$$D(k_0h)=(1+e^{-2k_0h})^{-1},\ \alpha=k_0(1+\mu^2)^{-1/2},$$

$$S(m,n)=\int_{-1}^{1}\int_{-1}^{0}\frac{\partial f}{\partial x}P_m(x)P_n(1+2z)\ dxdz.$$

We shall call $S(m,n)$ the shape factor of(m,n). Since the hull form is usually given as an offset, and for the obvious numerical reasons, it is preferred to integrate the above by parts with respect to x. Assuming that $f=0$ at $x=\pm 1$, the shape factor can be rewritten as

$$S(m,n)=-\int_{-1}^{1}\int_{-1}^{0}f(x,z)P_m'(x)P_n(1+2z)\ dxdz.$$

Here, the superscript prime of P_m denotes the differentiation with respect to its argument. Since $P'_m(x)$ is odd for even m, $S(2m,n)$ vanishes if $f(x,z)$ is even in x,i.e., a hull is symmetric with respect to her midship.Considering that the usual ship form has a longitudinal symmetry approximately, $S(2m,n)$ is in general expected very small. We also note that $S(0,n)$vanishes for all n, and that $S(m,0)$ represents the longitudinal distribution of the displacement and $S(1,n)$ the vertical one. For example, it can be easily shown that $S(1,0)=-2C_B$,$S(2,0)=-6C_B\ x_1$,$S(1,1)=-2C_B-4C_B\ z_1$, where C_B is the block coefficient and x_1, z_1 are the LCB, VCB of a given ship, respectively, which can be used to check the validity of the developed numerical code for computing the shape factor.

Rewriting the real and the imaginary part of $I+iJ$ separately, we obtain

$$I^2+J^2=D^2\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}(-1)^{m+r}\{\bar{j}_{2m}(\alpha)\bar{j}_{2r}(\alpha)\bar{i}_n(\beta)$$

$$\cdot\,\bar{i}_s(\beta)S(2m,n)S(2r,s)$$

$$+\bar{j}_{2m+1}(\alpha)\bar{j}_{2m+1}(\alpha)\bar{i}_n(\beta)\bar{i}_s(\beta)S(2m+1,n)$$

$$\cdot\,S(2r+1,s)\},$$

where

$$\bar{j}_m(\alpha)=(2m+1)j_m(\alpha),$$

$$\bar{i}_n(\beta\,;k_0h_1)=(2n+1)e^{-\beta}\{1+(-1)^n e^{-k_0h_1}\}i_n(\beta).$$

Substituting this into the Michell integral and defining the hydrodynamic factor as

$$V(m,\ n,\ r,\ s)=\int_{\mu_0}^{\infty}D^2G\bar{j}_m(\alpha)\bar{j}_r(\alpha)\bar{i}_n(\beta)\bar{i}_s(\beta)\ d\mu,$$

we get the guadruple summation for C_W as

$$C_W=C_0\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\sum_{s=0}^{\infty}(-1)^{m+r}$$

$$\{V(2m,n,2r,s)\cdot S(2m,n)S(2r,s)$$

$$+V(2m+1,n,2r+1,s)\ S(2m+1,n)\ S(2r+1,s)\}.$$

As $\mu\rightarrow\infty$, $k_0\sim\mu^2$, and hence $\alpha\sim\mu^2,\beta\rightarrow\mu^2$. Thus, we can show that

$$D^2G\bar{j}_m(\alpha)\bar{j}_r(\alpha)\bar{i}_n(\beta)\bar{i}_s(\beta)\sim\mu^{-5},\ as\ \mu\rightarrow\infty.$$

As the integrand for the hydrodynamic factor diminishes very fast, there is little difficulty in computing it by a numerical quadrature such as the Simpson's rule, In computing the shape factor, it is necessary to have a good surface interpolant, since as m, n increase we need more and more values of the offset to carry out the numerical integration with an acceptable accuracy. For example, when $(m,n)=(5,5)$, we need at least 40 stations and 50 waterlines in order to have ten points between the consecutive zeros of the related Legendre polynomials. In developing our code we found that the interpolation of the hull offset by the bicubic B−spline(Barsky and Greenberg, 1980) offering a reasonably good accuracy for non-mathematical hull forms. Once the shape factor is obtained for a given ship, it can be stored and used for the computation of C_W for any depth of water. Given L,T and H, on the other hand, the hydrodynamic factor can be computed as a function of Fh once and for all, and can be stored for later use.

3. Numerical Results and Discussion

Wigley Hull

For some elementary hull forms, we can get the shape factor analytically in closed form, and we chose the following Wigley parabolic hull as our first computational example.

$$f(x,\ z)=(1-x^2)(1-z^2),$$
$$L=16m,\ B=1.6m,\ T=1m,\ S=37.98m^2.$$

Due to the longitudinal symmetry, every $S(2m,n)$ vanishes identically. Furthermore, since the $x-$derivative of $f(x,\ z)$ is linear in x and quadratic in z, all the shape factors are zero, except those corresponding to $m=1,\ n=0,1,2$ for which the values are given as

$$S(1,0)=-\frac{8}{9},\ S(1,1)=-\frac{2}{9},\ S(1,2)=\frac{2}{45}.$$

Note the fast decrease of absolute magnitude of the shape factor as n increases. Now, the quadruple summation for C_W becomes a double summation as

$$C_W=C_0\sum_{n=0}^{2}\sum_{s=0}^{2}V(1,n,1,s)S(1,n)S(1,s).$$

As the integration for the corresponding hydrodynamic factors can be performed as accurate as desired, we can obtain exact values of C_W for the Wigley parabolic hull, and we show our result for deep water in the Fig. 1 for the purpose of comparison with those presented in the Workshop on Ship Wave-resistance Computations (Bai, 1979). Shaded area in the Fig. 1 corresponds to the range of experimental data, and we conjecture that the large value of L/B, namely 10, is responsible for the excellent agreement shown.Considering that in the workshop of 1979 people produced many different values of C_W even when they used the same Michell formula, we can acknowledge how tremendously the development of the computer in the mean time has benefited us. By the way, all the computational results presented in this study were done on a 486(33MHz)based personal computer.

Fig. 2 exhibits the variation of C_W with Fn for 4 different values of H, i.e., $2T$, $4T$, $8T$ and L. We note that C_W for $H=L$ is hardly distinguishable from that for $H=\infty$. C_W gets its maximum at $Fh=1.0$, 0.94, 0.71, 0.5 in each case, respectively. Thus, the greater H becomes the lower

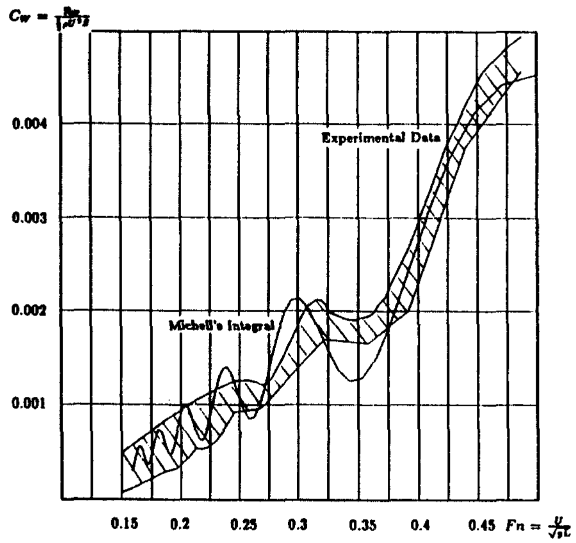

Fig. 1 Comparison of the computed wave resistance coefficient and the experimental data(Bai, 1979) for the Wigley hull in deep water

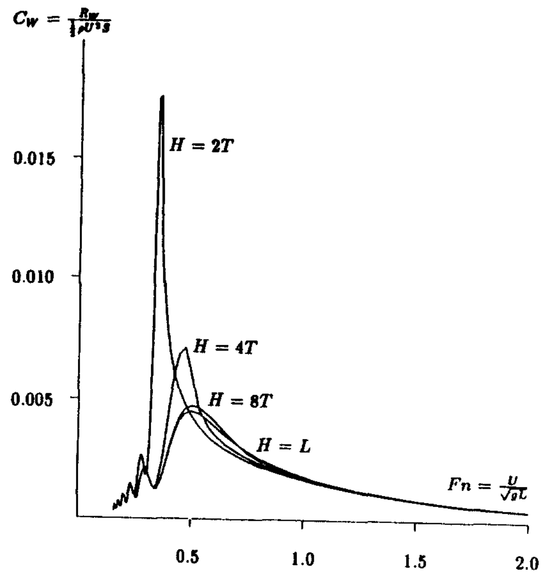

Fig. 2 Variation of the wave resistance coefficient with the Froude number for $H=2T,4T,8T$ and L for the Wigley hull

Fh gets where C_W has its maximum. As H decreases, the maximum of C_W increases rapidly for $H<8T(=L/2)$. This is because the disturbance caused by the ship near $Fh=1.0$ is relatively stronger for smaller H. As anticipated, at very low and at very high Fn, C_W does not change much as H, since the effect of the water depth upon C_W is insignificant in those ranges of Fn.

Fig. 3 shows how C_W varies with H for 4 different values of Fn, namely, 0.3, 0.4, 0.5 and 0.7. Here, we observe a general trend that when a ship runs with a constant speed while the depth of water is decreasing, her resistance increases first until it gets its maximum at a H corresponding to $Fh=1.0$ approximately, and then decreases rather rapidly for supercritical speeds, except for the range of very small H at the end, say H being less than one and half times the draft, where C_W increases again though not much. Since C_W for deep water has its so-called the last hump near $Fn=0.5$, C_W for $Fn>0.5$ is less than that for Fn=0.5 for all depths of water as can be observed in the Fig. 3.

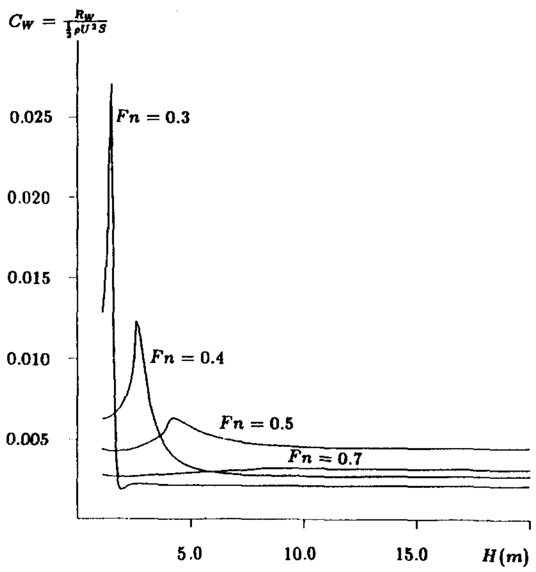

Fig. 3 Variation of the wave resistance coefficient with the depth of the water for $Fn=0.3, 0.4, 0.5$ and 0.7 for the Wigley hull

Series 60 C_B 0.6

As an example of the practical hull form, we took the Series 60 C_B 0.6(4210W), which was one of the models also chosen at the workshop in 1979(Bai, 1979). In computing the shape factor numerically for a nonmathematical hull form, we need to use a surface interpolant as stated, and we employed the bicubic B-spline method of Barsky and Greenberg(1980).

Table 1 Particulars and the shape factor of the Series 60 C_B 0.6

$L_{wl}=123.962m$, $B=16.2550m$, $T=6.50140m$, $S=2534.39\,m^2$

$n=0,5$	Shape Factor					
$S(0,n)$	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
$S(1,n)$	-1.176150	-0.087000	0.022361	-0.013911	0.005126	-0.002503
$S(2,n)$	-0.047546	-0.034387	-0.018882	0.000038	-0.001591	0.000165
$S(3,n)$	0.311060	-0.054798	-0.014995	-0.006143	-0.001464	-0.000660
$S(4,n)$	0.043169	-0.006848	-0.012639	-0.007137	-0.002300	-0.001049
$S(5,n)$	0.079700	0.055864	-0.014216	-0.003628	-0.003582	-0.001361
$S(6,n)$	0.059304	0.013811	0.004604	-0.003607	-0.003692	-0.000949
$S(7,n)$	-0.024000	-0.005850	0.007977	-0.000562	-0.002699	-0.000928

Full convergence test for the shape factor was not possible due to the time limit, and $max.(m,n)=(7,5)$ was taken, which means that the hull form is represented in the 6th order polynomial in x and 5th in z, respectively. As input data in computing the shape factor for the Series 60, an offset with the 21 stations and 16 waterlines was prepared, and the interpolant generated a hull form at 201 stations and 101 waterlines. We show in the Table 1 a set of so attained values of the shape factor along with the principal particulars of the Series 60 C_B 0.6. Since C_B is 0.6, $S(1.0)$ is expected to be -1.2, and our numerical value is -1.176(2% error). We could have improved the result by taking more offset points, however, we decided to take the above error as acceptable, for our aim at the moment is to find out the relative change in the wave resistance.

Fig. 4 displays the comparison of the computed C_W and the experimental results taken from Bai(1979). Unlike the previous example calculation for the Wigley hull, the agreement is not so good, and among other possible

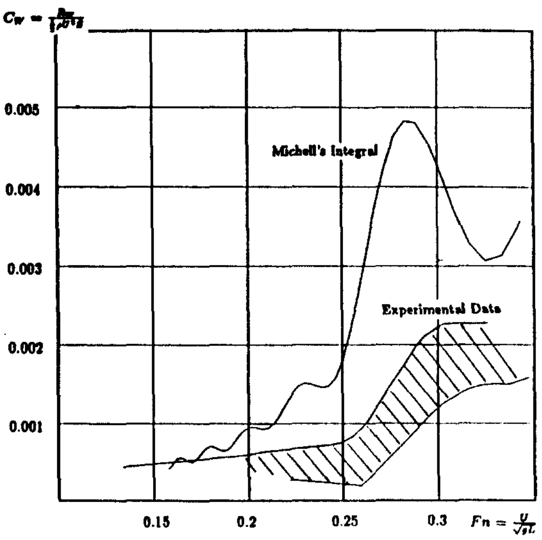

Fig. 4 Comparison of the computed wave resistance coefficient and the experimental data(Bai, 1979) for the Series 60 C_B 0.6 in deep water

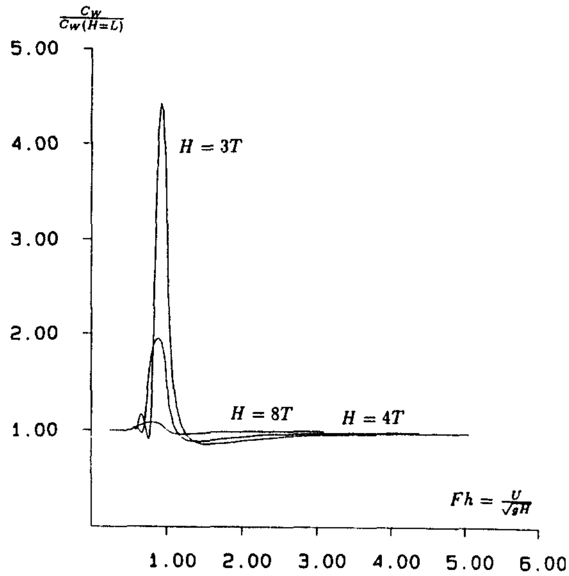

Fig. 5b Ratio of the wave resistance coefficient at a specific depth and that at $H=L$ vs. the depth Froude number for $H=3T$,$4T$,and $8T$ for the Series 60 C_B 0.6

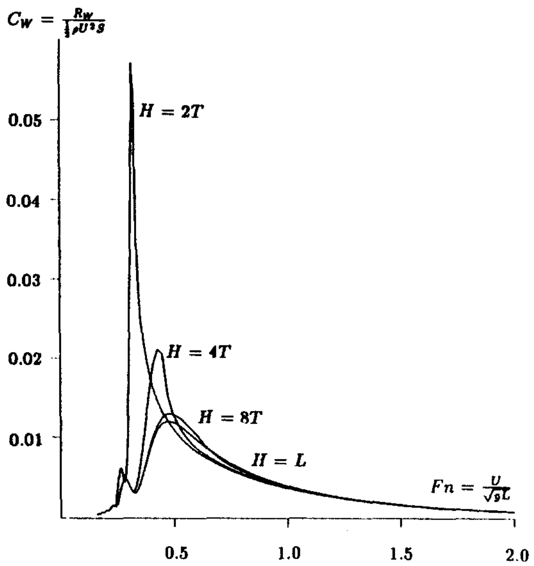

Fig. 5a Variation of the wave resistance cofficient with the Froude number for $H=2T$,$4T$,$8T$ and L for the Series 60 C_B 0.6

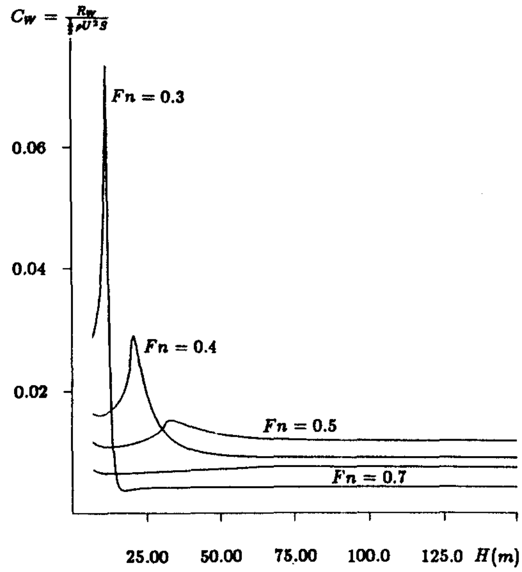

Fig. 6 Variation of the wave resistance coefficient with the depth of the water for $Fn=0.3$,0.4,0.5 and 0.7 for the Series 60 C_B 0.6

reasons for the difference we point out the smallness of the ratio $L/B(=7.626)$. That is, we contend the confirmation of the well-known fact that the thin ship theory gives less accurate result for smaller L/B ratio, which is equivalent to saying that we still lack a reliable way of predicting the absolute value of the wave resistance of a ship with smaller and practical values of L/B ratio (Wehausen, 1973).

Fig. 5a demonstrates the variation of C_W with Fn for $H=2T$, $4T$, $8T$ and L. We can see the same tendency as shown in the Fig. 2, but now with bigger values of C_W for the Series 60. In order to show the relative change of C_W as H varies, the similar numerical results are presented in a different way in the Fig. 5b, where the ordinate is taken as the ratio of C_W for a specific depth and that for $H=L$, and the abscissa as Fh. This shows the very similar characteristics with the result of Millward and Bevan(1986), though they chose the Wigley parabolic hull as their mathematical model.

Fig. 6 shows the change of C_W with H for $Fn=0.3$, 0.4, 0.5 and 0.7. As expected from the previous figures, we can observe the same pattern as in the Fig. 3 for the Wigley hull. We note that in the case of the Series 60, C_W for $Fn=0.7$ is now even smaller than for $Fn=0.4$ for all depths.

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