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A Deformation Model of a Bag-Finger Skirt and the Motion Response of an ACV in Waves

by

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Bag-Finger형 스커트의 변형모델과 규칙파중 공기부양선의 운동해석

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Abstract

In this paper, the effects of a skirt deformation on the responses of an Air Cushion Vehicle in waves are investigated. The air in the bag and in the plenum chamber is assumed to be compressible and to have a uniform instantaneous pressure distribution in each volume. The free surface deformation is determined in the framework of linear potential theory by replacing the cushion pressure with the pressure patch moving uniformly with an oscillating strength. And the bag-finger skirt is assumed to be deformed due to the pressure disturbance while its surface area remained constant. The restoring force and moment due to the deformation of bag-finger skirt from the equilibrium shape is included in the equations of heave and pitch motions.

The numerical results of motion responses due to various ratios of the bag and cushion pressure or bag-to-finger depth ratios are shown.

요 약

본 논문에서는 규칙파중에서 공기부양선의 운동응답에 미치는 스커트의 변형에 대한 연구가 이루어졌다. Bag과 부양실내의 공기는 압축성 유체로 또한 각각의 체적내에서 압력은 공간상으로 일정하다고 가정하였다. 부양압력에 의한 자유표면의 변형은 선형포텐셜 이론을 사용하였으며, 부양압력을 진동하며 일정속도로 전진하는 압력면으로 대치하여 구하였다. Bag-Finger형 스커트는 표면적이 변화하지 않는 상태에서 변형한다고 가정하여, 압력변화와 자유표면 상승에 대한 변형모델을 제시하였고, 스커트의 복원력과 복원모멘트가 공기부양선의 상하동요, 종동요에 미치는 영향을 해석하였다.

Bag내의 압력과 부양압력을 변화시키며 또 스커트의 모양을 변화시키면서 파중에서 공기부양선의 운동응답을 계산하였다.

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1. Introduction

Since the first Air Cushion Vehicle, applying the idea of Christopher Cockerell, the SR.N1, was launched in England at 1959, many ACV's have been built and used for various applications because of their many advantages such as high speed and amphibiousness etc.. Theoretical and experimental works to analyze a craft ride quality have been performed by many researchers throughout the world. But analytical models for computing the response of hovercraft have not yet been developed sufficiently to be used as a design tool owing to the absence of adequate experimental data on some of the related mechanism[1-3].

The analysis of responses of an ACV over regular waves was started by Reynolds[4]. He developed a linearized equation of motion by considering a single-plenum craft with single-degree of freedom in heave. A quadratic expression for the fan characteristics, the incompressible Bernoulli equation and the usual equations of continuity were used in this analysis. Later Reynolds et al.[5] extended this work to include pitch motion in addition to heave by adopting a craft with a transverse skirt. The pressure deviations of the fore and aft compartment from their equilibrium values were used to formulate the equation of motion for pitch. The important assumptions included in both papers were that the skirt hemline makes no contact with the water surface and the wavy surface is rigid.

The effect of the presence of the water surface upon the perturbations of pressure in the plenum in a surface effect ship was examined by Breslin[6], Kim and Tsakonas[7], and in an ACV by Doctors[8, 9]. Breslin assumed that the deformation of the water surface participates in the generation of the bubble pressure in conjunction with the actions of seals, fans, etc., and that the deformation of the wave surface under the oscillatory rectangular pressure patch, having an infinite beam, in uniform translation be used to display the way in which the motion of the water surface participates in the determination of the pressure variations in the plenum air. His work was later extended to three-dimension by Kim and Tsakonas. They evaluated the wave elevation, the escape area at the stern and the volume induced by an oscillatory rectangular patch in uniform translation for the entire range of the speed frequency parameter τ of practical interest, from very low to considerably high. Earlier than the Kim and Tsakonas, Doctors had developed the same analysis to evaluate the hydrodynamic influence, and he applied this result to the motion of Air Cushion Vehicle which was taken by Reynolds. The hydrodynamic influence was felt through the alteration of the air gap under the skirt due to water deflection and a change in the effective flux balance of air in the cushion, which was assumed to be incompressible. Also he evaluated the nonlinear effect on the motion responses of the craft for different wave heights. He extended his previous work to higher Froude numbers and encounter frequencies of practical interest[10], and to include the effect of compressibility of the air by considering only the accumulation term in the continuity equations for the chambers.

Rhee and Lee[10] made a similar analysis to evaluate the responses of an ACV in uniform translation over regular waves, in which the effects of the height and inclination of the skirt on the motion responses were examined. They evaluated the hydrodynamic influence due to cushion pressure by referring to the works by Doctors[9], but developed a numerical approach that was valid for the entire range of the parameter τ and for a polygonal pressure patch by use of Stoke's theorem. In the dynamic analysis of the air flow in chamber and duct, the adiabatic and isentropic flow was applied directly to the equations of the mass conservation.

The object of this paper is to present a method for analyzing the skirt deformation due to pressure variations and surface elevations, and to evaluate the heave and pitch responses of an ACV with a deformable bag and finger skirt in uniform translation over regular waves. A model for the deformation of the bag-finger skirt is proposed. The hydrodynamic influence and the air flow are considered in line with Rhee and Lee[10], but the skirt deformation, its effects on the air flow and skirt forces are included.

The heave and pitch response of an ACV with bag-finger skirt to regular waves are calculated for different ratios of the bag and cushion pressure and for different shapes of bag-finger skirt by using the linear equations of motion. Our results show that the shape of bag-finger skirt has an important effect on the motion responses in the neighborhood of the helmholtz resonance.

2. Model of Skirt Deformation

A bag and finger type skirt, which is shown in Fig. 1, is considered in this study. The skirt deformation is assumed to depend on the restoring coefficient, the pressure in the bag and in the plenum chamber, and the free surface elevation. An analysis of the restoring force of the bag is given in Appendix for a simplified model.

In this chapter, the skirt deformation due to the pressure changes and the free surface elevation is examined.

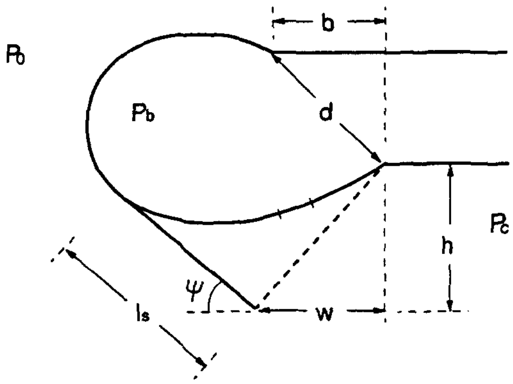

Fig. 1 Schematic view of a bag-finger skirt

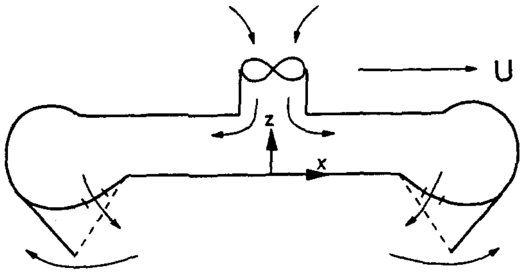

Fig. 2 Model of a craft and coordinate system

2.1 Deformation due to the Pressure Changes in the Bag

When the pressure in the bag P_b increases, the bag moves downward. The pressure is understood as a gage pressure hereafter. Suppose that pressure P_b is increased by ΔP_b and that the bag is deformed by $\Delta\psi$. The upward force variation acting on the bag is

$$-b\Delta P_b,$$

and on the finger is

$$-P_c l_s \cos(\psi+\Delta\psi) + P_c l_s \cos\psi \simeq P_c l_s \sin\psi\Delta\psi,$$

where P_c is the pressure in the plenum chamber. The restoring force of the bag is expressed as,

$$P_b d\Delta\psi.$$

From the equilibrium condition of forces, we can obtain the angular deformation $\Delta\psi$ as

$$\Delta\psi = \frac{-b\Delta P_b}{P_b d - P_c l_s \sin\psi}. \tag{2.1}$$

Since the force acting on the bag and its counterpart on the structure cancel each other, the force acting on the skirt system due to the pressure increase ΔP_b becomes

$$P_b d\Delta\psi + b\Delta P_b = -b\left(\frac{1}{D} - 1\right)\Delta P_b, \tag{2.2}$$

where D is defined as

$$D \equiv 1 - \frac{P_c l_s \sin\psi}{P_b d}. \tag{2.3}$$

From the angular displacement $\Delta\psi$, we obtain the vertical displacement of the lowest point of the finger,

$$\omega\Delta\psi = -\frac{wb}{P_b d D}\Delta P_b, \tag{2.4}$$

and the horizontal displacement of the point,

$$h\Delta\psi = -\frac{hb}{P_b d D}\Delta P_b. \tag{2.5}$$

2.2 Deformation due to the Pressure Changes in the Plenum Chamber

As the pressure in the plenum chamber P_c increases, the bag will deform in the upward direction. Suppose that the pressure P_c is increased by ΔP_c and that the bag is deformed by $\Delta\psi$. The upward force acting on the bag is changed by

$$\Delta P_c(w + l_s \cos\psi),$$

and on the finger is

$$-\Delta P_c l_s \cos\psi + P_c l_s \sin\psi\Delta\psi.$$

And the restoring force of the bag is

$$P_b d\Delta\psi.$$

Similarly, we can obtain the angular displacement $\Delta\psi$ as

$$\Delta\psi = \frac{w}{P_b d D}\Delta P_c. \tag{2.6}$$

Among the forces acting on the bag, $w\Delta P_c$ will be included in the force on the pressurized support area, so we omit the term $w\Delta P_c$ here. Therefore the upward force acting on the skirt system due to the pressure increase ΔP_c in the plenum chamber can be represented as

$$P_b d\Delta\psi - w\Delta P_c = w\left(\frac{1}{D} - 1\right)\Delta P_c. \tag{2.7}$$

The vertical displacement of the lowest point of the finger is

$$w\Delta\psi = \frac{w^2}{P_b dD}\Delta P_c, \tag{2.8}$$

and the horizontal displacement of the point,

$$h\Delta\psi = \frac{hw}{P_b dD}\Delta P_c. \tag{2.9}$$

(Positive sign corresponds to the outward direction.)

2.3 Deformation due to the Free Surface Elevation

As the free surface below the skirt system moves upward, the finger touches the surface and the downward force acting on the finger is decreased. In consequence, the bag experiences upward force and the bag deforms upward and lifts up the finger.

If the skirt system cannot be deformed, the upward force of the skirt due to the surface elevation can be represented as

$$P_c\frac{h_w}{\tan\psi}, \tag{2.10}$$

where h_w is the elevation of the surface.

Suppose that the bag is deformed by an amount of angular deformation $\Delta\psi$, then the restoring for of the bag is

$$P_b d\Delta\psi.$$

Furthermore, when the lower part of the finger is immersed, the upward force acting on the finger is

$$P_c(l_s\sin\psi\,\Delta\,\psi + \cos\psi\Delta l),$$

where Δl is a wetted length along the finger. Inspecting the skirt geometry, following relation for Δl and $\Delta\psi$ is obtained.

$$\Delta l = \frac{1}{\sin\psi}(h_w - w\Delta\psi). \tag{2.11}$$

where Δl must be positive. Thus $\Delta\psi$ must satisfy the following condition,

$$\Delta\psi \le \frac{hw}{w}. \tag{2.12}$$

From the equilibrium condition of forces, we can obtain $\Delta\psi$ as follows,

$$\Delta\psi = \frac{P_c\frac{hw}{\tan\psi}}{P_b d - P_c l_s\sin\psi + P_c\frac{hw}{\tan\psi}}. \tag{2.13}$$

In order to satisfy the condition (2.12), the following ineguality must hold,

$$P_b d - P_c l_s\sin\psi \ge 0,$$

which may be rewritten as

$$D = 1 - \frac{P_c l_s\sin\psi}{P_b d} \ge 0. \tag{2.14}$$

If (2.14) is not satisfied, there is no way to satisfy the equilibrium condition of forces, so the bag continues to undergo a deformation. However, this will not happen in real situation, and this just means that the skirt is in its unstable equilibrium state. When D equals to unity, the deformation of skirt due to the surface elevation will be small. And when D is zero, the skirt will be deformed in such a way that the lowest point of the finger always remains on the surface. As D becomes smaller, the skirt will deform more easily due to the surface elevation. D may be used as the criterion of the skirt responsiveness.

We may rewrite the angular displacement of the bag due to the surface elevation as

$$\Delta\psi = \frac{P_c\frac{hw}{\tan\psi}}{P_b dD + P_c\frac{w}{\tan\psi}}.$$

The upward force acting on the skirt system is then

$$P_b d\Delta\psi = P_c\frac{h_w}{\tan\psi}\;\frac{P_b d}{P_b dD + P_c\frac{w}{\tan\psi}} = \frac{h_w}{\tan\psi}\,[1 + D'], \tag{2.15}$$

where D' is defined as follows,

$$D' = \frac{1 - D - \frac{P_c}{P_b}\frac{w}{d}\frac{1}{\tan\psi}}{D + \frac{P_c}{P_b}\frac{w}{d}\frac{1}{\tan\psi}}. \tag{2.16}$$

Comparing this with (2.10), the upward skirt force is increased by the skirt response-force factor D'.

3. Mathematical Model

The heave and pitch responses of an Air Cushion Vehicle with a bag-finger skirt travelling at a speed of advance U in a regular waves are examined in this chapter. The coordinate system and the craft are shown in Figure 2. The origin of coordinate lies at midship and vertically at the top of plenum chamber.

3.1 Air Flows

The air pressure and density changes are assumed to have the adiabatic isentropic relationship, *i.e.*,

$$\rho_c = \rho_a \left(1 + \frac{P_c}{P_a}\right)^{1/\gamma}, \tag{3.1}$$

where ρ_a and P_a are the density and the pressure of air in atmospheric condition respectively, and ρ_c and P_c are in the plenum chamber, γ is the ratio of specific heat and is taken as 1.4.

Under the assumption that the air is compressible, and that the instantaneous pressure is uniform throughout the volume at any instant in time, the conservation of mass for the plenum chamber may be written as

$$\frac{d}{dt}(\rho_c V_c) = \dot{\rho}_c V_c + \rho_c \dot{V}_c = \rho_c Q_i - \rho_c Q_e, \tag{3.2}$$

where V_c is the volume of the plenum chamber, Q_i and Q_e the volumetric flow rate entering into and exiting from the plenum chamber, respectively. And the dot above a term stands for a derivative with respect to time. Using the equation (3.1), the above equation may be rewritten as

$$\frac{V_c}{\gamma(P_c + P_a)} \dot{P}_c = Q_i - Q_e - \dot{V}_c. \tag{3.3}$$

Similarly, the conservation of mass in the bag and duct may be written as

$$\frac{V_d}{\gamma(P_b + P_a)} \dot{P}_b = -\left(\frac{1 + P_c/P_a}{1 + P_b/P_a}\right)^{\delta} Q_i + Q_f, \tag{3.4}$$

where V_d and P_b are the volume and pressure in the bag and duct, respectively, Q_f the inlet volume flow rate into the bag and duct, and δ is the reciprocal of γ.

The flow rate through the fan Q_f and the pressure difference across the fan P_f (this is normally P_b in the absence of the secondary duct, hence, P_f will be the same as P_b hereafter) are assumed to have the relation below.

$$P_f = C_1 + C_2 Q_f + C_3 Q_f^2 \tag{3.5}$$

where C's are constants given from experiments of the fan characteristics.

The volumetric flow rates are assumed to be governed by the steady orifice flow law, then Q_i and Q_e can be represented as follows,

$$Q_i = \left(\frac{1 + P_b/P_a}{1 + P_c/P_a}\right)^{\delta} kA_i \sqrt{2(P_b - P_c)/\rho_a (1 + P_b/P_a)^{\delta}}, \tag{3.6}$$

$$Q_e = kA_e \sqrt{2P_c/\rho_a (1 + P_c/P_a)^{\delta}}. \tag{3.7}$$

where k is an orifice flow discharge coeffcient, A_i the inlet orifice area into the plenum chamber and A_e the escape area under the skirt.

The rate of change of the plenum chamber is assumed to be

$$\dot{V}_c = A_c\{\dot{z} - x_c\dot{\theta}\} - \dot{V}_{cpw} + \dot{V}_w + \dot{V}_s, \tag{3.8}$$

where A_c is the pressurized support area, x_c the centroid of A_c, and z and θ denote the heave and pitch displacements. And V_{cpw} is the volume change due to the free surface deformation (will be explained in the next section), V_w due to incident waves and V_s due to skirt deformations. The escape area under the skirt may be written as

$$A_e = A_{eo} + \eta_s \int_l (z - x\theta - \zeta_{pw} - \zeta_w)\, dl + \eta_s A_{es}, \tag{3.9}$$

where A_{eo} denotes the escape area at the equilibrium state, ζ_{pw} and ζ_w are the free surface elevation due to the cushion pressure and the incident waves respectively, A_{es} is the escape area due to the skirt deformation. The integral has to be carried out along the cushion perimeter. Since the escape area does not change proportionally to the relative motion responses, η_s is introduced to evaluate the escape area properly.

The deviations of the variables from their equilibrium values are assumed to be small, and the equilibrium values are denoted by placing subscript 'o', henceforth. We linearize the inlet flow as follows,

$$Q_i = Q_{ipc} P_c + Q_{ipb} P_b, \tag{3.10}$$

where

$$Q_{ipc}=-\frac{\rho_{do}}{\rho_{co}}kA_i\sqrt{\frac{2(P_{bo}-P_{co})}{\rho_{do}}}\left(\frac{\delta}{P_{co}+P_a}+\frac{1}{2(P_{bo}-P_{co})}\right),$$

$$Q_{ipb}=\frac{\rho_{do}}{\rho_{co}}kA_i\sqrt{\frac{2(P_{bo}-P_{co})}{\rho_{do}}}\left(\frac{\delta}{2(P_{bo}+P_a)}+\frac{1}{2(P_{bo}-P_{co})}\right).$$

And,

$$Q_i'=kA_i\sqrt{\frac{2(P_b-P_c)}{\rho_d}}$$

$$=Q_{ipc}'P_c+Q_{ipb}'P_b, \tag{3.11}$$

where

$$Q_{ipc}'=-kA_i\sqrt{\frac{2(P_{bo}-P_{co})}{\rho_{do}}}\frac{1}{2(P_{bo}-P_{co})},$$

$$Q_{ipb}'=kA_i\sqrt{\frac{2(P_{bo}-P_{co})}{\rho_{do}}}\left(\frac{1}{2(P_{bo}-P_{co})}-\frac{\delta}{2(P_{bo}-P_a)}\right).$$

And the escape area,

$$A_e=A_{ez}z+A_{e\theta}\theta+A_{epc}P_c+A_{epb}P_b+A_{ew}\zeta, \tag{3.12}$$

where ζ is the amplitude of the incident wave and

$$A_{ez}=\eta_s\int_l dl,$$

$$A_{e\theta}=-\eta_s\int_l xdl,$$

$$A_{epc}=\eta_s\int_l\frac{w^2}{P_{bo}dD}dl-\eta_s\int_l\zeta_{pw}dl,$$

$$A_{epb}=-\eta_s\int_l\frac{wb}{P_{bo}dD}dl,$$

$$A_{ew}=-\eta_s\int_l\zeta_w dl.$$

ζ_{pw} and ζ_w are the surface elevations due to the cushion pressure and of the incident wave, respectively. And the outlet flow becomes

$$Q_e=Q_{ez}z+Q_{e\theta}\theta+Q_{epc}P_c+Q_{epb}P_b+Q_{ew}\zeta, \tag{3.13}$$

where

$$Q_{ez}=k\sqrt{2P_{co}/\rho_{co}}A_{ez},$$

$$Q_{e\theta}=k\sqrt{2P_{co}/\rho_{co}}A_{e\theta},$$

$$Q_{epc}=k\sqrt{2P_{co}/\rho_{co}}\left(A_{epc}+A_{eo}\left[\frac{1}{2P_{co}}-\frac{\delta}{2(P_{co}+P_a)}\right]\right),$$

$$Q_{epb}=k\sqrt{2P_{co}/\rho_{co}}A_{epb},$$

$$Q_{ew}=k\sqrt{2P_{co}/\rho_{co}}A_{ew}.$$

And the time rate of volume change is

$$\dot V_c=V_z\dot z+V_\theta\dot\theta+V_{pc}\dot P_c+V_{pb}\dot P_b+V_w\dot\zeta, \tag{3.14}$$

where

$$V_z=V_c-\eta_s\int_l(w^2+h^2)\frac{P_{co}(1+D')}{P_{bo}d\tan\psi}dl,$$

$$V_\theta=-A_ox_c+\eta_s\int_l(w^2+h^2)\frac{P_{co}(1+D')}{P_{bo}d\tan\psi}xdl,$$

$$V_{pc}=-A_{cpw}+\int_l(w^2+h^2)\frac{w}{P_{bo}dD}dl+\eta_s\int_l(w^2+h^2)\frac{P_{co}(1+D')}{P_{bo}d\tan\psi}\zeta_{pw}dl,$$

$$V_{pb}=-\int_l(w^2+h^2)\frac{b}{P_{bo}dD}dl,$$

$$V_w=-A_{cw}+\eta_s\int_l(w^2+h^2)\frac{P_{co}(1+D')}{P_{bo}d\tan\psi}\zeta_w dl,$$

whre V_{cpw} is the volume change due to the free surface deformation excited by the cushion pressure and V_{cw} due to the incident wave (both will be given in the next section).

3.2 Free Surface Deformation

The deformation of free surface due to the cushion pressure was obtained by Rhee and Lee[10]. We use their results, and just write down the assumptions and methods of calculation only here.

The water is assumed to be incompressible and inviscid, and the water depth is infinite. The cushion pressure is replaced with the pressure patch which oscillates and translates with a constant speed on the otherwise calm water. The shape of the pressure patch is restricted within a polygonal one. The free surface deformation is obtained using a linear potential theory. The resulting free surface deformation is proportional to the applied pressure, *i.e.*

$$\zeta_{pw}\cdot P_c$$

The free surface elevation of the regular head waves incoming from the positive x-axis is

$$\zeta_a e^{i(k[x+Ut]+wt)} = \zeta_a e^{i(kx+w_e t)} = \zeta_w \cdot \zeta, \tag{3.15}$$

$$w_e = w + Uk\ ,\ k = w^2/g$$
$$\zeta_w = e^{ikx}\qquad ,\ \zeta = \zeta_a e^{iw_e t}$$

where ζ_a is a wave amplitude, k the wave number, w a circular frequency of incoming wave and w_e an encountering frequency.

The escape area due to the free surface deformation is obtained by integrating the free surface elevations along the skirt perimeter, and the volume changes V_{cpw}, V_{cw} are obtained by integrating the free surface elevation over the cushion area. The integration over the cushion area is transformed to the integral along the skirt perimeter by the Stoke's theorem.

3.3 Skirt Forces

In chapter 2, the force acting on the skirt is analyzed locally. Each element of the skirt is assumed to move independently, and the frictional force due to the contact of the finger with the water surface is neglected.

Forces and moments of the skirt system can be obtained by integrating the local forces.

$$F_s = \int_l dF_V,$$
$$M_s = -\int_l x dF_V + \int_l (z_G + h)\sin\beta dF_H, \tag{3.16}$$

where dF_V and dF_H are the vartical and horizontal force component of the skirt system respectively, and β is the angle of skirt hemline to the positive x-direction.

From the results in chapter 2, the vertical force of the skirt system can be obtained as follows,

$$F_s = F_z z + F_\theta\theta + F_{pc}P_c + F_{pb}P_b + F_w\zeta, \tag{3.17}$$

where

$$F_z = -\eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}dl$$
$$F_\theta = \eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}xdl$$
$$F_{pc} = \int_l \frac{w}{D}dl + \eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}\zeta_{pw}dl$$
$$F_{pb} = -\int_l \frac{b}{D}dl$$
$$F_w = \eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}\zeta_w dl.$$

The bow-down pitching moment is

$$M_c = M_z z + M_\theta\theta + M_{pc}P_c + M_{pb}P_b + M_w\zeta, \tag{3.18}$$

where

$$M_z = \eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}xdl - \eta_s P_{co}\int_l (z_G+h)\sin\beta dl$$
$$M_\theta = -\eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}x^2 dl - \eta_s P_{co}\int_l (z_G+h)\sin\beta x dl$$
$$M_{pc} = -\int_l \frac{w}{D}xdl - \eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}\zeta_{pw}xdl$$
$$+\ \eta_s P_{co}\int_l (z_G+h)\sin\beta\zeta_{pw}dl$$
$$M_{pb} = -\int_l \frac{b}{D}xdl$$
$$M_w = -\eta_s P_{co}\int_l \frac{1+D'}{\tan\psi}\zeta_w x dl + \eta_s P_{co}\int_l (z_G+h)\sin\beta\zeta_w dl,$$

where z_G is the vertical position of the center of gravity.

3.4 Equations of Motions

The heave and pitch equations of motion about the origin of coordinate is

$$m\ddot{z} - mx_G\ddot{\theta} = A_c P_c + F_s, \tag{3.19}$$
$$I\ddot{\theta} - mx_G\ddot{z} = -A_c x_c P_c + M_s, \tag{3.20}$$

whrer m is the mass of the craft and I the moment of inertia, x_G the longitudinal position of the center of gravity.

The equations of conservation of mass in the plenum chamber and the bag and duct are

$$C_{pc}\dot{P}_c = Q_i - Q_e - \dot{V}_c, \tag{3.21}$$
$$C_{pb}\dot{P}_b = -Q_i' + Q_f, \tag{3.22}$$

where

$$C_{pc} = \frac{\delta V_c}{P_{co} + P_a}\ ,\ C_{pb} = \frac{\delta V_d}{P_{bo} + P_a}. \tag{3.23}$$

The craft motions can be obtained by solving these four equations simultaneously. Using the variables appeared in the preceding sections, we may rewrite the equations of motions as

$$V_z\dot{z} + Q_{ez}z + V_\theta\dot{\theta} + Q_{e\theta}\theta + (C_{pc} + V_{pc})\dot{P}_c + (Q_{epc} - Q_{ipc})P_c$$
$$+ V_{pb}\dot{P}_b + (Q_{epb} - Q_{ipb})P_b = -V_w\dot{\zeta} - Q_{ew}\zeta, \tag{3.24}$$

$$Q_{ipc}'P_c + C_{pb}\dot{P}_b + \left(Q_{ipb}' - \frac{1}{C_2 + 2C_3Q_{fo}}\right)P_b = 0, \tag{3.25}$$

$$m\ddot{z}-F_z z-mx_G\ddot{\theta}-F_\theta\theta-(A_c+F_{pc})P_c-F_{pb}P_b=F_w\zeta, \quad (3.26)$$

$$-mx_G\ddot{z}-M_z z+I\ddot{\theta}-M_\theta\theta+(A_c x_c+M_{pc})P_c-M_{pb}P_b=M_w\zeta, \quad (3.27)$$

4. Numerical Results

To investigate the effects of a skirt deformation on the motion responses of an ACV in waves, the heave and pitch response of a Plenum-Chamber Type ACV with a bag-finger type skirt in regular head waves have been calculated in a framework of a linear theory. The schematic views of the craft and the bag-finger particulars are given in Table 1 and 2. The craft is assumed to have a constant speed in waves, and the motion responses are calculated in a frequency domain at cushion length based Froude numbers of 1.0 and 1.5. The cushion length of the craft is 20 *m*. In all figures, the motion responses of the craft having the same principal particulars in Table 1, but with a rigid skirt, are shown as a solid line for reference. The heave response is nondimensionalized by the incident wave amplitude and the pitch response by the maximum slope of the incident wave.

Table 1 Principal particulars of the craft and coefficients

B/L	0.5	A_i/L^2	0.01
$m/\rho_w L^3$	0.006	V_d/L^3	0.0125
$I/\rho_w L^5$	3.25×10^{4}	k	0.6
x_G/L	0.05	$C_1/\rho_w gL$	0.04
x_G/L	0	$C_2\sqrt{L^3/g\rho_w^2}$	0.0
h/B	0.1	C_3L^4/ρ_w	−30.

Table 2 Particulars of the bag-finger skirt

l_s/B	0.07	w/B	0.02
b/B	0.05	ψ	45°
d/B	0.1	η_s	1

In Fig. 3 and 4, the effect of a ratio of bag pressure to cushion pressure on the motion response is shown. In the calculation, the ratio of bag pressure to cushion pressure is obtained by changing the inlet area. The calculated pressure in the bag, P_b is 6257, 4369, and 3484 (N/m^2), and these values correspond to the nondimensionalized inlet area, A_i/L^2, 0.005, 0.01, and 0.015, respectively, while the cushion pressure is kept constant of the value of 2412 (N/m^2) for these cases. And the skirt responsiveness factor D is 0.8092, 0.7267, 0.6573 and the skirt response-force factor D' 0.1283, 0.1946, 0.2566, respectively. From these figures, the increase of the bag pressure to cushion pressure ratio decreases the skirt repsponse-force factor and increases the heave responses.

In Fig. 5 and 6, the motion responses are calculated for various d/B's. The calculations are carried out fot the values of 0.07, 0.10, 0.13, and D and D' are obtained as 0.6096, 0.7267, 0.7898 and 0.3032, 0.1946, 0.1432, respectively. The heave response is increased with the value of d/B, but the increment is not so significant.

In Fig. 7 and 8, the motion responses are calculated for various b/B's. For the values of b/B, 0.02, 0.05, 0.08, D and D' are not changed and their values are 0.7267 and 0.1946, respectively. The motion response increases as b/B decreases.

Fig. 9 and 10 show the effect of w/B on the motion responses. In this case, D can not be changed and D' has the values of 0.3761, 0.1946, 0.05534 according to

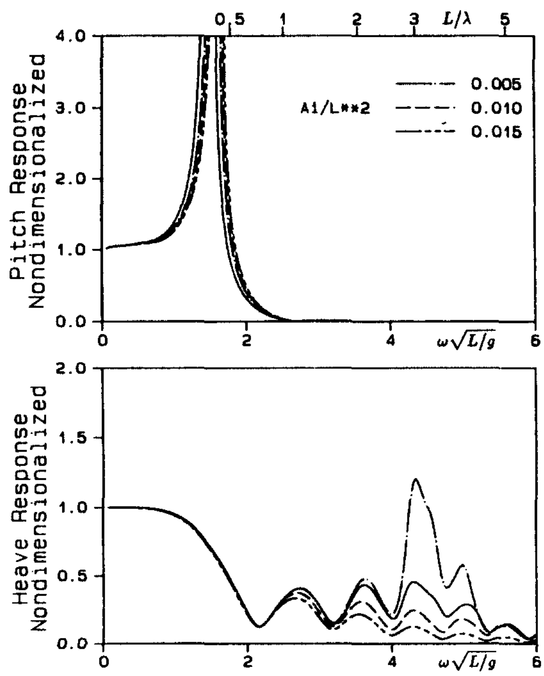

Fig. 3 Motion responses with various A_i/L^2's at F_n=1.0. solid line is for the rigid skirt

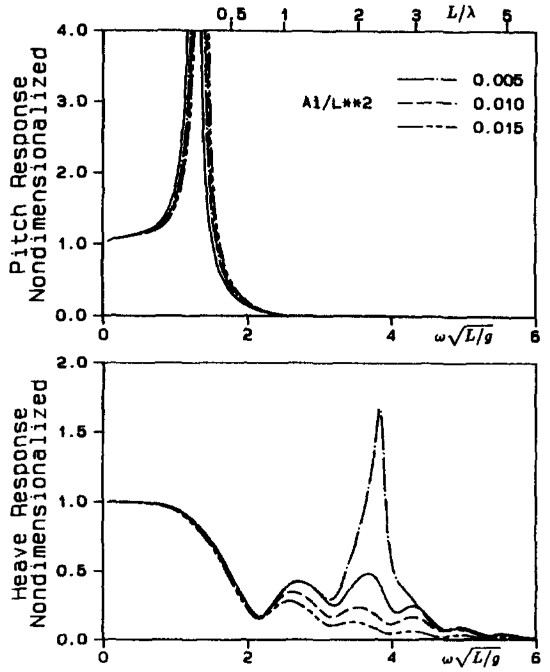

Fig. 4 Motion responses with various A_l/L^2's at F_n=1.5. solid line is for the rigid skirt

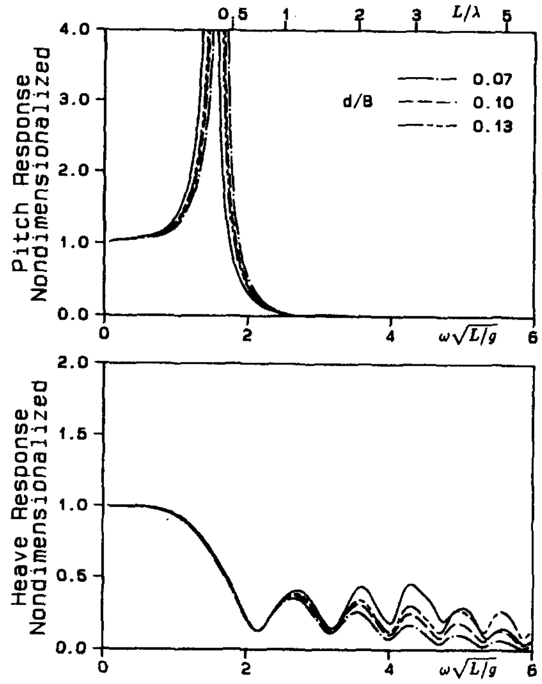

Fig. 5 Motion responses with various d/B's at F_n=1.0. solid line is for the rigid skirt

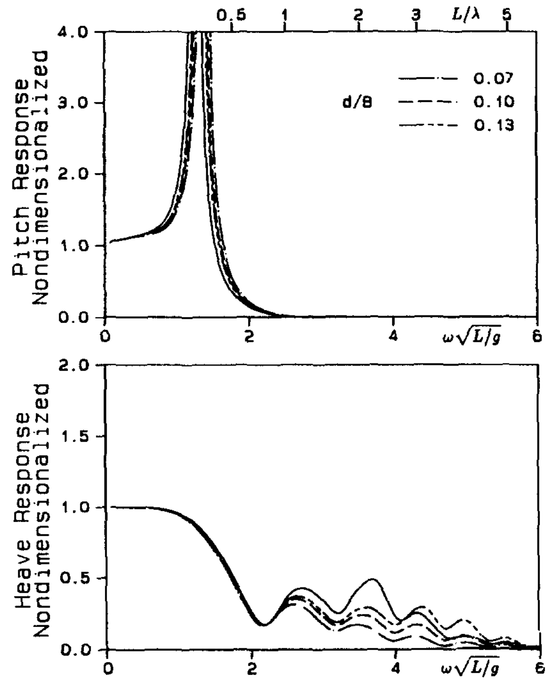

Fig. 6 Motion responses with various d/B's at F_n=1.5. solid line is for the rigid skirt

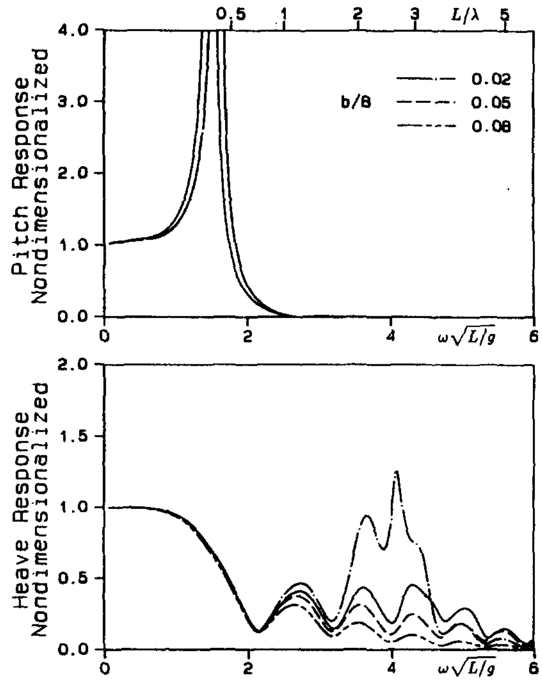

Fig. 7 Motion responses with various b/B's at F_n=1.0. solid line is for the rigid skirt

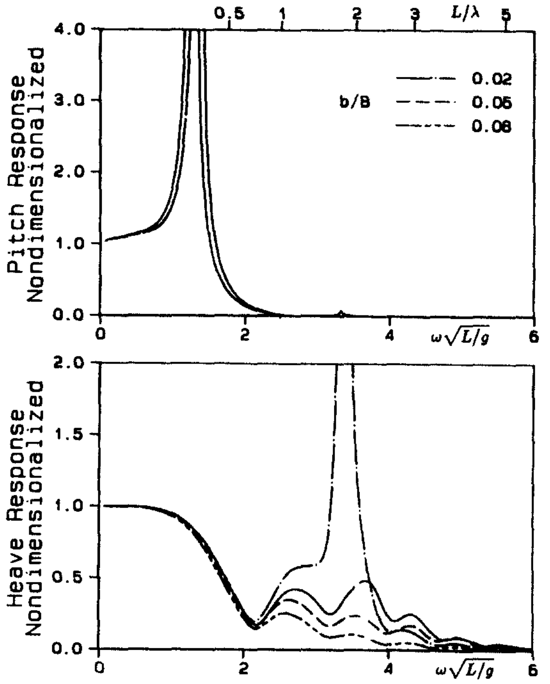

Fig. 8 Motion responses with various b/B's at F_n=1.5. solid line is for the rigid skirt

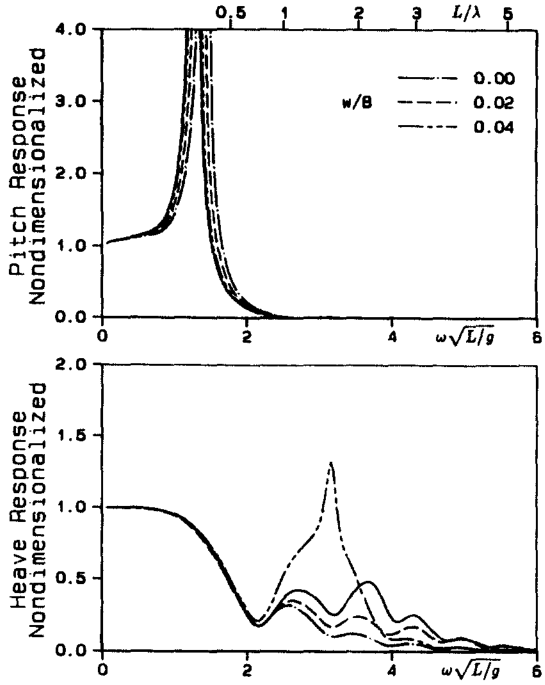

Fig. 10 Motion responses with various w/B's at F_n=1.5. solid line is for the rigid skirt

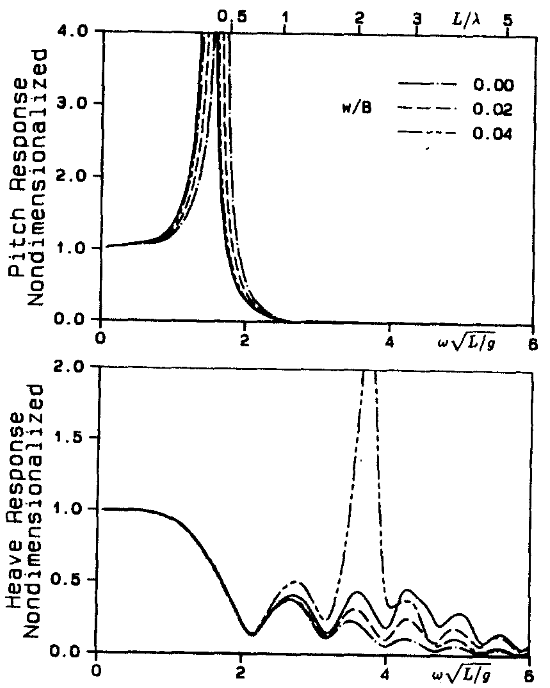

Fig. 9 Motion responses with various w/B's at F_n=1.0. solid line is for the rigid skirt

the different values of w/B of 0.0, 0.02, 0.04, respectively. The motion response increases as w/B.

Above results may be summarized as follows : The skirt response-force factor D' can be used to predict the craft motion. The motion response grows much as D' decreases, and the values of D' can be easily changed by changing the values of w/B.

On the other hand, the skirt responsiveness factor D can be used to predict the skirt deformation, as D decreases, the skirt deformation becomes large, and a negative value of D indicates that the unstable skirt deformation may happen.

However, since D and D' are not affected by b/B, a new formulation for D and D' which includes this effect seems to be needed.

Conclusions

A simplified model has been developed to explain the skirt deformation due to the pressure change in the bag and in the plenum chamber and that due to the water surface elevation. This model can also predict the skirt force.

A series of numerical calculations for the craft motion has been carried out for the heave and pitch response for different values of inlet area and for various skirt parameters. Through this investigation, our findings are :

1. The motion response grows much as the skirt response-force factor D' decreases.

2. To reduce the motion response by increasing D', increase of the inlet area, decrease of d/B or decrease of w/B is needed.

3. The skirt deforms more easily as the skirt responsiveness factor D decreases. To reduce D, increase of the inlet area or decrease of d/B is needed.

The skirt responsiveness factor D and the skirt response-force factor D' can be used to predict the skirt responsiveness and the craft motion respectively. We hope that the comparison with experimental study can be made, and that an additional factor which can include the effect of b/B can be found.

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Appendix. The Restoring Coefficient of a Bag

The cross section of the simplified model of a bag which is used to formulate the restoring forces is shown below.

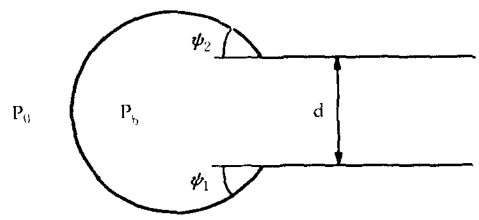

Fig. for appendix

The bag is assumed to be attached to the main structures with hinge, and thus only the restoring force, not the moment, can be transmitted to the structure. The lateral force acting on the bag can be represented as follows,

$$T(\cos\psi_2 + \cos\psi_1) = d(P_b - P_0). \qquad (A.1)$$

where T is the tensile force of the bag and can be regarded constant along the bag, and d is the distance between two points where the bag is attached to the structure. The upward force acting on the bag can be given as

$$T(\sin\psi_2 - \sin\psi_1) = F_0. \quad (A.2)$$

Wince the surface area of the bag is presumed constant through the entire period of motion, the variation of a upward force ΔF will cause angular deformations $\Delta\psi_1$ and $\Delta\psi_2$ at higher and at lower hinge points. Then the upward force becomes

$$F_0 + \Delta F = T(\sin(\psi_2 + \Delta\psi_2) - \sin(\psi_1 + \Delta\psi_1)). \quad (A.3)$$

Thus ΔF becomes

$$\Delta F = T(\sin(\psi_2 + \Delta\psi_2) - \sin(\psi_1 + \Delta\psi_1) - \sin\psi_2 + \sin\psi_1). \quad (A.4)$$

If we suppose $\Delta\psi_2 = -\Delta\psi_1 = \Delta\psi \ll 1$, then

$$\Delta F = T(\cos\psi_2 + \cos\psi_1)\Delta\psi. \quad (A.5)$$

Substituting (A.1) into (A.5), we obtain

$$\Delta F = d(P_b - P_0)\Delta\psi \quad (A.6)$$

Thus the restoring coefficient of a bag due to the angular deformation can be written as

$$\frac{dF}{d\psi} = d(P_b - P_0). \quad (A.7)$$