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AUTOCORRELATION FUNCTION STRUCTURE OF BILINEAR TIME SERIES MODELS ¹

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ABSTRACT

The autocorrelation function structures of bilinear time series model $BL(p, q, r, s)$, $r \geq s$ are obtained and shown to be analogous to those of $ARMA(p, l)$, $l = \max(q, s)$. Simulation studies are performed to investigate the adequacy of Akaike information criteria for identification between $ARMA(p, l)$ and $BL(p, q, r, s)$ models and for determination of orders of $BL(p, q, r, s)$ models. It is suggested that the model of having minimum Akaike information criteria is selected for a suitable model.

1. INTRODUCTION

Time series analysis has been focus on the linear models. However, there are many situations in which the linear models are not appropriate to represent real time series. Therefore, it is natural to ask if there exist non-linear models which provide a better fit to reality. One of the non-linear time series models suggested in recent years is the bilinear model.

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The general bilinear model of order p, q, r , and s , denoted by $BL(p, q, r, s)$, is defined by

$$X_t = \sum_{i=1}^p a_i X_{t-i} + e_t + \sum_{j=1}^q c_j e_{t-j} + \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} e_{t-j}, \quad (1.1)$$

where $\{e_t\}$ is a sequence of independent and identically distributed random variables with mean 0 and variance $\sigma^2 < \infty$.

Subclasses of the model (1.1) have been considered in many works since Granger and Andersen (1978) have introduced them. Stationarity, invertibility, and ergodicity for subclasses of the model (1.1) have been subsequently studied in Subba Rao (1981), Quinn (1982), Bhaskara Rao et al. (1983), and Akamanan et al. (1986). Estimations of parameters have been considered in Pham and Tran (1981), Subba Rao and Gabr (1984), and Won Kyung Kim et al. (1990).

It is well known that the bilinear model $BL(p, 0, p, 1)$ has a similar autocorrelation structure to that of the autoregressive-moving average model $ARMA(p, 1)$ (Subba Rao, 1981). This fact makes identification problem arise. Granger and Anderson (1978) suggested the autocorrelation function of the squared process $\{X_t^2\}$ for identification, while Kumar (1986) considered the third-order moments. However, the autocorrelation structures of the general bilinear model $BL(p, q, r, s)$ have not been investigated.

In this paper, we study the autocorrelation structure of the bilinear model $BL(p, q, r, s)$, $r \geq s$, and perform some simulation studies to investigate the adequacy of Akaike information criteria (AIC) for identification between ARMA and BL models and for determination of orders of BL models.

2. PRELIMINARY

The general bilinear model (1.1) can be classified by three subclasses. If $b_{ij} = 0$ for all $i \neq j$ in (1.1), the bilinear model is called diagonal. If $b_{ij} = 0$ for all $i < j$, the model is called superdiagonal. In the case of diagonal and superdiagonal models, the e_{t-j} occurs after the independent X_{t-i} in the multiplicative terms with non-zero coefficients. This fact makes analysis somewhat easier. On the other hand, the model is said to be subdiagonal if $b_{ij} = 0$ for all $i > j$. In this case, analysis is very difficult since the X_{t-i} occurs strictly after the e_{t-j} .

In this paper, we restrict our attention to diagonal and superdiagonal models which are assumed to be stationary. The stationarity conditions for the models $BL(p, 0, p, q)$ and $BL(p, q, r, s)$, $r \geq s$, are obtained in Bhaskara Rao et al. (1983) and Akamanan et al. (1986), respectively.

3. AUTOCORRELATION STRUCTURE

The autocorrelation structures of stationary diagonal and superdiagonal bilinear models are obtained in the following theorems.

Lemma 1. For the diagonal bilinear model $DBL(0, q, s, s)$, $q \geq 0, s \geq 1$,

$$X_t = e_t + \sum_{i=1}^q c_i e_{t-i} + \sum_{j=1}^s b_{jj} X_{t-j} e_{t-j}, \quad (3.1)$$

the autocorrelation function satisfies the following equation.

$$\rho(k) = \gamma(0)^{-1} \left\{ \sum_{i=k}^q c_i E(e_{t-i} X_{t-k}) + \sum_{j=1}^s b_{jj} E(e_{t-j} X_{t-j} X_{t-k}) - \mu^2 \right\}, \quad k = 1, 2, \dots \quad (3.2)$$

where $\gamma(0) = E(X_t^2) - \mu^2$ and $\mu = E(X_t) = \sigma^2 \sum_{j=1}^s b_{jj}$.

If $k > \max(s, q)$, then we have $\rho(k) = 0$. (3.3)

Proof. Multiplying X_{t-k} on both sides of (3.1) and taking expectation gives

$$E(X_t X_{t-k}) = \sum_{i=k}^q c_i E(e_{t-i} X_{t-k}) + \sum_{j=1}^s b_{jj} E(e_{t-j} X_{t-j} X_{t-k}), \quad k = 1, 2, \dots \quad (3.4)$$

If $k > \max(s, q)$, then we have

$$E(X_t X_{t-k}) = \sigma^2 \mu \sum_{j=1}^s b_{jj}. \quad (3.5)$$

From the definition of autocorrelation function, i.e.,

$$\rho(k) = \gamma(k)/\gamma(0) = [E(X_t X_{t-k}) - \mu^2]/\gamma(0), \quad (3.6)$$

the proof is completed.

Lemma 2. For the diagonal bilinear model $DBL(p, 0, s, s)$, $p \geq 1, s \geq 1$,

$$X_t = \sum_{i=1}^p a_i X_{t-i} + e_t + \sum_{j=1}^s b_{jj} X_{t-j} e_{t-j}, \quad (3.7)$$

the autocorrelation function satisfies the following equation.

$$\begin{aligned} \rho(k) &= a_1 \rho(k-1) + a_2 \rho(k-2) + \dots + a_p \rho(k-p) \\ &+ \gamma(0)^{-1} \left\{ \sum_{j=1}^s b_{jj} E(X_{t-j} e_{t-j} X_{t-k}) - \mu^2 (1 - a_1 - a_2 - \dots - a_p) \right\}, \quad k = 1, 2, \dots, s \end{aligned}$$

$$\text{where } \mu = E(X_t) = \sigma^2 \sum_{j=1}^s b_{jj} / (1 - a_1 - a_2 - \cdots - a_p) \quad (3.8)$$

$$\rho(k) = a_1 \rho(k-1) + a_2 \rho(k-2) + \cdots + a_p \rho(k-p), \quad k \geq s+1. \quad (3.9)$$

Proof. Multiplying X_{t-k} on both sides of (3.7) and taking expectation gives

$$E(X_t X_{t-k}) = \sum_{i=1}^p a_i E(X_{t-i} X_{t-k}) + \sum_{j=1}^s b_{jj} E(e_{t-j} X_{t-j} X_{t-k}), \quad k = 1, 2, \dots \quad (3.10)$$

Therefore, (3.8) can be obtained from subtracting μ^2 and dividng by $\gamma(0)$ on both sides of (3.10). For $k \geq s+1$, we have

$$E(e_{t-j} X_{t-j} X_{t-k}) = \sigma^2 \mu, \quad j = 1, 2, \dots, s \quad (3.11)$$

where μ is given in (3.8).

Hence, the proof is completed.

Theorem 1. For the diagonal bilinear model DBL(p,q,s,s), $p \geq 1, q \geq 0, s \geq 1$,

$$X_t = \sum_{i=1}^p a_i X_{t-i} + e_t + \sum_{j=1}^q c_j e_{t-j} + \sum_{j=1}^s b_{jj} X_{t-j} e_{t-j}, \quad (3.12)$$

the autocorrelation function satisfies the following.

$$\rho(k) = a_1 \rho(k-1) + a_2 \rho(k-2) + \cdots + a_p \rho(k-p) + A_k, \quad k=1,2,\dots \quad (3.13)$$

where

$$A_k = \gamma(0)^{-1} \left\{ \sum_{i=k}^q c_i E(e_{t-i} X_{t-k}) + \sum_{j=1}^s b_{jj} E(e_{t-j} X_{t-j} X_{t-k}) - \mu^2 (1 - a_1 - a_2 - \cdots - a_p) \right\}$$

$$\text{and } \mu = \sigma^2 \sum_{j=1}^s b_{jj} / (1 - a_1 - a_2 - \cdots - a_p).$$

Particularly, we have that for $k > \max(s, q)$,

$$\rho(k) = a_1 \rho(k-1) + a_2 \rho(k-2) + \cdots + a_p \rho(k-p). \quad (3.14)$$

Proof. The expectation of cross product $X_t X_{t-k}$ is given by

$$E(X_t X_{t-k}) = \sum_{i=1}^p a_i E(X_{t-i} X_{t-k}) + \sum_{j=k}^q c_j E(e_{t-j} X_{t-k})$$

$$+ \sum_{j=1}^s b_{jj} E(e_{t-j} X_{t-j} X_{t-k}), \quad k = 1, 2, \dots \quad (3.15)$$

$$\text{If } k \geq q + 1, \text{ then } \sum_{j=1}^q c_j E(e_{t-j} X_{t-k}) = 0. \quad (3.16)$$

$$\text{If } k \geq s + 1, \text{ then } \sum_{j=1}^s b_{jj} E(e_{t-j} X_{t-j} X_{t-k}) = \mu \sigma^2 \sum_{j=1}^s b_{jj}. \quad (3.17)$$

Subtracting μ^2 from both sides of (3.15) and dividing $\gamma(0)$ completes the proof.

Lemma 3. For the superdiagonal bilinear model SBL(0,q,r,1), $q \geq 1, r \geq 2$,

$$X_t = e_t + \sum_{i=1}^q c_i e_{t-i} + \sum_{j=1}^r b_{j1} X_{t-j} e_{t-1}, \quad (3.18)$$

the autocorrelation function is obtained as follows.

$$\rho(k) = \gamma(0)^{-1} \left\{ \sum_{i=k}^q c_i E(e_{t-i} X_{t-k}) + \sum_{j=1}^r b_{j1} E(X_{t-j} e_{t-1} X_{t-k}) - \mu^2 \right\}, \quad k = 1, 2, \dots \quad (3.19)$$

$$\text{where } \mu = b_{11} \sigma^2. \quad (3.20)$$

$$\text{If } q=0, \text{ then we have } \rho(k) = 0, k \geq 2. \quad (3.21)$$

Proof . The proof is similar to that of Theorem 1.

Lemma 4. For the superdiagonal model SBL(0,q,r,s), $q \geq 0, 1 \leq s \leq r$,

$$X_t = e_t + \sum_{j=1}^q c_j e_{t-j} + \sum_{i \geq j=1}^r \sum_{j=1}^s b_{ij} X_{t-i} e_{t-j} \quad (3.22)$$

the autocorrelation function is obtained as follows.

$$\rho(k) = \gamma(0)^{-1} \left\{ \sum_{j=1}^q c_j E(e_{t-j} X_{t-k}) + \sum_{i \geq j=1}^r \sum_{j=1}^s b_{ij} E(X_{t-i} e_{t-j} X_{t-k}) - \mu^2 \right\} \quad k=1,2,\dots \quad (3.23)$$

$$\text{where } \mu = \sigma^2 \sum_{j=1}^s b_{jj}. \quad (3.24)$$

$$\text{If } k > \max(q,s), \text{ then we have } \rho(k) = 0. \quad (3.25)$$

Proof. The expectation of cross product $X_t X_{t-k}$ is given by

$$E(X_t X_{t-k}) = \sum_{j=1}^p c_j E(e_{t-j} X_{t-k}) + \sum_{i \geq 1}^r \sum_{j=1}^s b_{ij} E(X_{t-i} e_{t-j} X_{t-k}) \quad k = 1, 2, \dots, q \quad (3.26)$$

If $k > \max(q, s)$, then

$$E(e_{t-j} X_{t-k}) = 0, \quad j = 1, 2, \dots, q \quad (3.27)$$

$$\text{and } \sum_{i \geq 1}^r \sum_{j=1}^s b_{ij} E(X_{t-i} e_{t-j} X_{t-k}) = \sigma^2 \mu \sum_{j=1}^s b_{jj}. \quad (3.28)$$

Hence the proof is completed.

Theorem 2. For the superdiagonal model $\text{SBL}(p, q, r, s)$, $s \leq r$

$$X_t = \sum_{i=1}^p a_i X_{t-i} + e_t + \sum_{j=1}^q c_j e_{t-j} + \sum_{i \geq 1}^r \sum_{j=1}^s b_{ij} X_{t-i} e_{t-j}, \quad (3.29)$$

the following equations are satisfied.

$$\begin{aligned} \rho(k) = & a_1 \rho(k-1) + \dots + a_p \rho(k-p) + \gamma(0)^{-1} \left\{ \sum_{i=1}^q c_i E(e_{t-i} X_{t-k}) \right. \\ & \left. + \sum_{i \geq 1}^r \sum_{j=1}^s b_{ij} E(X_{t-i} e_{t-j} X_{t-k}) - \mu^2 (1 - a_1 - a_2 - \dots - a_p) \right\}, \quad k = 1, 2, \dots \end{aligned} \quad (3.30)$$

$$\text{where } \mu = \sigma^2 \sum_{j=1}^s b_{jj} / (1 - a_1 - a_2 - \dots - a_p). \quad (3.31)$$

Particularly, we have that for $k > \max(q, s)$,

$$\rho(k) = a_1 \rho(k-1) + a_2 \rho(k-2) + \dots + a_p \rho(k-p). \quad (3.32)$$

Proof. The proof is similar to that of lemma 4 except including terms of $a_i \rho(k-i)$, $i = 1, 2, \dots, p$.

Remark 1. From the above lemmas and theorems, the autocorrelation structures of bilinear models are similar to those of ARMA models. i.e.,

- (i) $\text{DBL}(0, q, s, s)$, $q \geq 0$, $s \geq 1 \equiv \text{MA}(l)$, $l = \max(q, s)$
- (ii) $\text{DBL}(p, 0, s, s)$, $p \geq 1$, $s \geq 1 \equiv \text{ARMA}(p, s)$
- (iii) $\text{DBL}(p, q, s, s)$, $p \geq 1$, $q \geq 0$, $s \geq 1 \equiv \text{ARMA}(p, l)$, $l = \max(q, s)$
- (iv) $\text{SBL}(0, q, r, 1)$, $q \geq 1$, $r \geq 2 \equiv \text{MA}(q)$
- (v) $\text{SBL}(0, q, r, s)$, $q \geq 0$, $1 \leq s \leq r \equiv \text{MA}(l)$, $l = \max(q, s)$

(vi) $\text{SBL}(p,q,r,s)$, $p \geq 1$, $q \geq 0$, $1 \leq s \leq r \equiv \text{ARMA}(p,l)$, $l=\max(q,s)$

From these autocorrelation structures, it is seen that the first parameter p of $\text{BL}(p,q,r,s)$ can be regarded as autocoregressive parameter and that the larger of the second and fourth parameters can be regarded as the moving average parameter.

Remark 2. The autocorrelation structures of the bilinear models $\text{DBL}(0,2,1,1)$, $\text{DBL}(0,1,2,2)$ and $\text{DBL}(0,2,2,2)$ are all the same. However, there is no reason to select $\text{DBL}(0,2,2,2)$ in fitting a model because of rule of parsimony.

4. SIMULATION STUDY

In this section, some simulation studies are performed to investigate the practical application of results in section 3 and the adequacy of Akaike information criteria for identification between ARMA and BL models and for choice of orders of BL models.

We consider $\text{ARMA}(1,1)$, $\text{BL}(1,0,1,1)$, $\text{MA}(2)$, $\text{DBL}(0,0,2,2)$ and $\text{BL}(0,1,1,1)$ models which are assumed to be invertible. Fifty replications of length 500 are generated from each model under the assumption that the e_t 's are $N(0,1)$. Using the Newton-Raphson estimation procedure, ARMA and BL models are fitted. The Newton-Raphson procedure for $\text{BL}(p,q,r,s)$ can be carried out as follows.

For the model $(1,1)$, it is assumed that the model is invertible and the e_t 's are $N(0, \sigma^2)$. Further, it is assumed that there are realizations $\{X_1, X_2, \dots, X_n\}$. Then, the joint density function of $\{e_m, e_{m+1}, \dots, e_n\}$, where $m = \max(p,q,r,s) + 1$, is given by

$$(2\pi\sigma^2)^{\frac{-(n-m+1)}{2}} \exp[-2\sigma^{-2} \sum_{t=m}^n e_t^2]$$

Since the Jacobian of the transformation $(\frac{dX_t}{de_t})$ is unity, the likelihood function of $\{X_m, X_{m+1}, \dots, X_n\}$ is the same as the joint density function of $\{e_m, e_{m+1}, \dots, e_n\}$. Maximizing the likelihood function is minimizing the sum of squares of errors, $S(\underline{\theta})$,

$$S(\underline{\theta}) = \sum_{t=m}^n e_t^2,$$

with respect to the parameters $\underline{\theta}' = (a_1, \dots, a_p, c_1, \dots, c_q, b_{11}, \dots, b_{rs})$. Denote $\theta_i = a_i, i = 1, \dots, p$, $\theta_{j+p} = c_j, j = 1, \dots, q$, $\theta_{i+j+p+q} = b_{ij}, i = 1, \dots, r, j = 1, \dots, s$, and $R = p + q + r + s$. Then the partial derivatives of $S(\underline{\theta})$ are given by

$$\frac{dS(\underline{\theta})}{d\theta_k} = 2 \sum_{t=m}^n e_t \frac{de_t}{d\theta_k}, \quad k = 1, 2, \dots, R$$

and

$$\frac{d^2 S(\theta)}{d\theta_k d\theta_l} = 2 \sum_{t=m}^n e_t \cdot \frac{de_t}{d\theta_k} \cdot \frac{de_t}{d\theta_l} + 2 \sum_{t=m}^n e_t \frac{d^2 e_t}{d\theta_k d\theta_l},$$

$$k = 1, 2, \dots, R, \quad l = 1, 2, \dots, R$$

where these partial derivatives of e_t satisfy the following recursive equations.

$$\begin{aligned} \frac{de_t}{da_k} &= -X_{t-k} - \sum_{j=1}^q c_j \frac{de_{t-j}}{da_k} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \frac{de_{t-j}}{da_k}, \\ \frac{de_t}{dc_k} &= -e_{t-j} - \sum_{j=1}^b c_j \frac{de_{t-j}}{da_k} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \frac{de_{t-j}}{dc_k}, \\ \frac{de_t}{db_{kl}} &= -\sum_{j=1}^p c_j \frac{de_{t-j}}{db_{kl}} - X_{t-k} e_{t-j} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \frac{de_{t-j}}{db_{kl}}, \\ \frac{d^2 e_t}{da_k da'_k} &= -\sum_{j=1}^q c_j \frac{d^2 e_{t-j}}{da_k da'_k} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \cdot \frac{d^2 e_{t-j}}{da_k da'_k}, \quad k' = 1, 2, \dots, R \\ \frac{d^2 e_t}{dc_k dc'_k} &= -\frac{de_{t-j}}{dc_k} - \frac{de_{t-j}}{dc'_k} - \sum_{j=1}^q c_j \frac{d^2 e_{t-j}}{dc_k dc'_k} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \cdot \frac{d^2 e_{t-j}}{dc_k dc'_k} \\ \frac{d^2 e_t}{db_{kl} db_{k'l'}} &= -\sum_{j=1}^q c_j \frac{de_{t-j}}{db_{kl} db_{k'l'}} - X_{t-k} \frac{de_{t-j}}{db_{kl}} - X_{t-k'} \frac{de_{t-j}}{db_{k'l'}} \\ &\quad - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \frac{d^2 e_{t-j}}{db_{kl} db_{k'l'}}, \quad l = 1, 2, \dots, R \\ \frac{d^2 e_t}{da_k dc_{k'}} &= -\frac{de_{t-j}}{dc_{k'}} - \sum_{j=1}^q c_j \frac{d^2 e_{t-j}}{da_k dc_{k'}} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \frac{d^2 e_{t-j}}{da_k dc_{k'}}, \\ \frac{d^2 e_t}{da_k db_{k'l'}} &= -\sum_{j=1}^q c_j \frac{d^2 e_{t-j}}{da_k db_{k'l'}} - X_{t-k} \frac{de_{t-j}}{da_k} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \frac{d^2 e_{t-j}}{da_k db_{k'l'}}, \\ \frac{d^2 e_t}{dc_k db_{k'l'}} &= -\frac{de_{t-j}}{db_{k'l'}} - \sum_{j=1}^q c_j \frac{d^2 e_{t-j}}{dc_k db_{k'l'}} - X_{t-k'} \frac{de_{t-j}}{dc_k} - \sum_{i=1}^r \sum_{j=1}^s b_{ij} X_{t-i} \frac{d^2 e_{t-j}}{dc_k db_{k'l'}} \end{aligned}$$

By using these recursive equations, we can evaluate the first-order and second-order derivatives for a given set of values of a_i, c_j , and db_{ij} under the assumption that

$$e_t = 0, \quad t = 1, 2, \dots, m-1$$

$$\frac{de_t}{d\theta_i} = 0, \quad t = 1, 2, \dots, m-1, \quad i = 1, 2, \dots, R$$

$$\text{and } \frac{d^2 e_t}{d\theta_i d\theta_j} = 0, \quad t = 1, 2, \dots, m-1, \quad i, j = 1, 2, \dots, R$$

Now let $G'(\underline{\theta})$ be the transpose of the vector of first-order derivatives, and let $H(\underline{\theta})$ be a matrix of second-order derivatives, i.e.,

$$\begin{aligned} \text{and } G'(\underline{\theta}) &= \left[\frac{dS(\underline{\theta})}{d\theta_1} \quad \frac{dS(\underline{\theta})}{d\theta_2} \quad \dots \quad \frac{dS(\underline{\theta})}{d\theta_k} \right] \\ H(\underline{\theta}) &= \begin{bmatrix} \frac{d^2 S(\underline{\theta})}{(d\theta_1)^2} & \dots & \frac{d^2 S(\underline{\theta})}{d\theta_1 d\theta_R} \\ \vdots & & \vdots \\ \frac{d^2 S(\underline{\theta})}{d\theta_R d\theta_1} & \dots & \frac{d^2 S(\underline{\theta})}{(d\theta_R)^2} \end{bmatrix} \end{aligned}$$

To minimize $S(\underline{\theta})$, we set the first order derivatives to zero. Expanding $G(\hat{\underline{\theta}})$ near $\hat{\underline{\theta}} = \underline{\theta}$ in Taylor series, we obtain

$$0 = [G(\hat{\underline{\theta}})] = G(\underline{\theta}) + H(\underline{\theta}) (\hat{\underline{\theta}} - \underline{\theta})$$

Rewriting this equation, we have

$$\hat{\underline{\theta}} - \underline{\theta} = -H^{-1}(\underline{\theta})G(\underline{\theta})$$

Hence the Newton-Raphson equation is given by

$$\underline{\theta}^{(k+1)} = \underline{\theta}^{(k)} - H^{-1}(\underline{\theta}^{(k)})G(\underline{\theta}^{(k)})$$

where $\underline{\theta}^{(k)}$ is the estimate obtained at the k-th iteration stage. It is known that the Newton-Raphson estimates usually converge, but they may not correspond to the global minimum of $S(\underline{\theta})$.

There are many criteria which identify ARMA models, for example, Akaike information criteria, Bayesian information criteria, and final prediction error. In this paper, we consider Akaike information criteria to identify between ARMA and BL models and to determine orders of BL models. Akaike information criteria for BL(p,q,r,s) is given by

$$\text{AIC} = (n - m + 1) \log \hat{\sigma}^2 + 2(p + q + rs)$$

$$\text{where } \hat{\sigma}^2 = (n - m + 1)^{-1} \sum_{t=m}^n \hat{e}_t^2$$

From the generated realizations of a ARMA(1,1) model with a set of coefficients $\phi_1 = 0.5, \theta_1 = 0.3$ and the other set of coefficients $\phi_1 = -0.6, \theta_1 = 0.2$, ARMA(1,1) and BL(1,0,1,1) models are fitted for each set. The initial estimates for computing Newton-

Raphson estimates are selected as the coefficient plus 0.2. Table 1 shows the average AIC values of 50 replications for ARMA(1,1) and BL(1,0,1,1) models. In both sets of coefficients, the average AIC values are shown to be less in ARMA(1,1) model than in BL(1,0,1,1) model. Conversely, if realizations are generated from BL(1,0,1,1) models, then the average AIC values are less in BL(1,0,1,1) models than in ARMA(1,1) models.

Table 1. Average AIC value* for ARMA(1,1) and BL(1,0,1,1) models

Data generated model	Fitted model	ARMA(1,1)	BL(1,0,1,1)
ARMA(1,1)			
$\phi_1 = 0.5$ $\theta_2 = 0.3$		40.75	65.92
$\phi_1 = -0.6$ $\theta_1 = 0.2$		38.58	43.63
BL(1,0,1,1)			
$a_1 = 0.5$ $b_{11} = 0.3$		189.62	34.43
$a_1 = -0.6$ $b_{11} = 0.2$		112.56	39.84

* All the average AIC values are significant.

In table 2, the same analysis is done for MA(2) models and BL(0,0,2,2) models. In case that the realizations are generated from MA(2) models, the average AIC values of MA(2) models are shown to be less than those of BL(0,0,2,2) models in both sets of coefficients. On the contrary, the average AIC values are shown to be less in BL(0,0,2,2) models than in MA(2) models if realizations are generated from BL(0,0,2,2) models.

It is suggested from these simulation studies that the model of having minimum AIC value is selected for a suitable model.

Table 2. The average AIC values* for MA(2) and BL(0,0,2,2) models

Data generated model	Fitted model	MA(2)	BL(0,0,2,2)
MA(2)			
$\theta_1 = -0.4$ $\theta_2 = -0.3$		39.24	148.47
$\theta_1 = 0.5$ $\theta_2 = -0.2$		37.36	166.22
BL(0,0,2,2)			
$b_{11} = -0.4$ $b_{22} = -0.3$		311.91	40.06
$a_{11} = 0.5$ $b_{11} = -0.2$		369.67	39.74

* All the average AIC values are significant.

To investigate the adequacy of AIC for choice of orders of BL models, three BL models are fitted to the realizations which are generated from each model. Table 3 shows the average AIC values of 50 replications. In case that BL(0,1,1,1) model is fitted, the Newton-Raphson estimation procedure does not usually converge. Convergency is very sensitive to initial estimates of coefficients. Hence the average AIC values have large standard error and they are not significant in BL(0,1,1,1) models.

It is seen from table 3 that the model of generating realizations has the minimum AIC values for any set of coefficients. These results suggest that the Akaike information criteria can be applicable to determine orders of bilinear models.

Table 3 Average AIC values* for BL models

Data generated model	Fitted model	BL(1,0,1,1)	BL(0,0,2,2)	BL(0,1,1,1)
BL(1,0,1,1)				
$a_1 = 0.5 \quad b_{11} = 0.3$		36.37	189.24	**
$a_1 = -0.6 \quad b_{11} = 0.2$		38.07	267.65	**
BL(0,0,2,2)				
$b_{11} = -0.4 \quad b_{22} = -0.3$		157.80	41.51	**
$b_{11} = 0.5 \quad b_{22} = -0.2$		260.39	38.28	**
BL(0,1,1,1)				
$b_{11} = 0.5 \quad c_1 = -0.3$		113.17	172.93	40.27
$b_{11} = -0.4 \quad c_1 = -0.2$		53.02	110.74	40.06

* All AIC values are significant except **.

** Unsignificant because of large standard error.

5. CONCLUSION

In this paper, the autocorrelation structures of bilinear time series model BL(p,q,r,s) $r \geq s$, are obtained and shown to be analogous to those of ARMA(p,ℓ), $\ell = \max(q,s)$. From these results, it is seen that the first parameter p of BL(p,q,r,s) $r \geq s$, can be regarded as the autoregressive parameter and the larger of the second and fourth parameters is regarded as the moving average parameter. Some simulation studies are performed to investigate the adequacy of AIC criteria for identification between ARMA and BL models and for choice of orders of BL models. It is suggested that Akaike information criteria can

be applicable to determine orders of bilinear models and the model of having minimum AIC value is selected for a suitable model.

REFERENCES

- (1) Akamanam, S.I., Rao, B.M. and Subramanyam, K.(1986). On the ergodicity of bilinear time series models. *Journal of Time Series Analysis*, 7, 157-163.
- (2) Bhaskara Rao, M., Subba Rao, T. and Walker, A.M. (1983). On the existence of some bilinear time series models. *Journal of Time Series Analysis*, 4, 95-110.
- (3) Granger, C.W.J. and Andersen, A.P.(1978). *An Introduction to Bilinear Time Series Models*. Vandenhoeck and Rpurecht, Gottingen.
- (4) Kumar, K.(1986). On the identification of some bilinear time series models. *Journal of Time Series Analysis*, 7, 117-122.
- (5) Quinn, B.G.(1982). Stationarity and invertibility of simple bilinear models. *Stochastic Process and Application.*, 12, 225-230.
- (6) Subba Rao, T(1981). On the theory of bilinear time series model. *Journal of Royal Statistical Society. B*, 43, 244-255.
- (7) Subba Rao, T. and Gabr, M.M.(1984). *An Introduction to Bispectral Analysis and Bilinear Time Series Models*. Lecture Notes in Statistics, Vol.24. Berlin : Springer-Verlag.
- (8) Tuan Dinh Pham and Lanh Tat Tran(1981). On the first-order bilinear time series model. *Journal of Applied Probability*, 18, 617-627.
- (9) Won Kyung Kim, Billard,L. and Basawa,I.V.(1990) Estimation for the first-order diagonal bilinear time series model. *Journal of Time Series Analysis*, 11, 215-229.